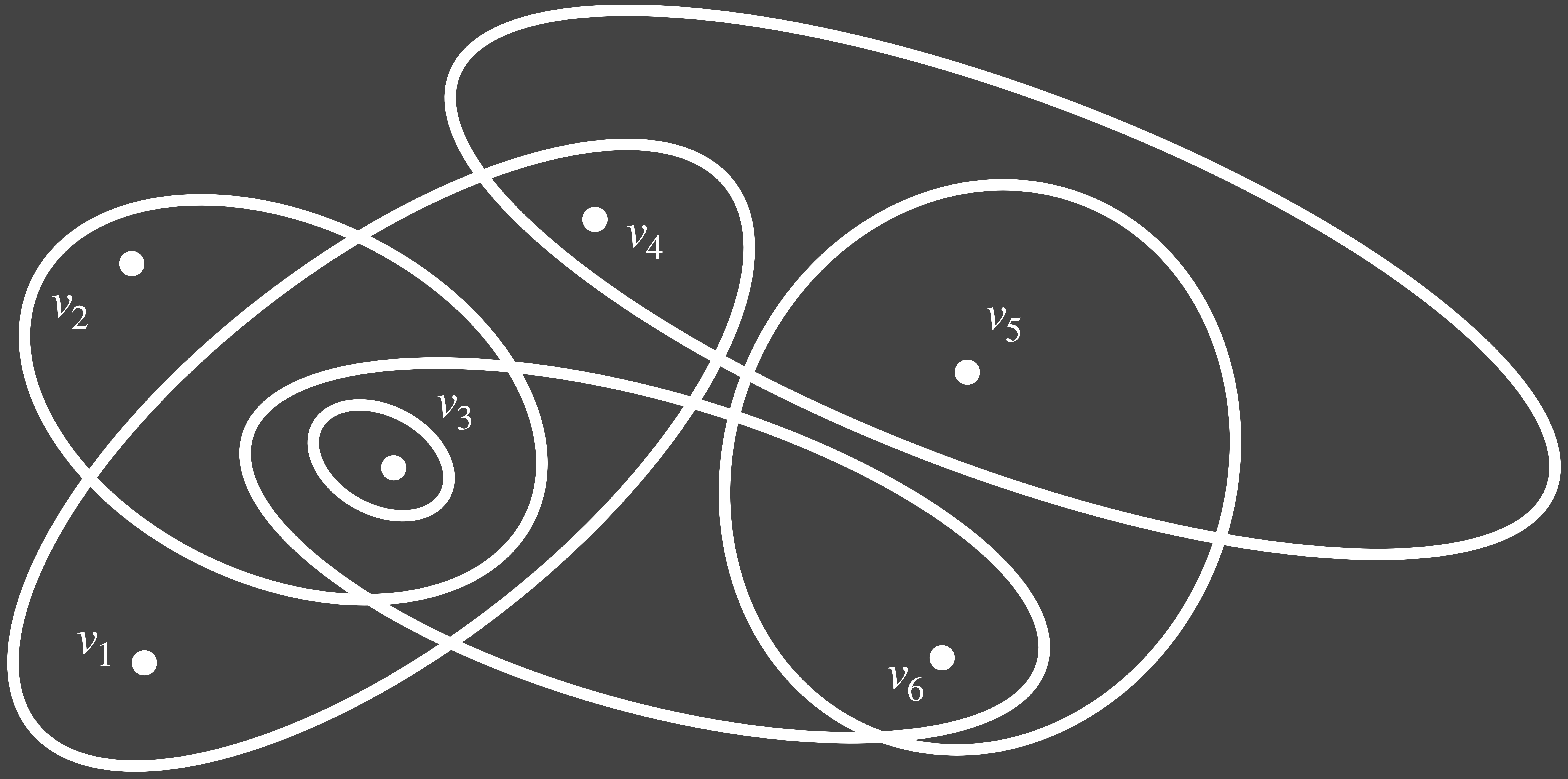
Random Order Set Cover is as Easy as Offline

To appear in FOCS 2021

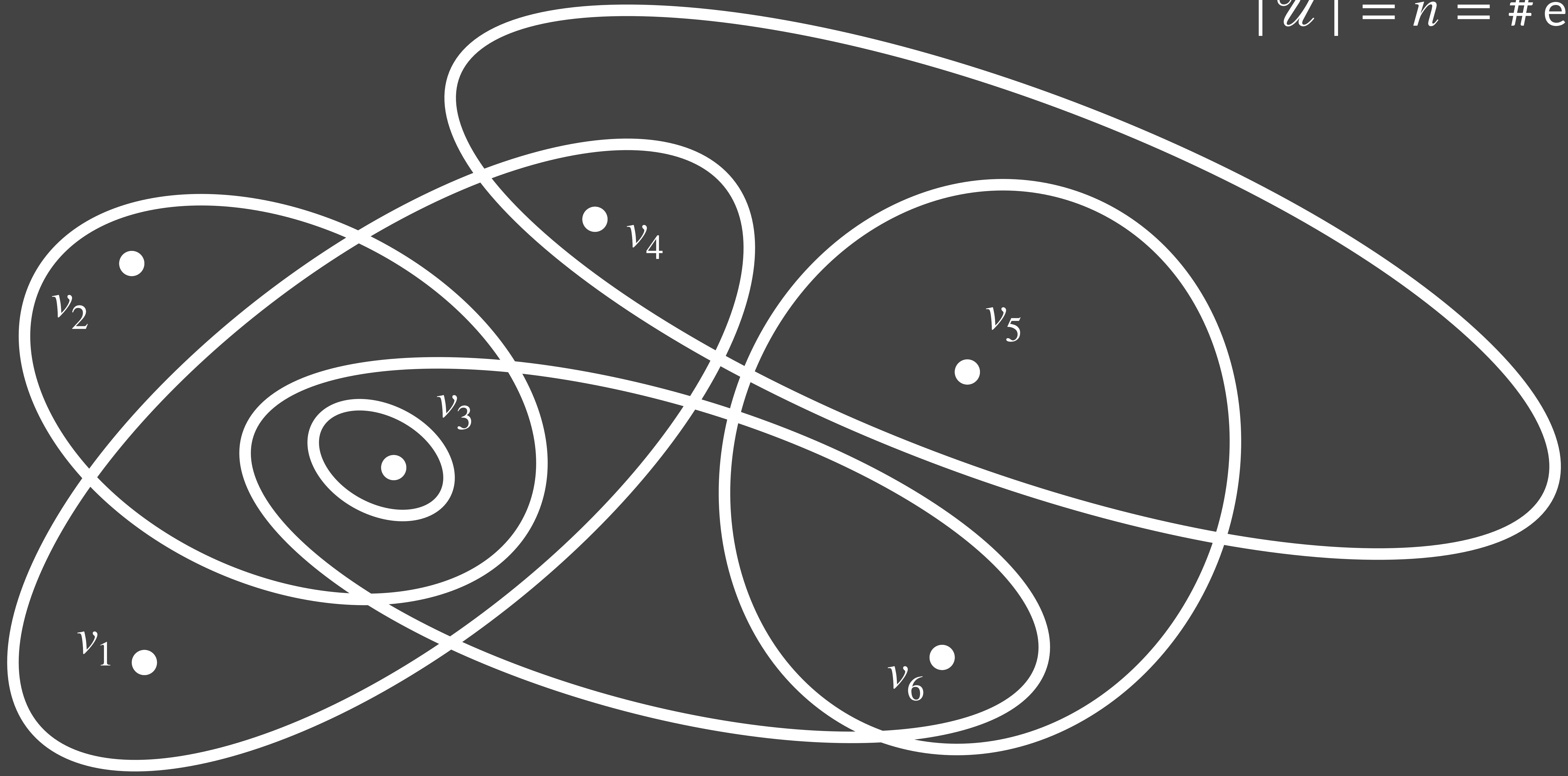
Anupam Gupta (CMU), Greg Kehne (Harvard), and **Roie Levin** (CMU)

Set Cover



Set Cover

$|\mathscr{U}| = n = \text{\# elements}$



Set Cover

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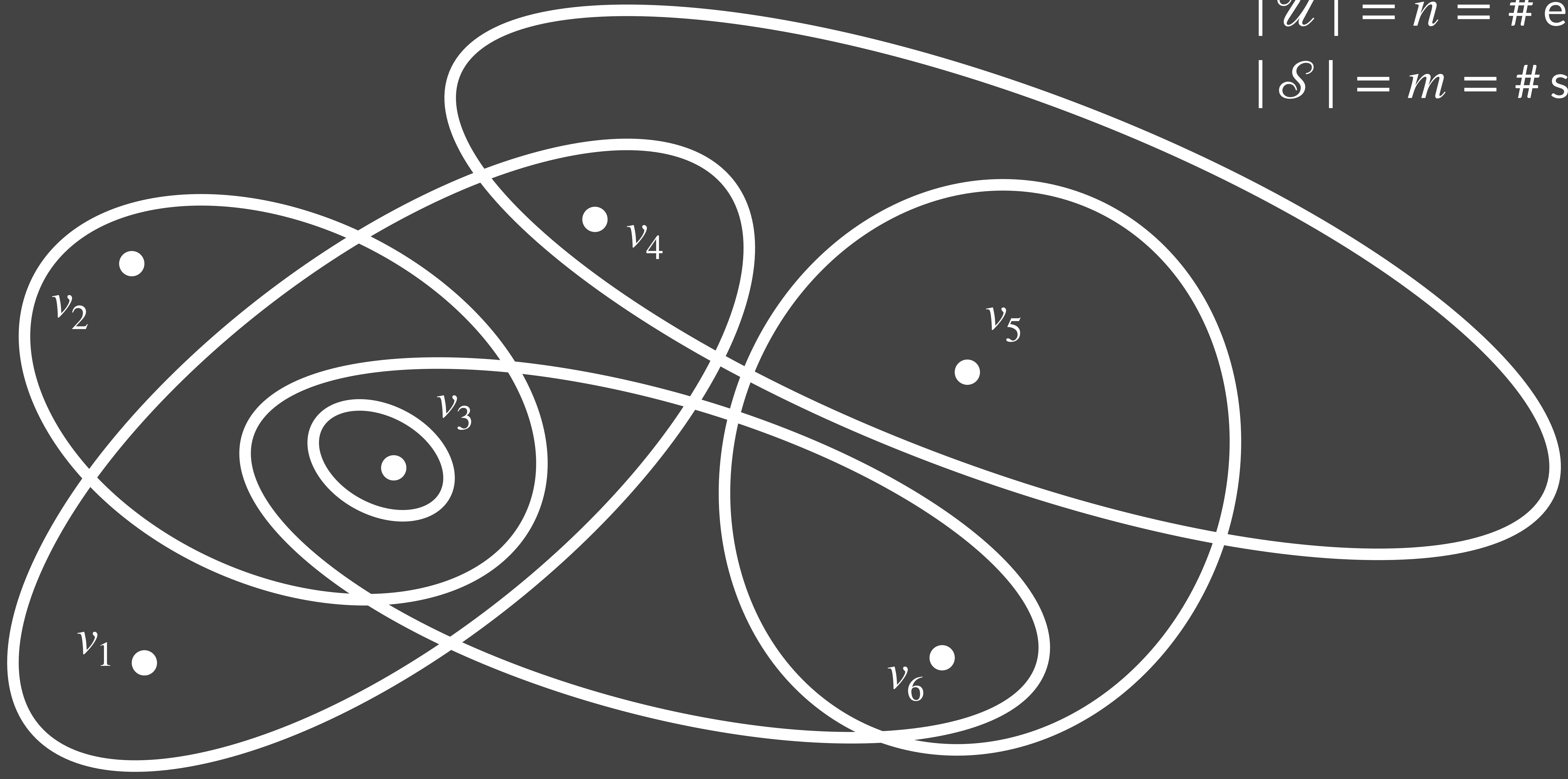

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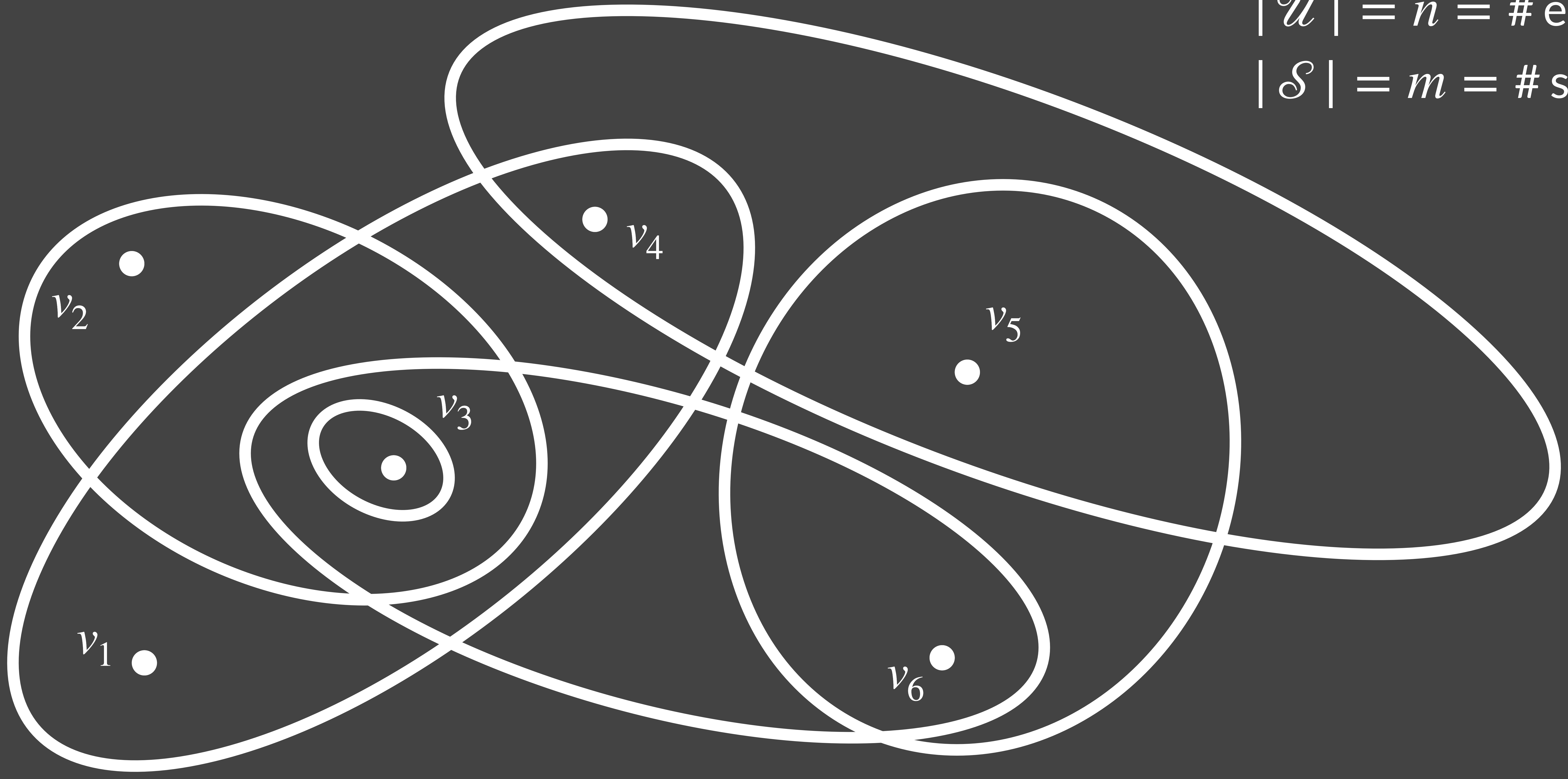

Goal: pick smallest # sets to cover all elements.

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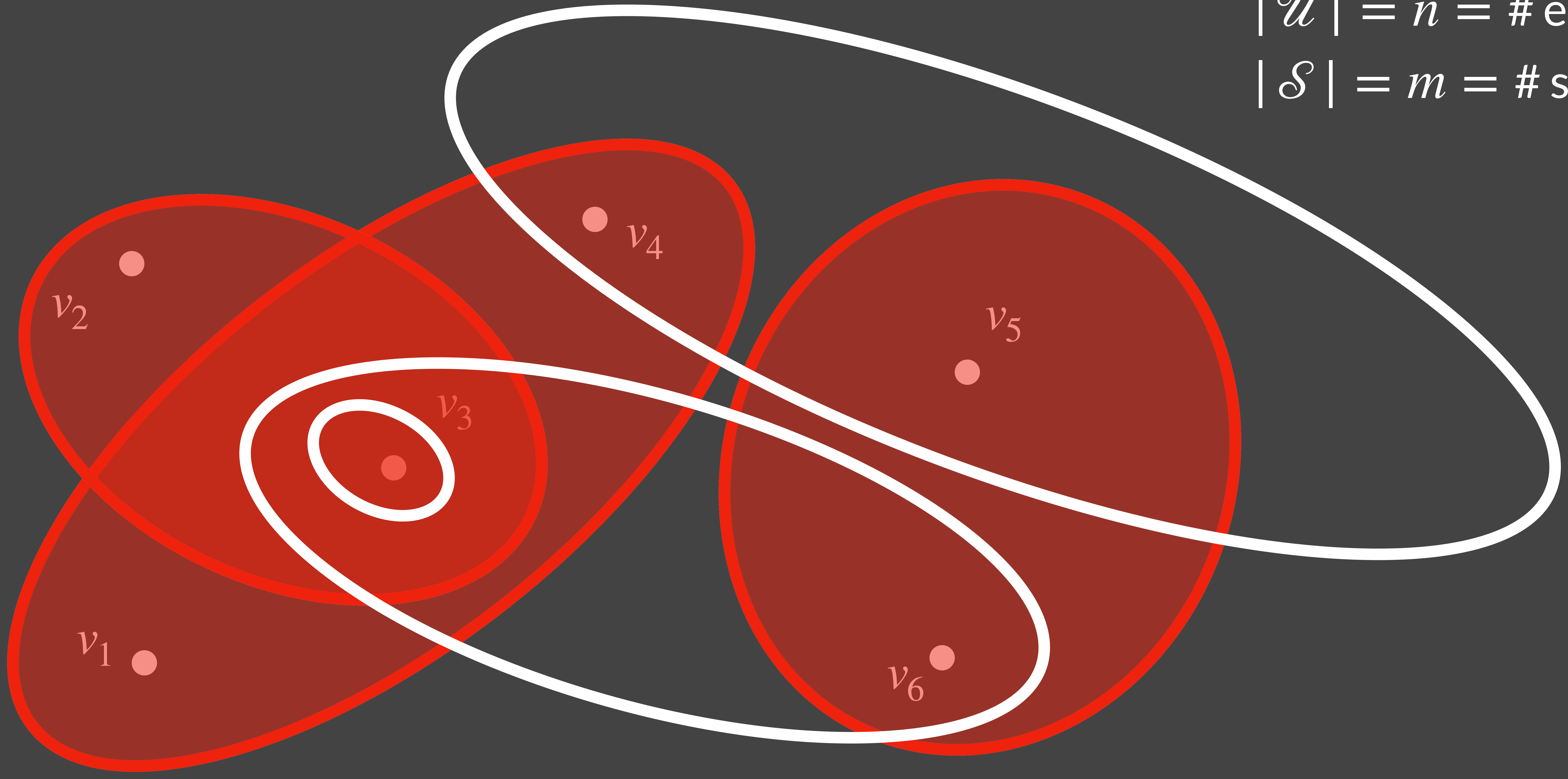

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Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

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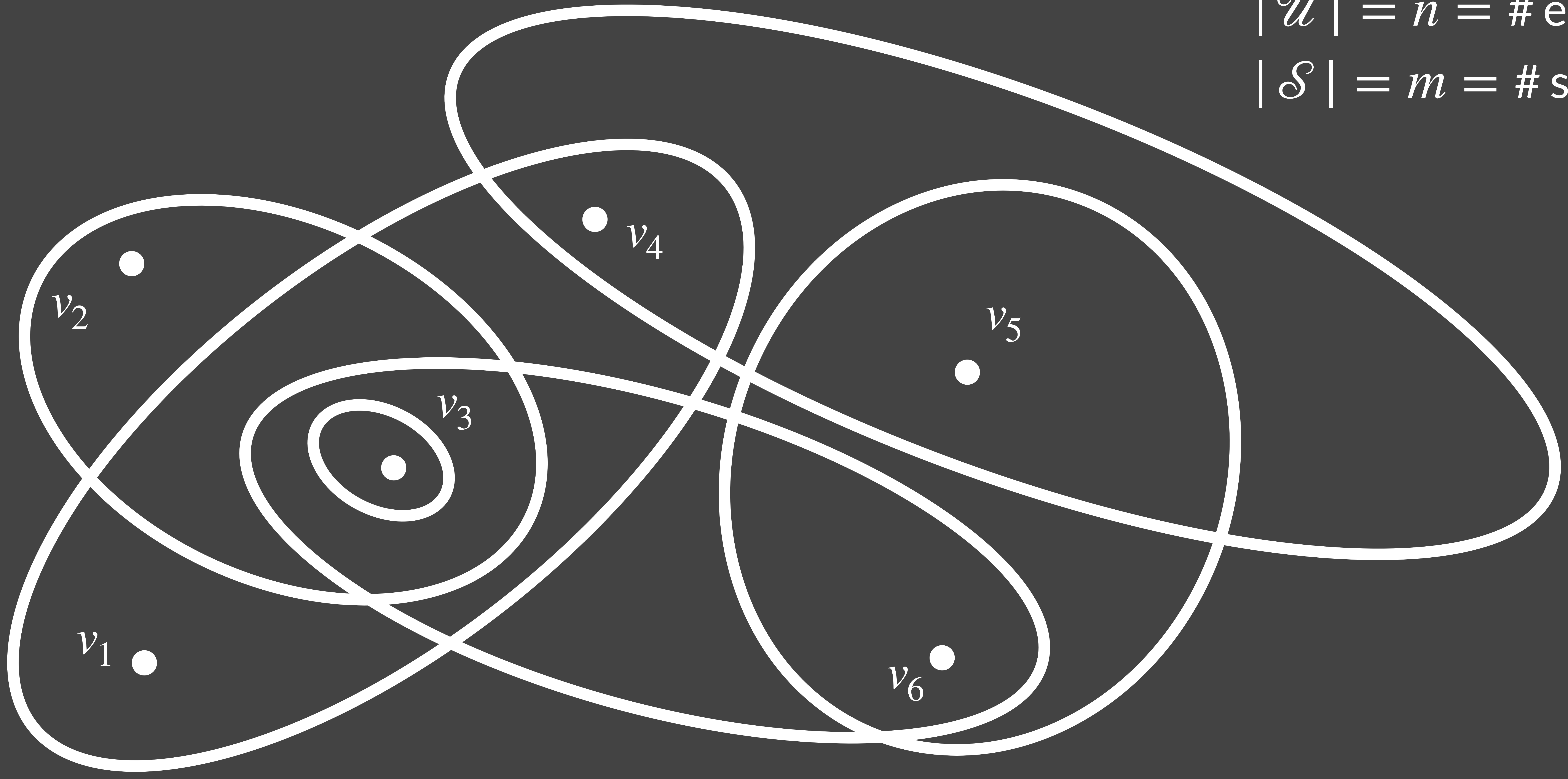

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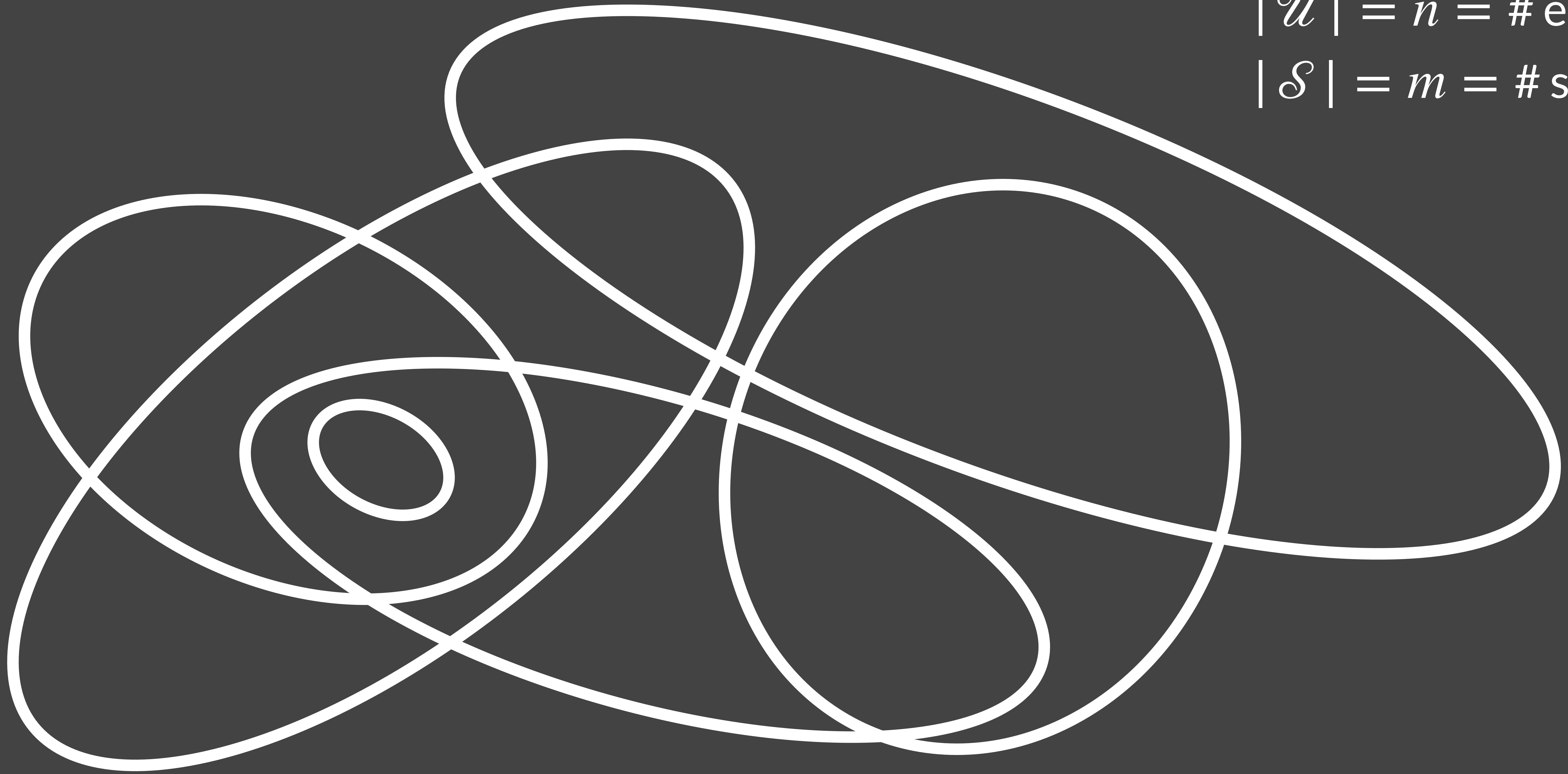

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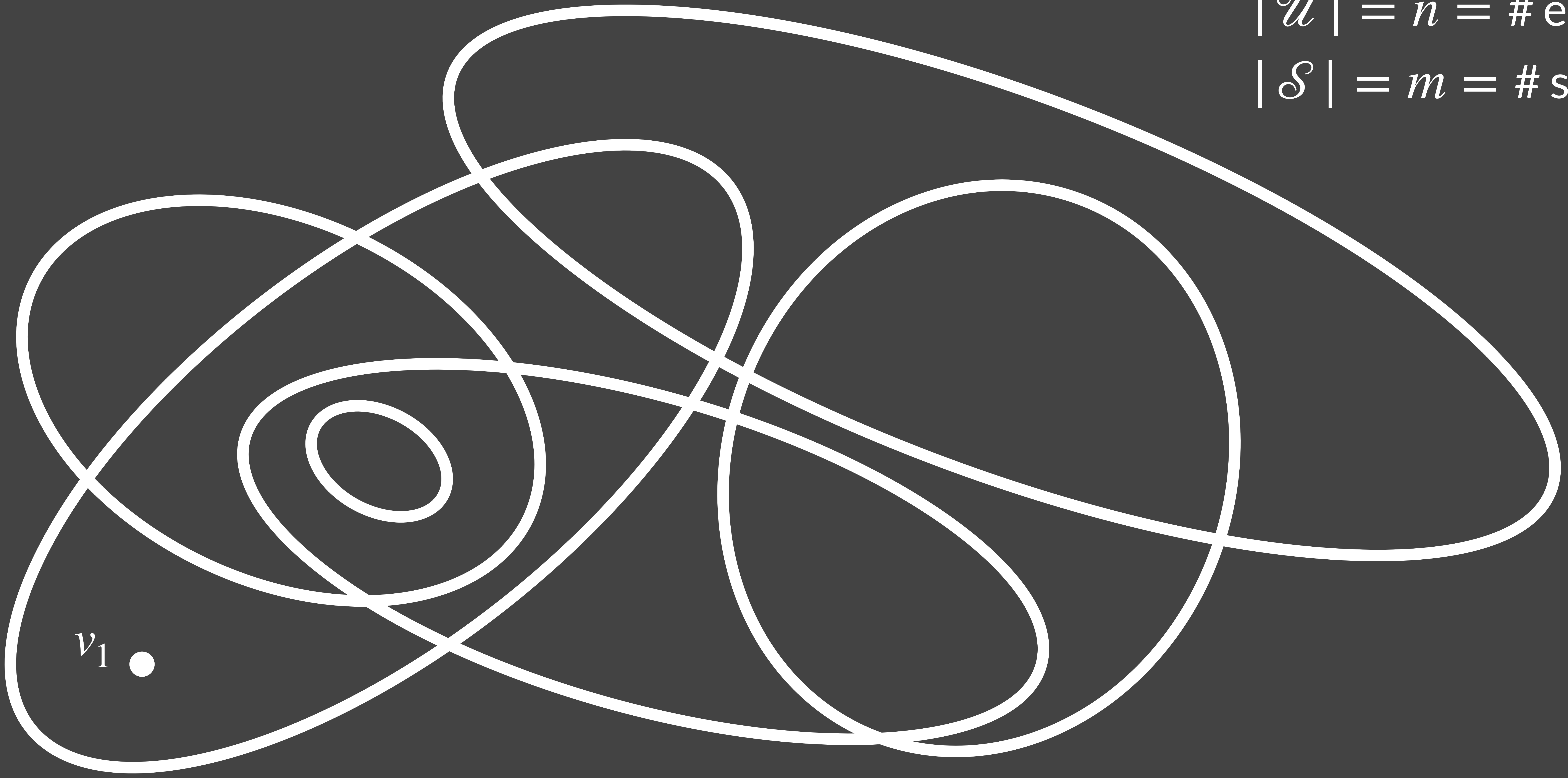

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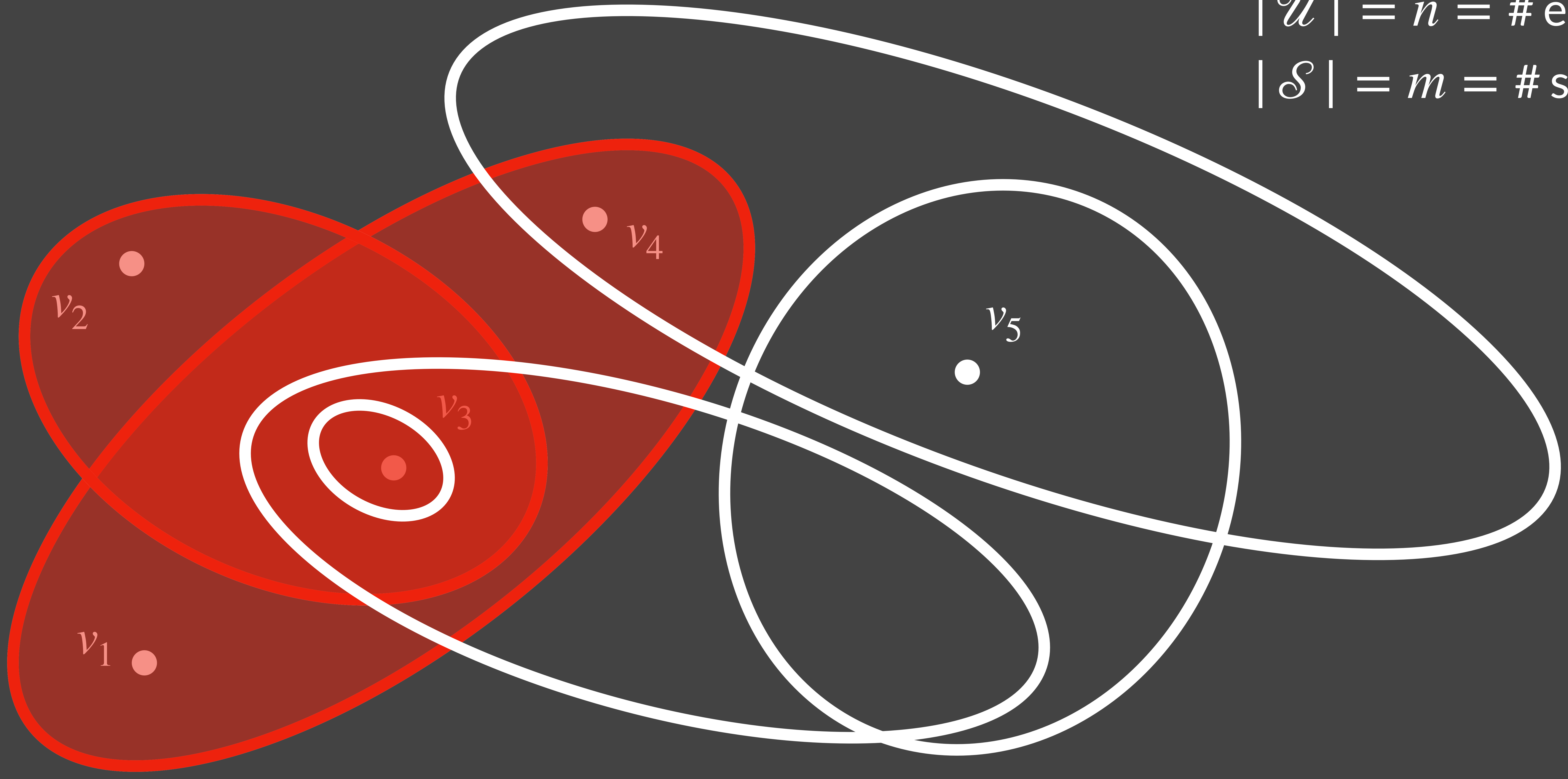

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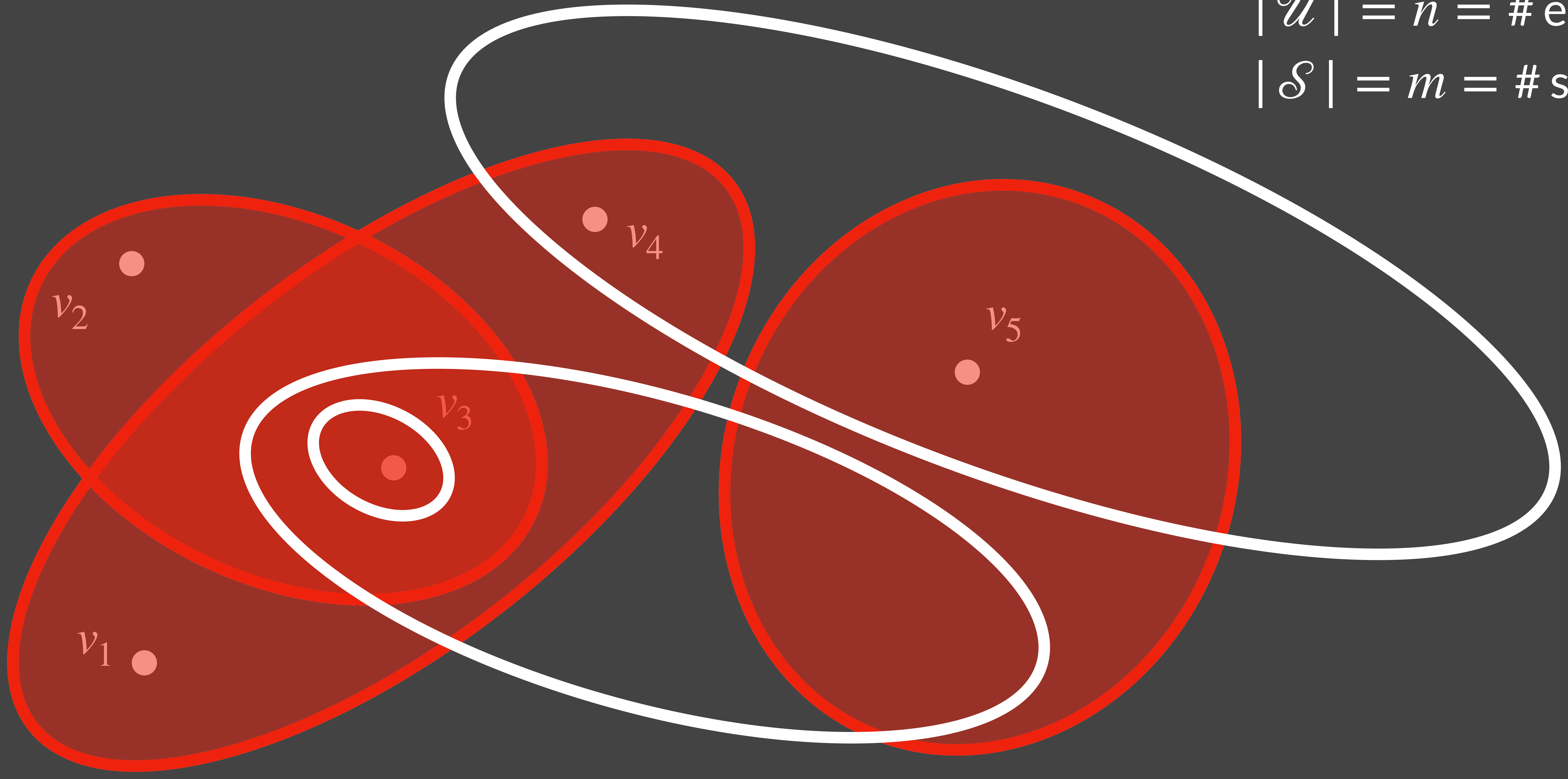

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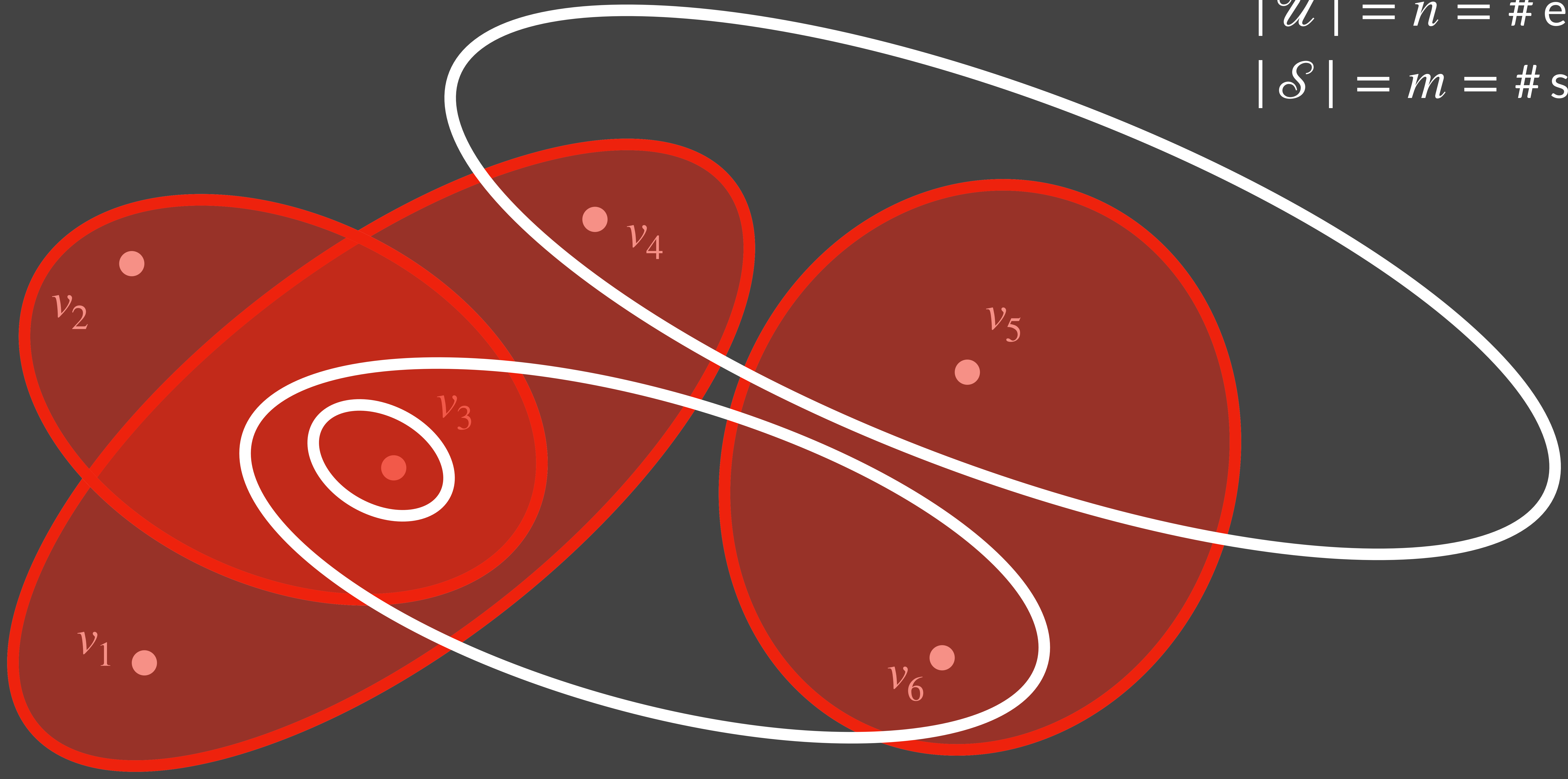

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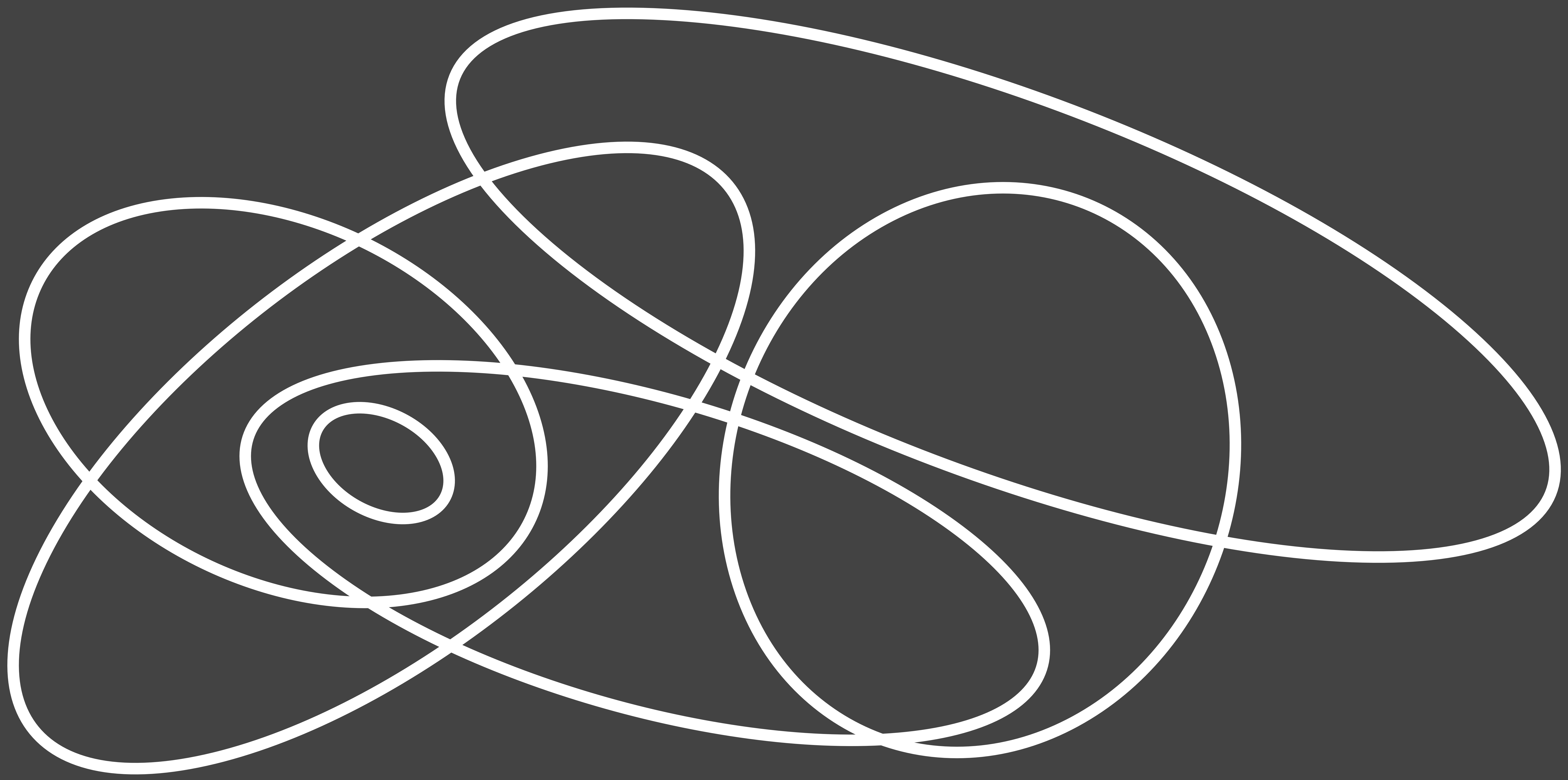
Random Order (RO) Set Cover



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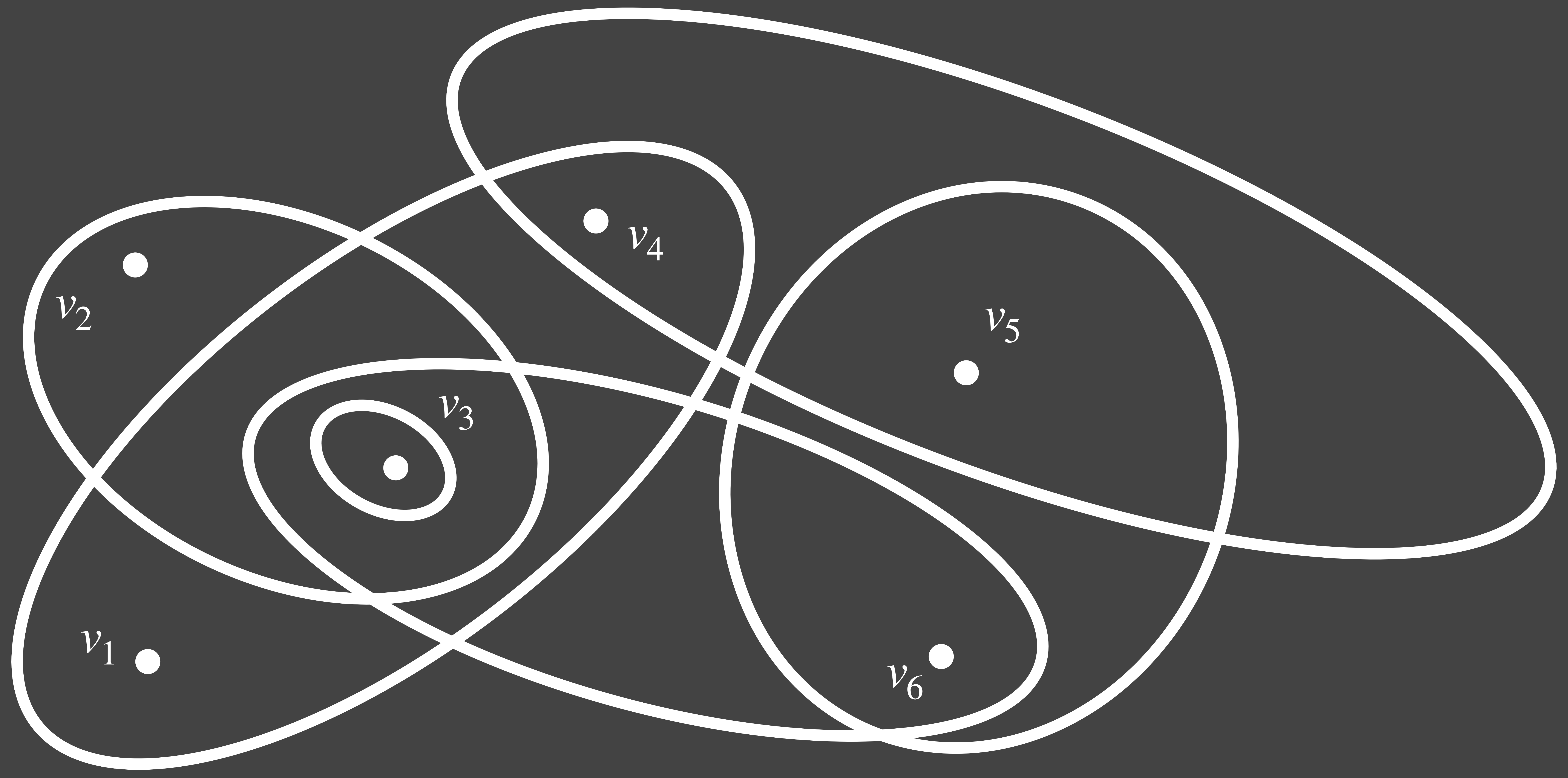

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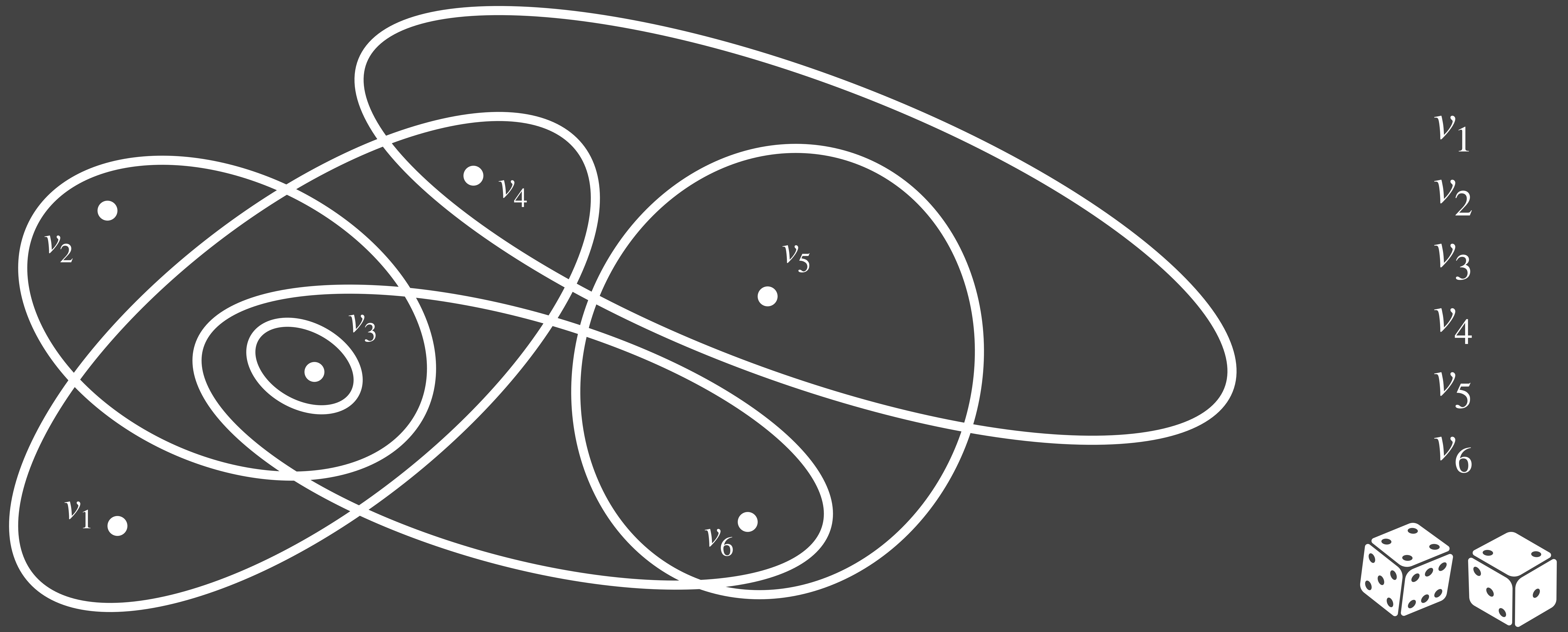

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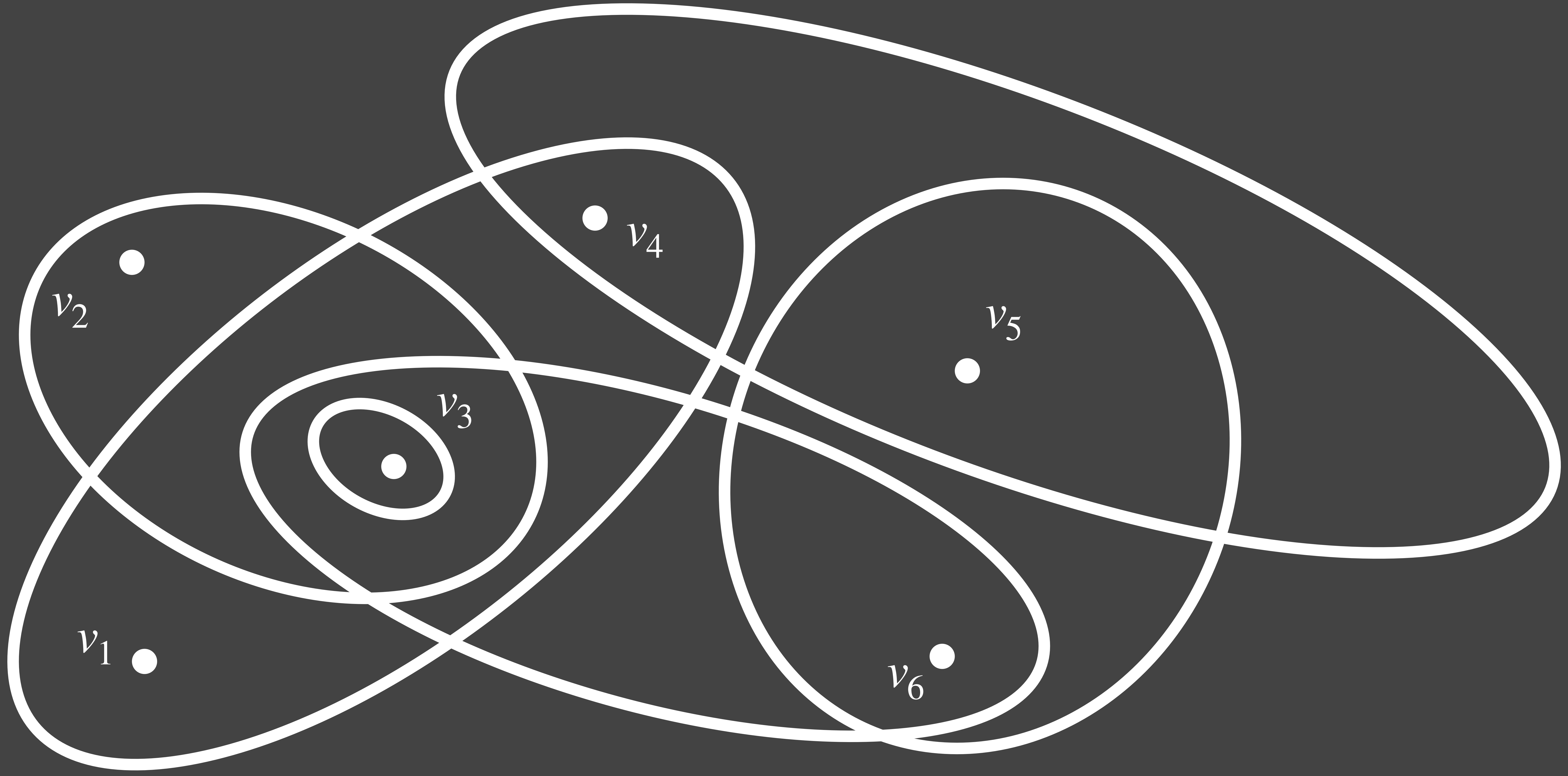

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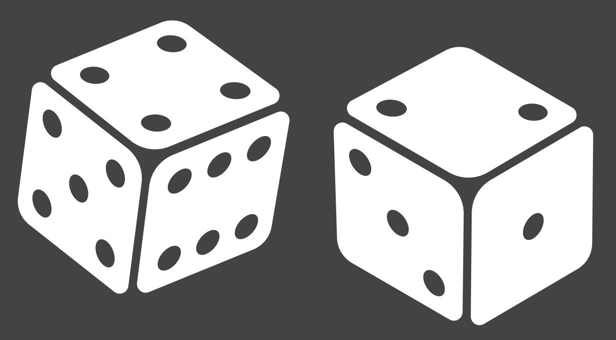

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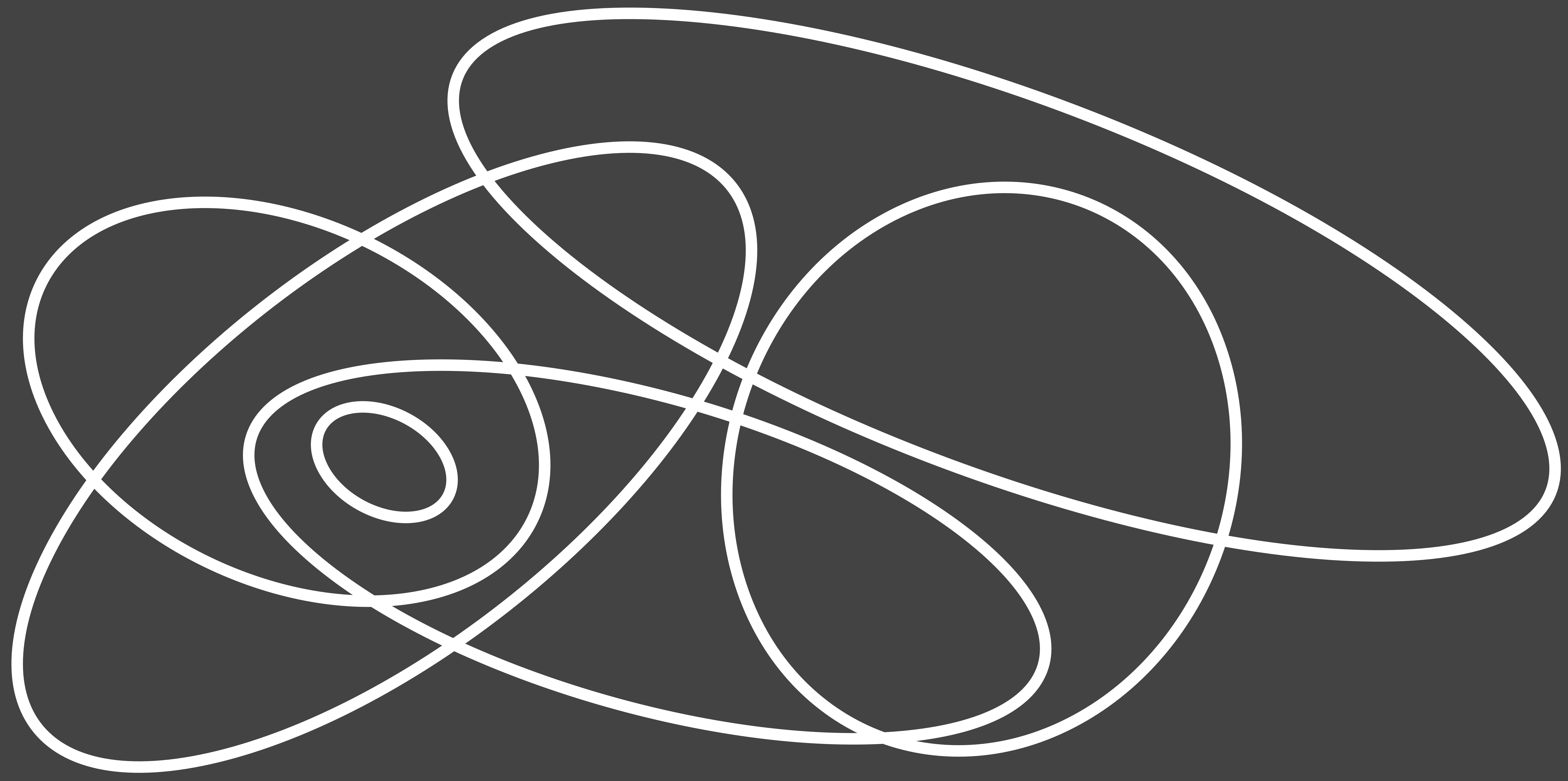

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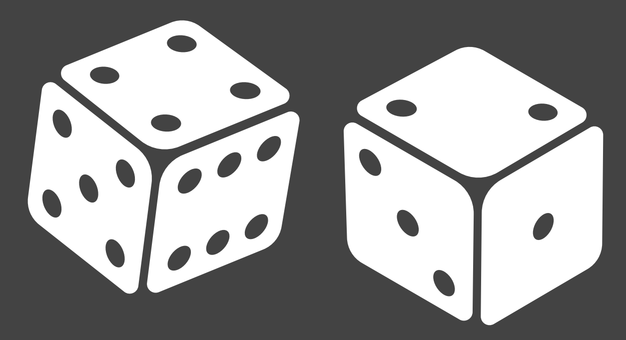

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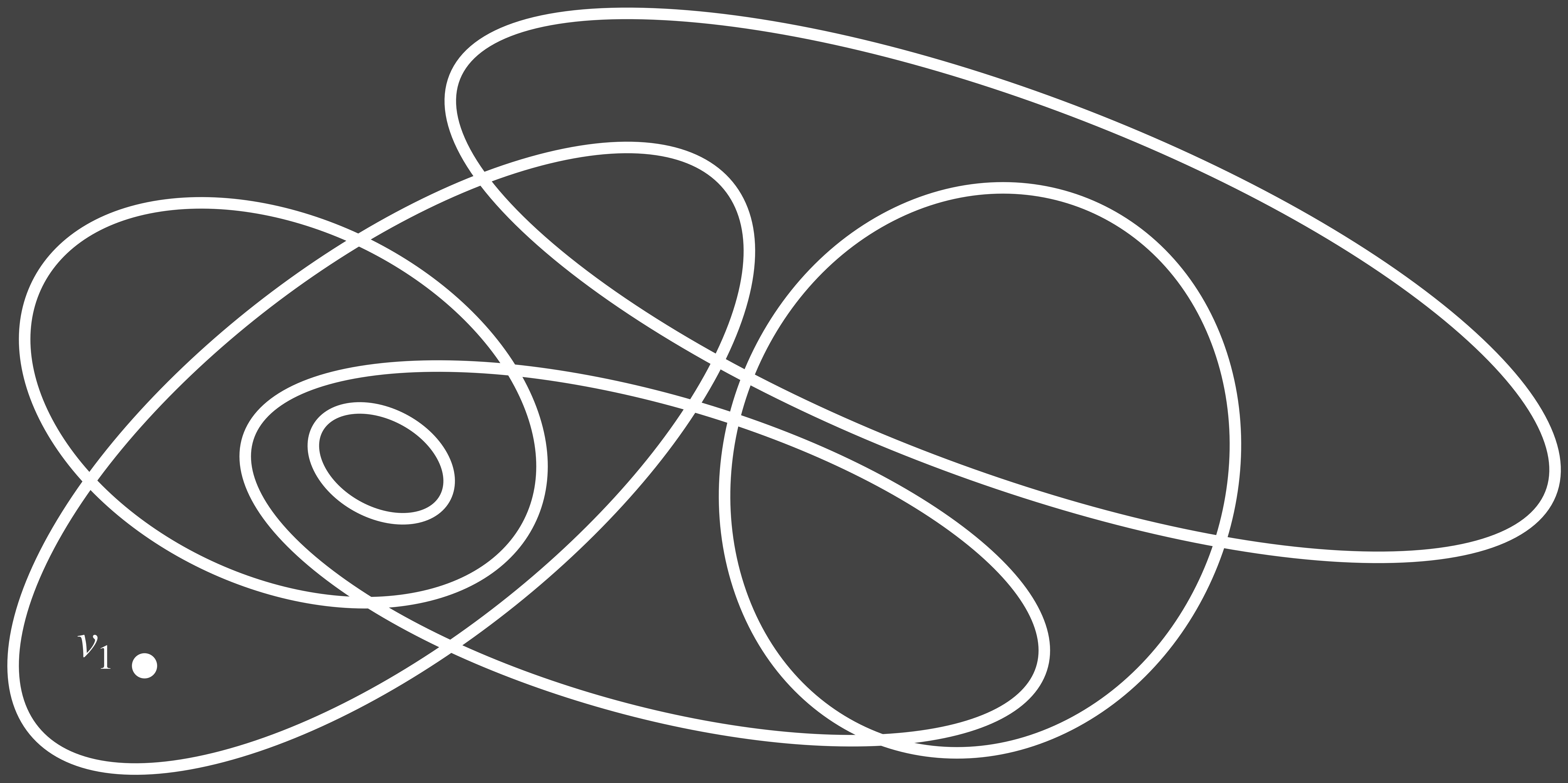

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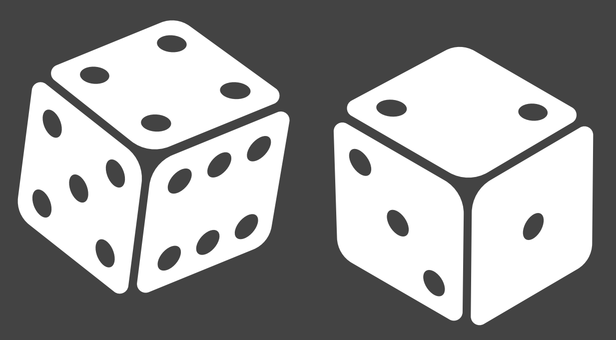

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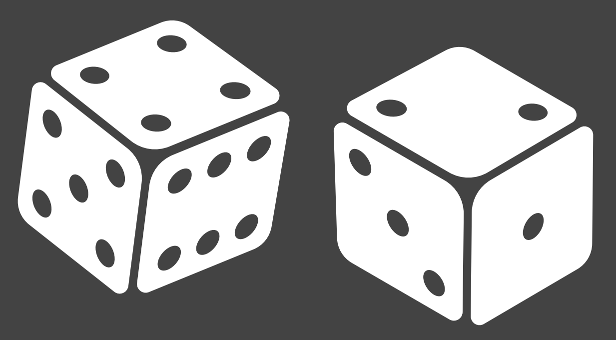

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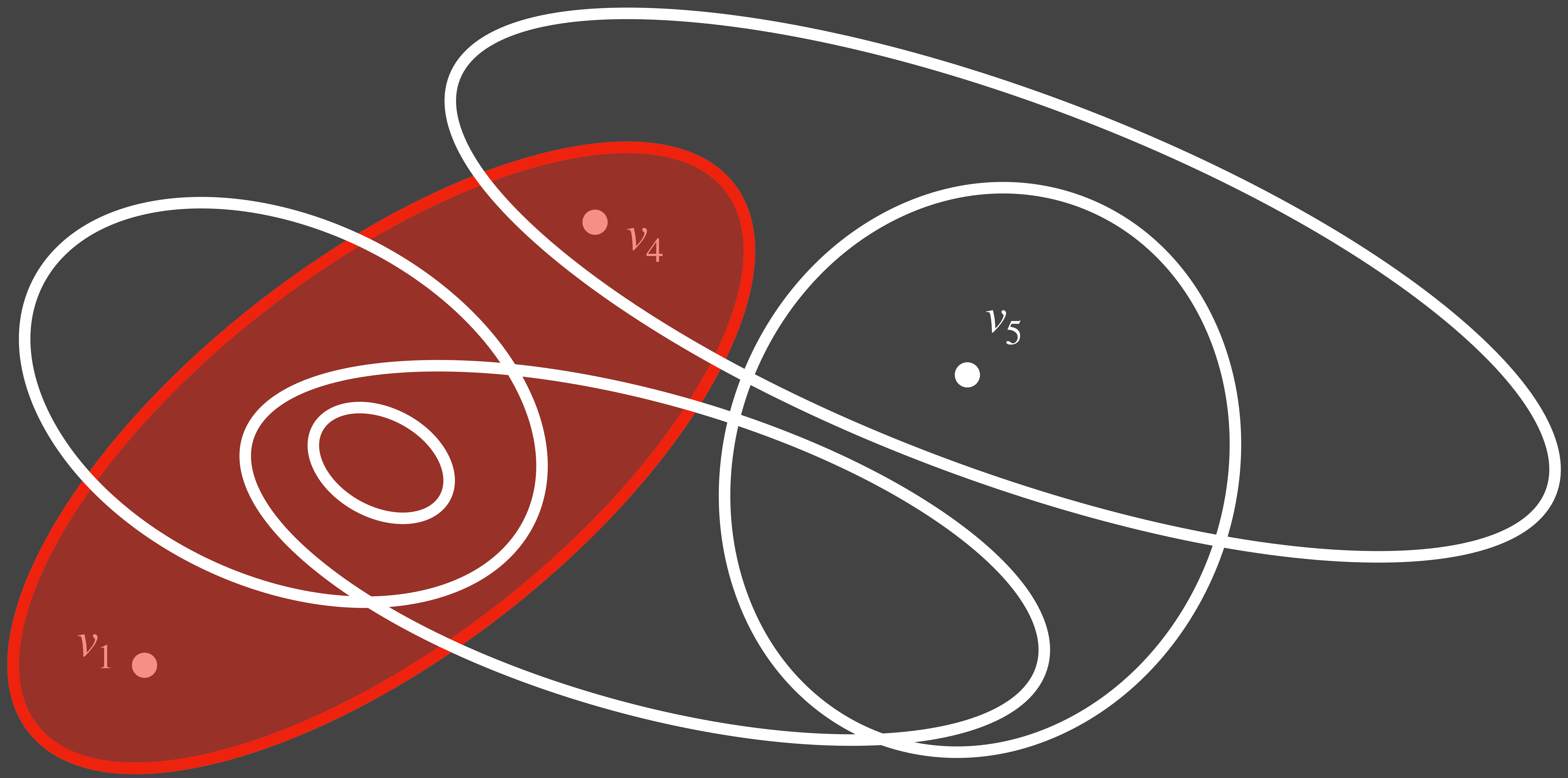

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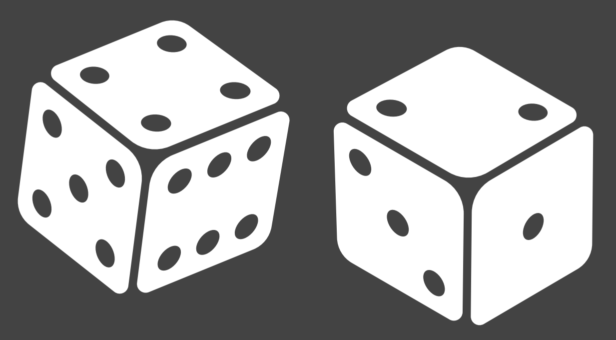

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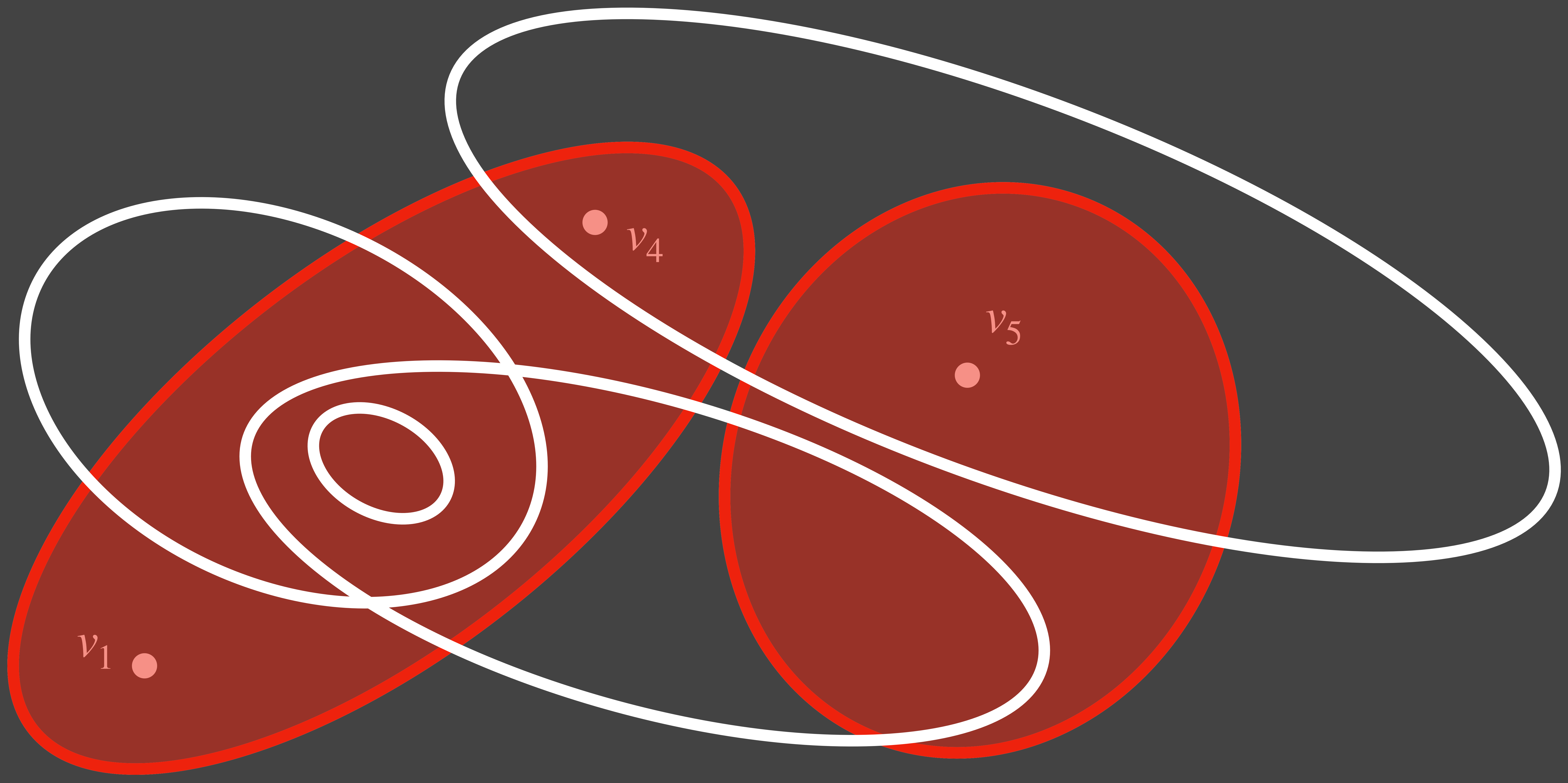

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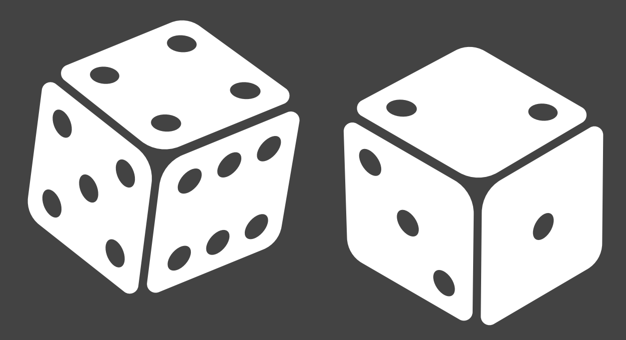

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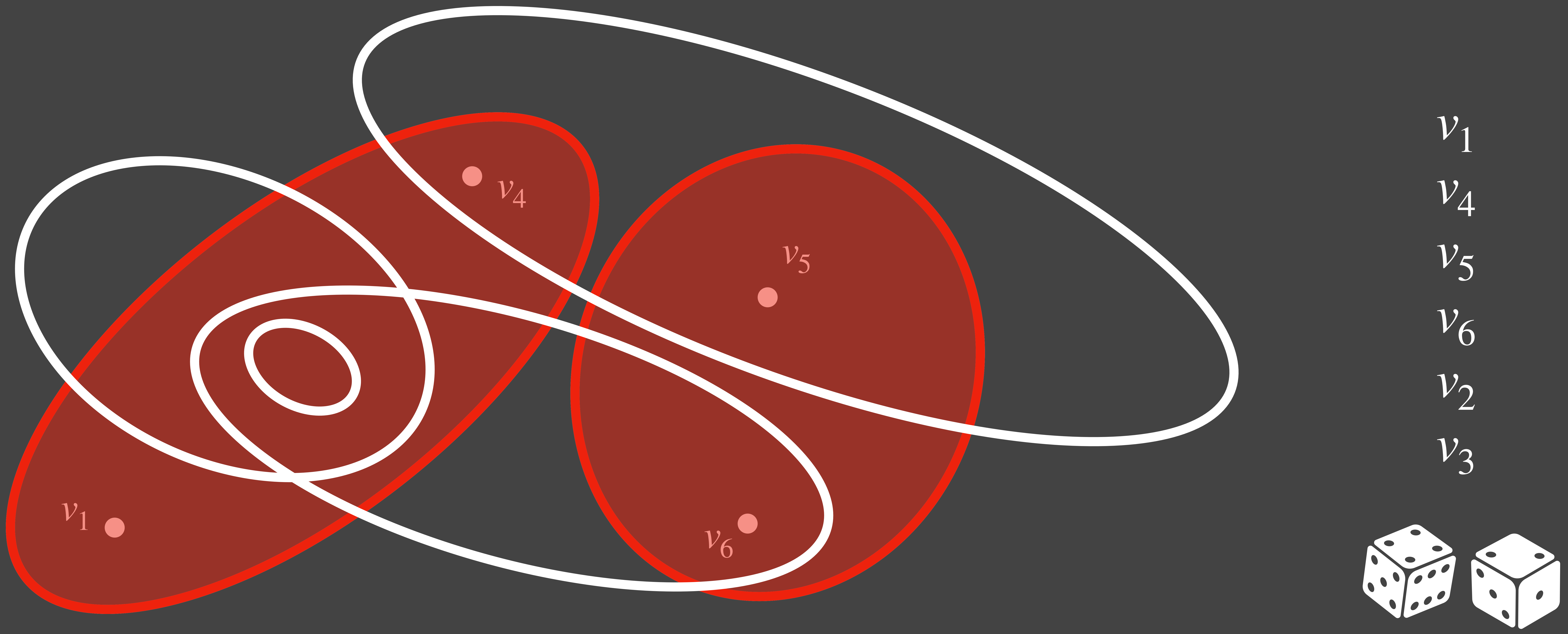

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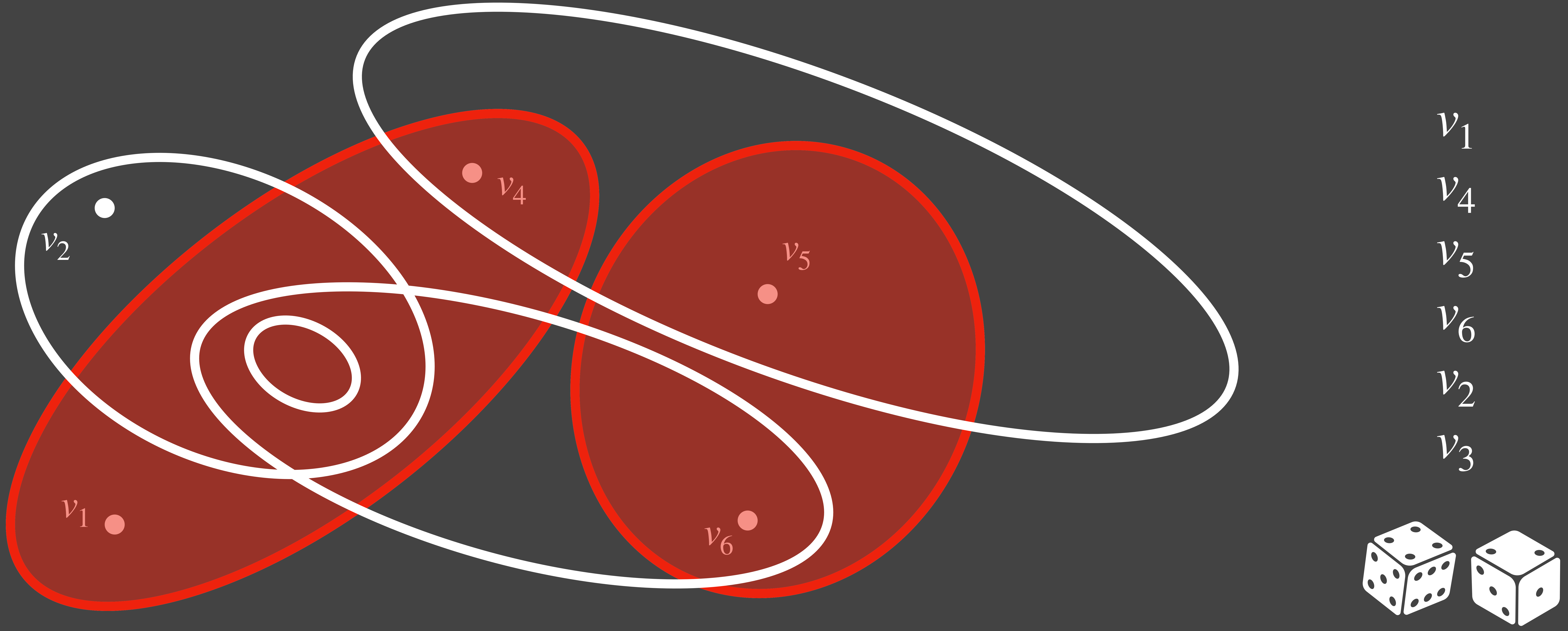

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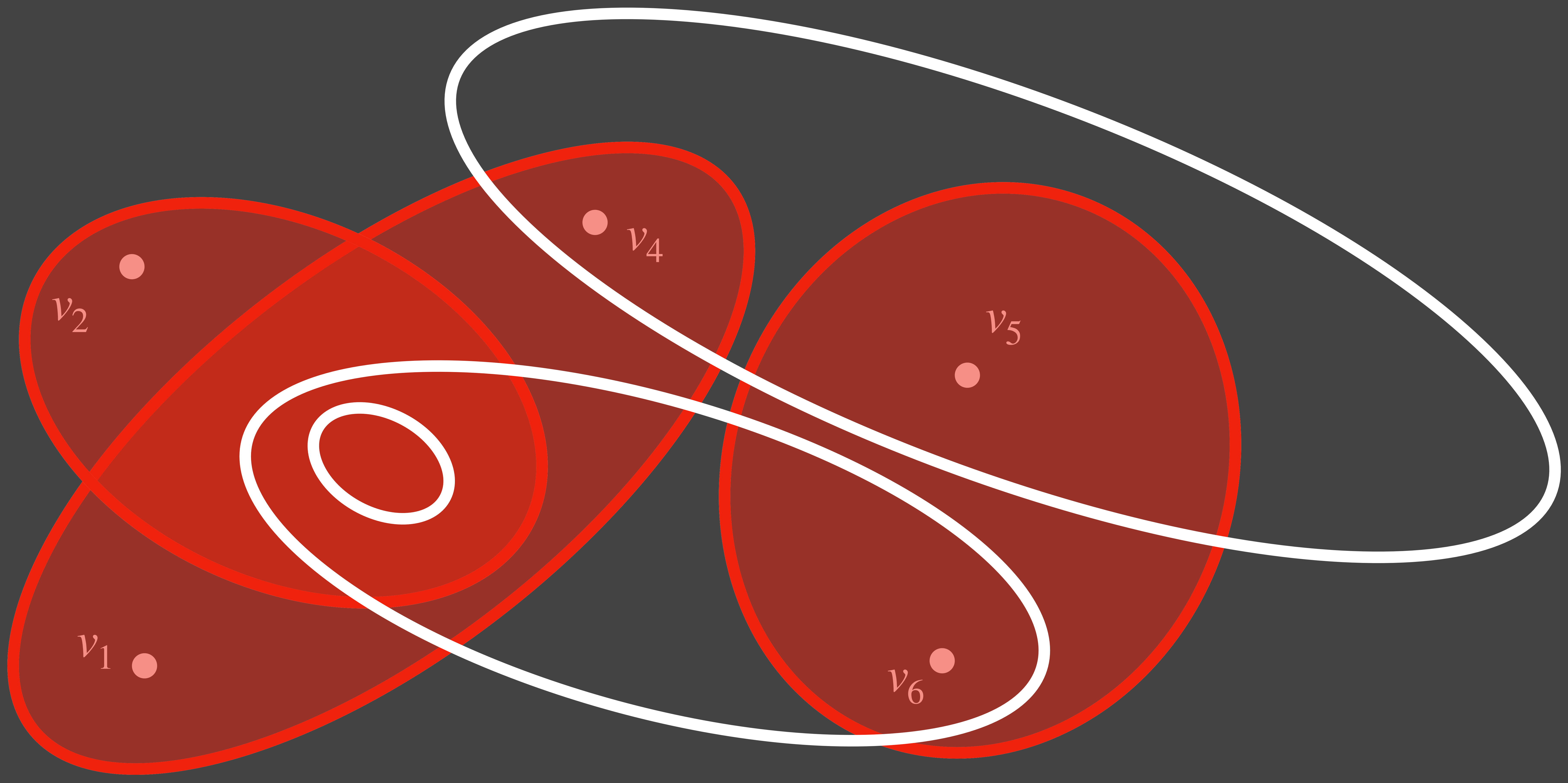

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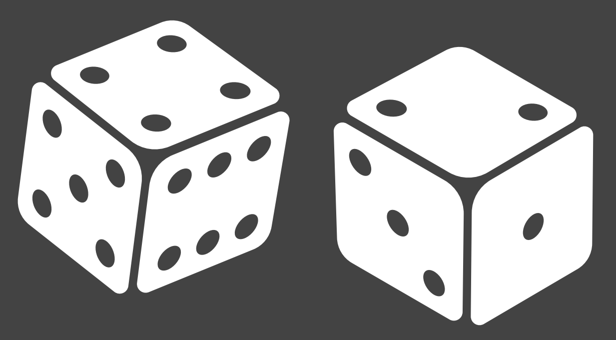

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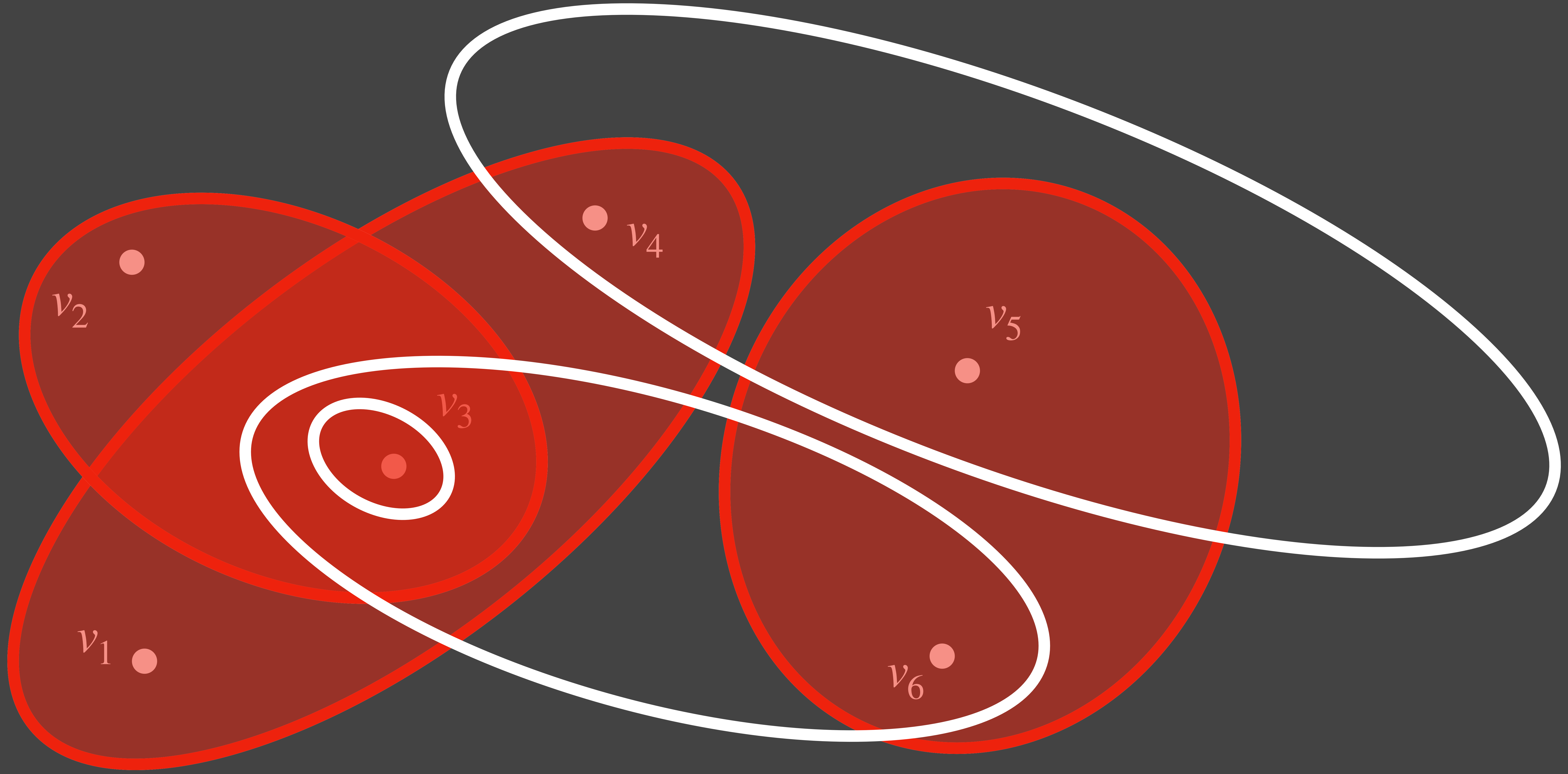

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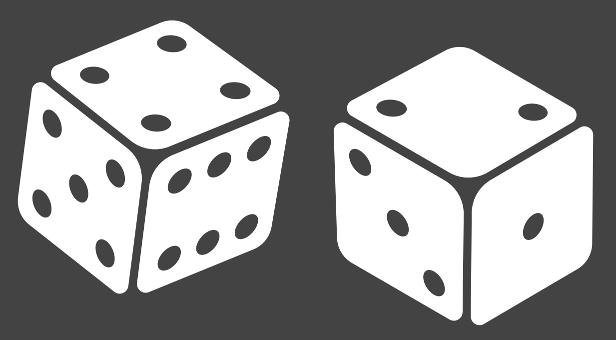

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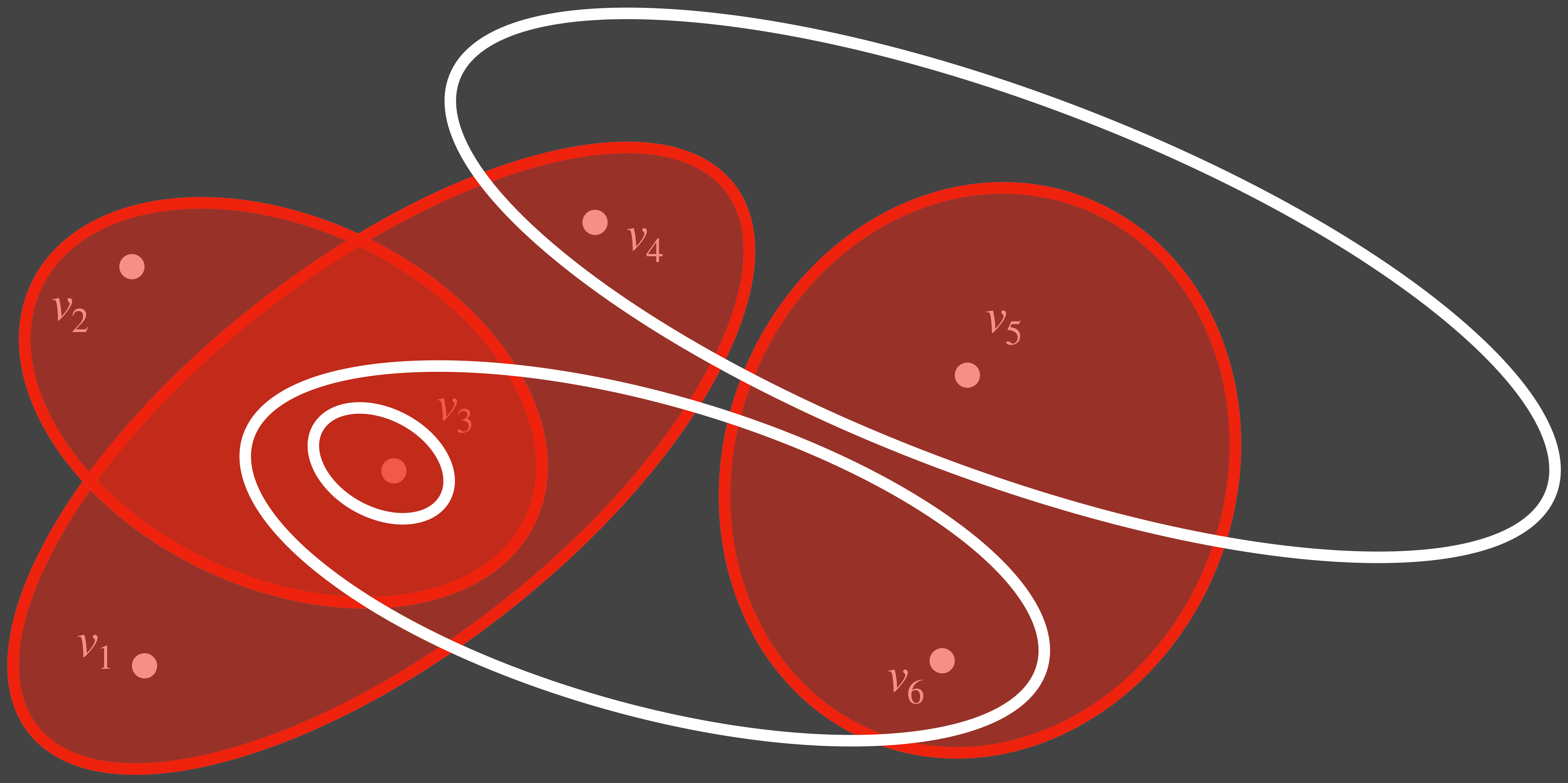

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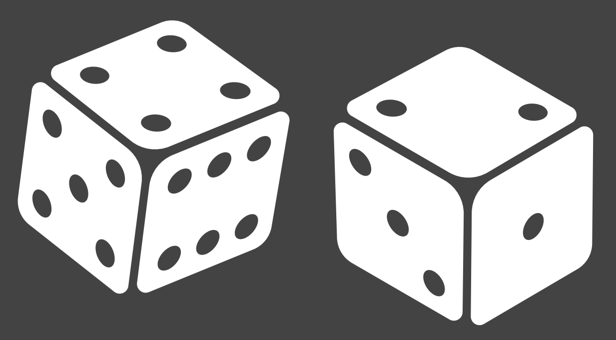

Is RO more like Offline or Online?

What is known?

$m =$ # sets
$n =$ # elements

Offline	log n + 1 [Johnson74],[Lovasz75], [Chvatal79]
Adversarial Online	O(log n log m) [Alon+03] [BuchbinderNaor09]
Stochastic	O(log (m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]
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Some reasons to believe $o(\log n \log m)$ not possible…

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Theorem [Gupta Kehne **L.** 21]:

There is a ***randomized poly time*** *algorithm for RO Covering IPs with expected competitive ratio* $O(\log mn)$.

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New algorithm! We show how to learn distribution & solve at same time.

RO Covering IPs

$$\min \quad c^\intercal x$$

$$a_1^\intercal x \geq 1$$
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$$a_3^\intercal x \geq 1$$
$$a_4^\intercal x \geq 1$$
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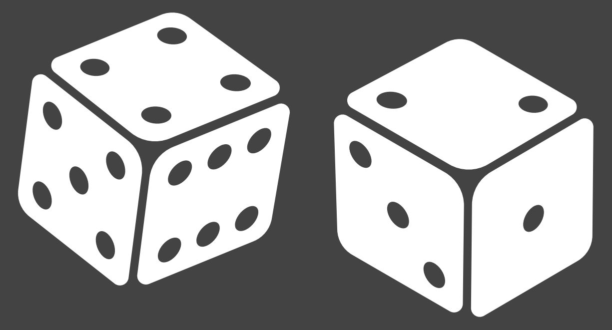
$$x \in \mathbb{Z}_{\geq 0}^m$$

RO Covering IPs



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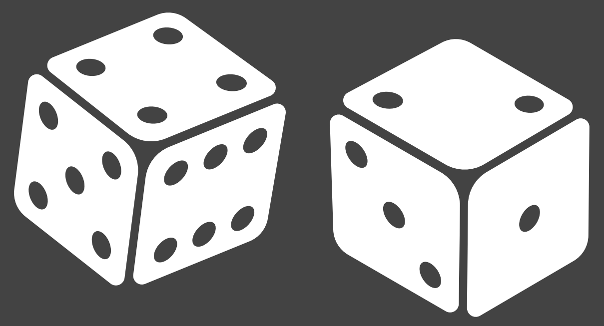

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RO Covering IPs

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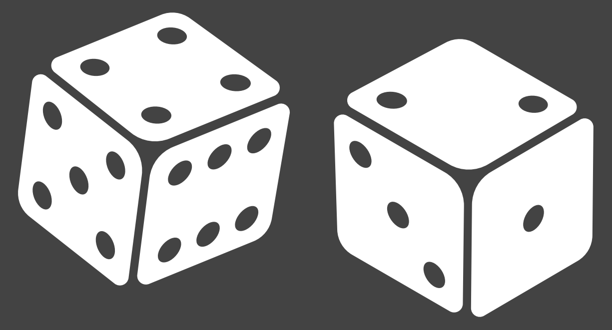
RO Covering IPs



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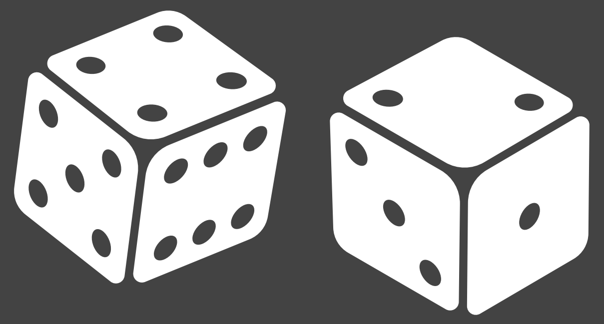

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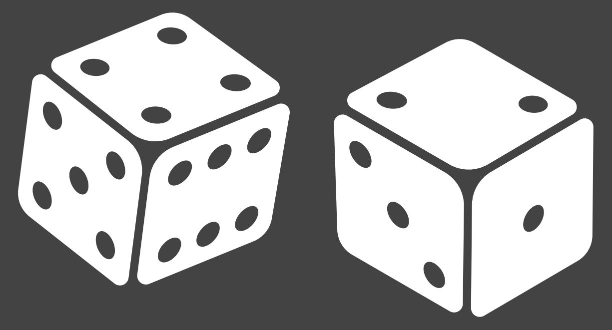

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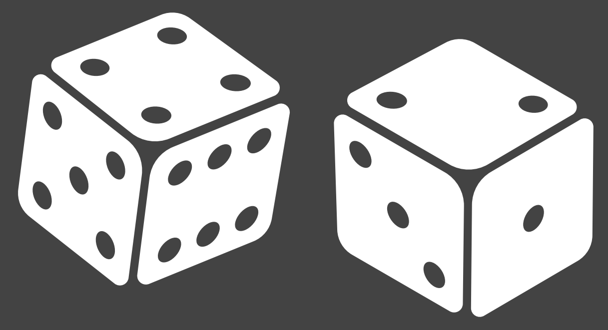

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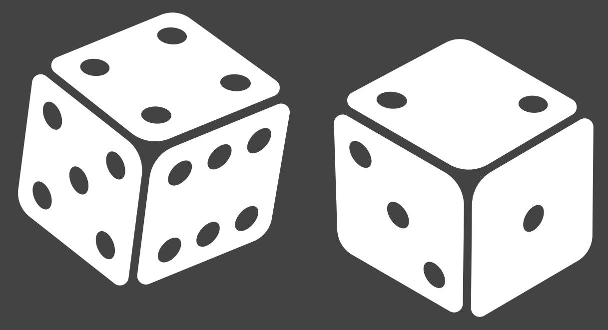

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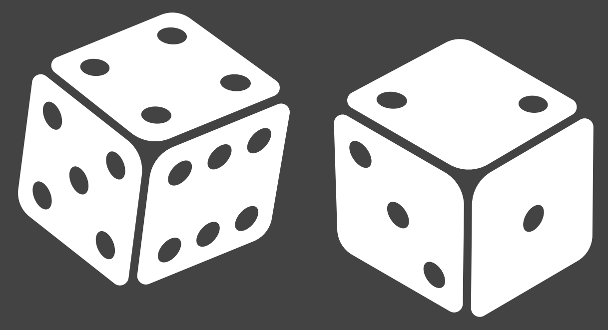
Goal: Maintain feasible solution x that is *monotonically* increasing.

RO Covering IPs



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Set Cover is the special case where constraint matrix A is 0/1.

Talk Outline

- ➜ Intro
- Previous Work
- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time
- Extensions & Lower Bounds

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Intro

Previous Work

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

Extensions & Lower Bounds

How [Alon+ 03] works

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2 Stage algorithm!

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(I) Solve LP Online.

(II) Round Online.

How [Alon+ 03] works

2 Stage algorithm!

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$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

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Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

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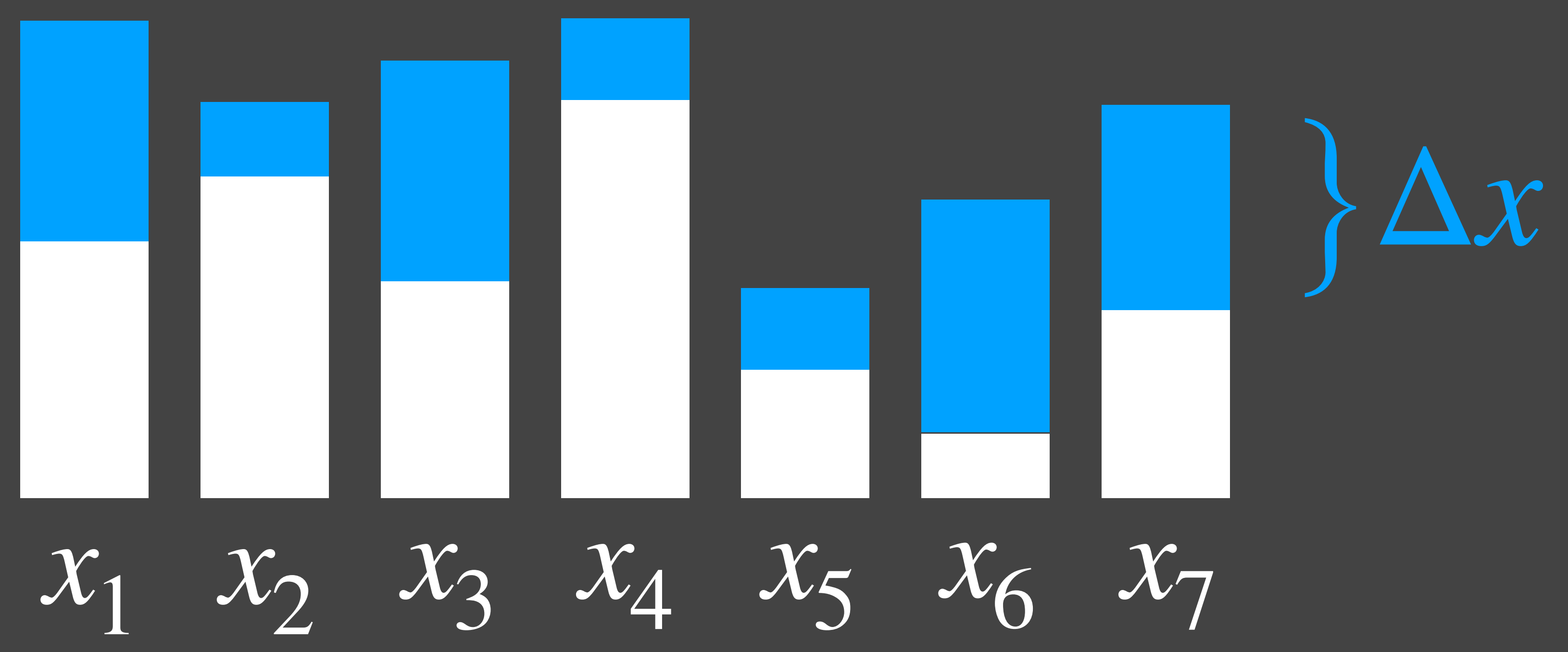

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$$\begin{aligned} &\min \sum_{S} x_S \\ \forall v \in \mathcal{U}: &\quad \sum_{S \ni v} x_S \geq 1 \\ \forall S \in \mathcal{S}: &\quad x_S \geq 0 \end{aligned}$$

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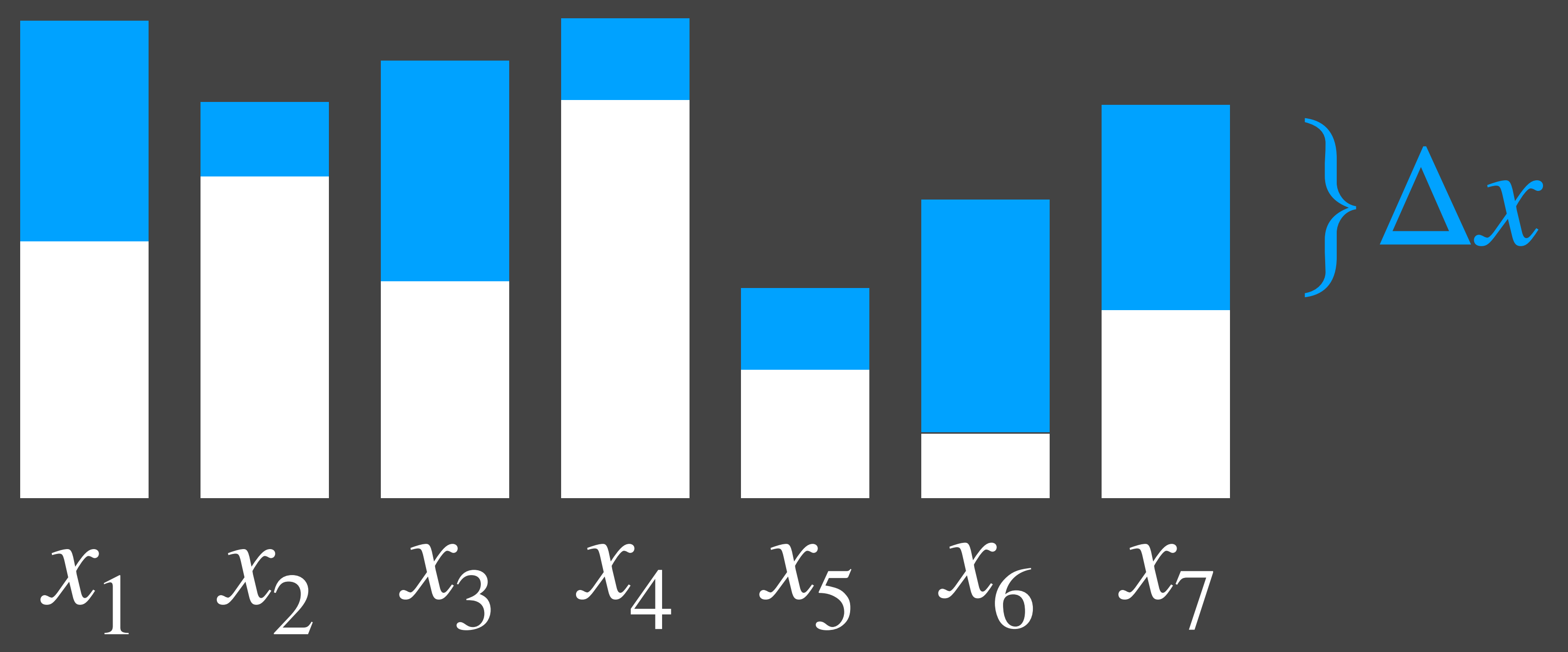

Take S with prob. $\propto \Delta x_S$.

How [Alon+ 03] works

2 Stage algorithm!

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$$\min \sum_S x_S$$

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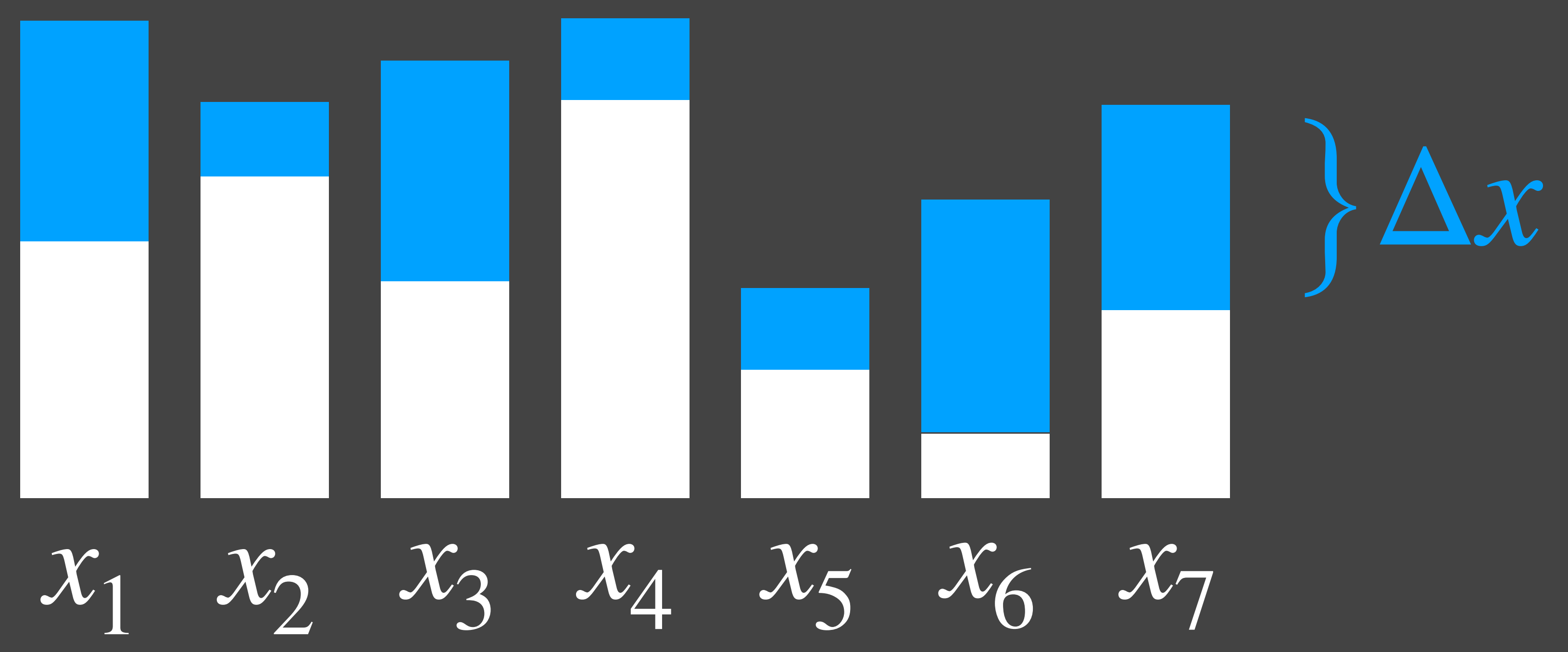

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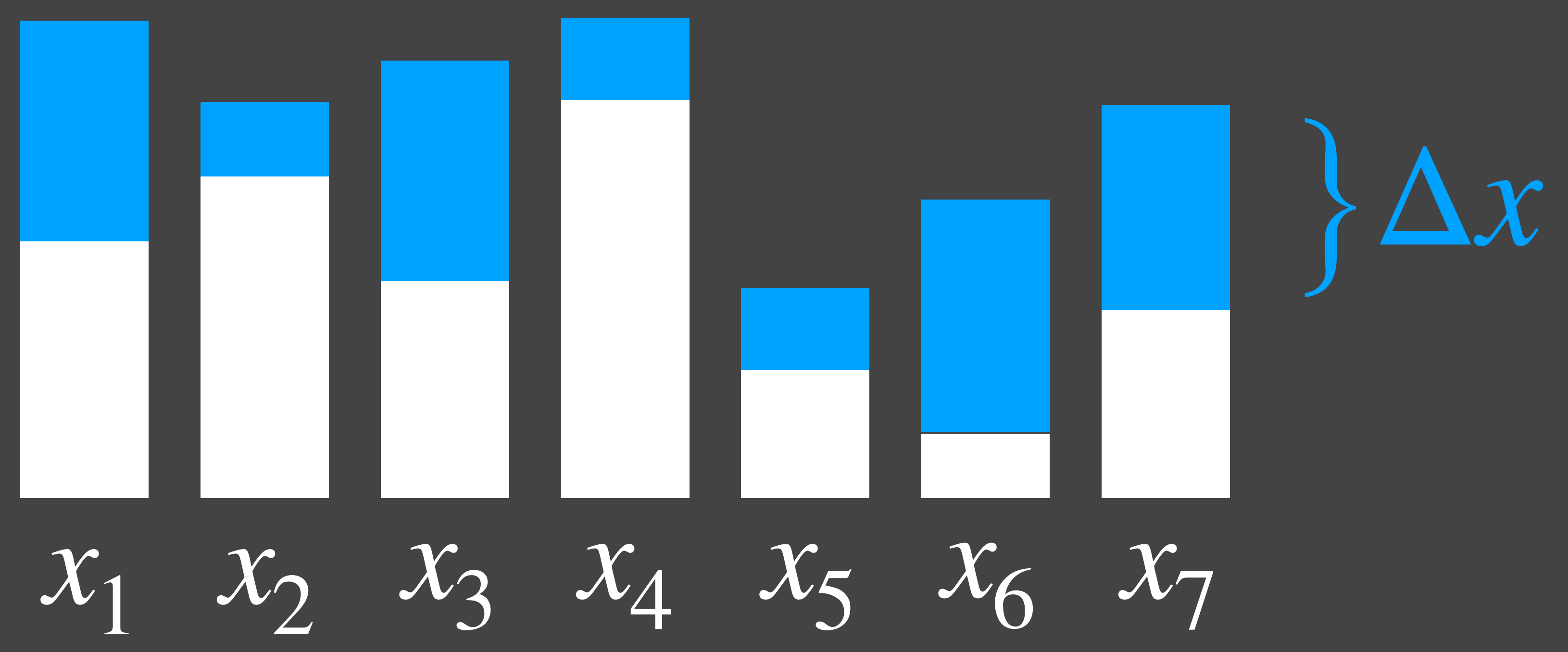

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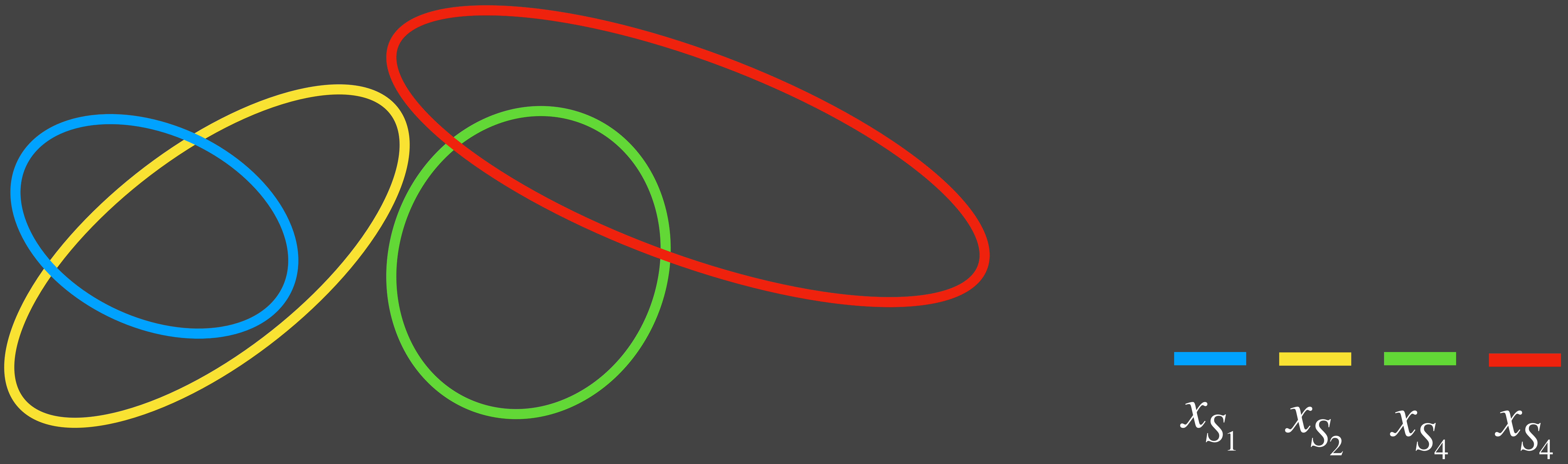
Expected Cost: $O(\log n \log m) \cdot \mathrm{OPT}$

Online LP Solver of [Alon+ 03]



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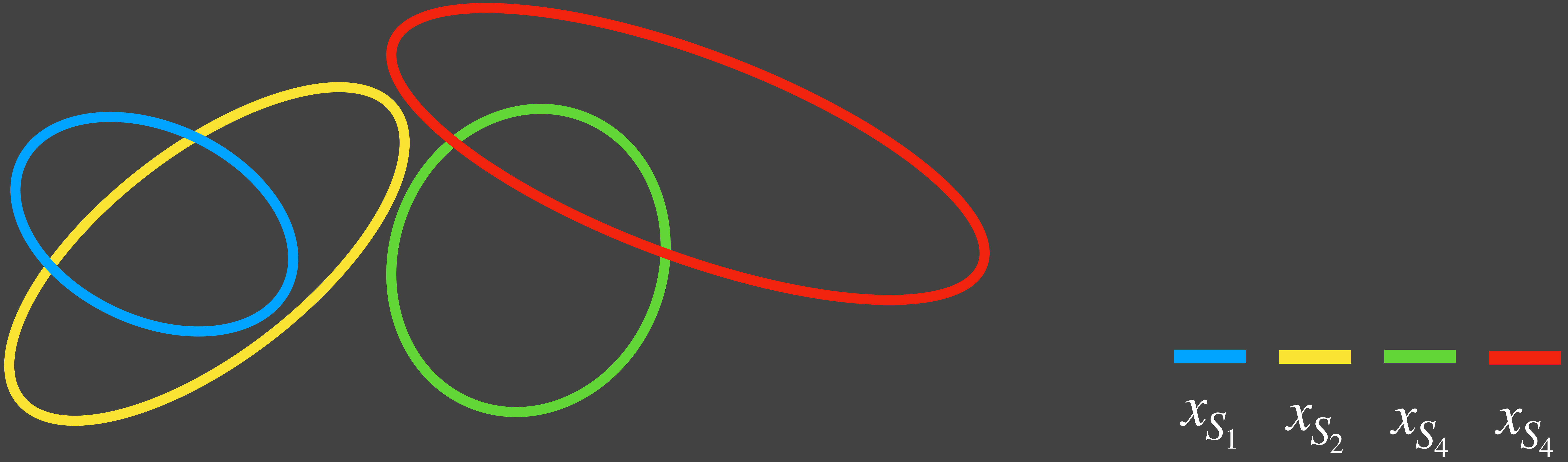

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While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

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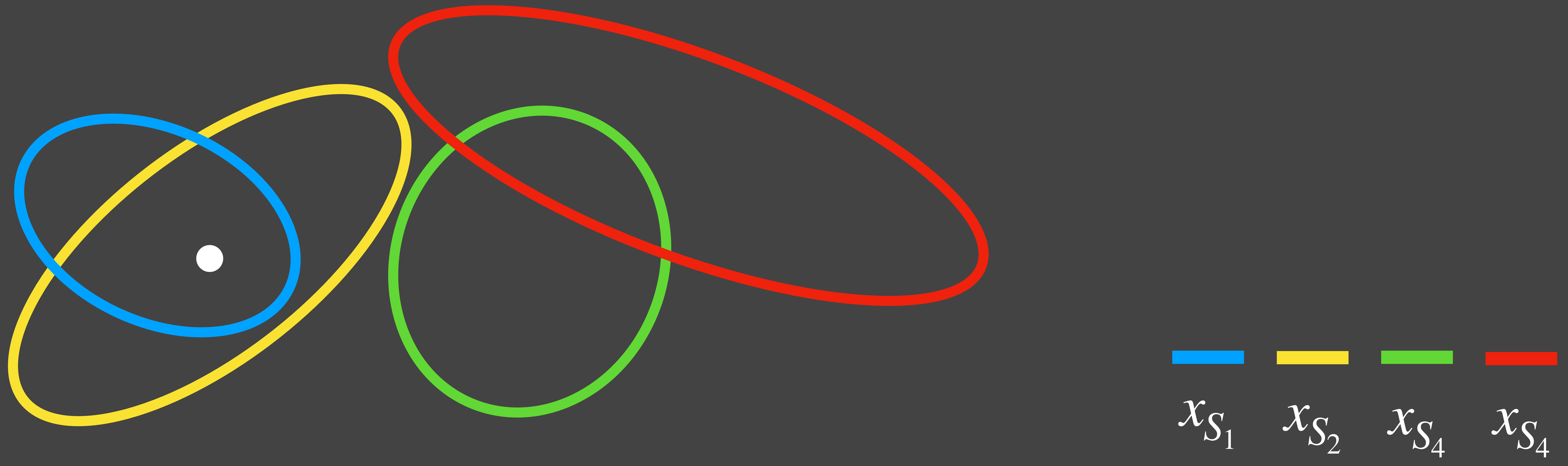

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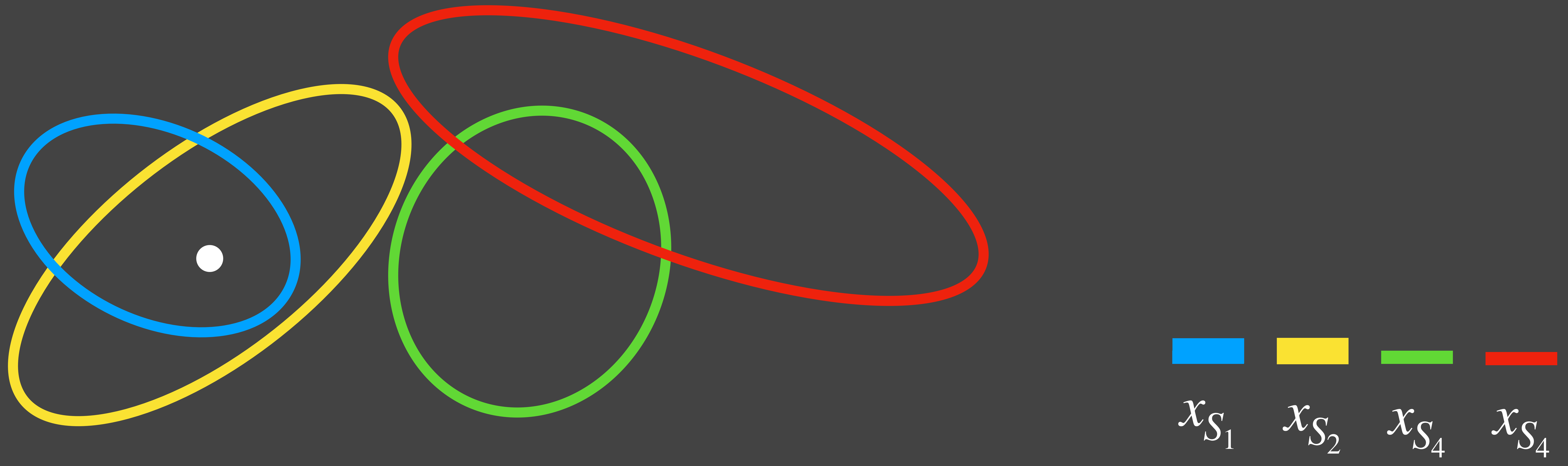

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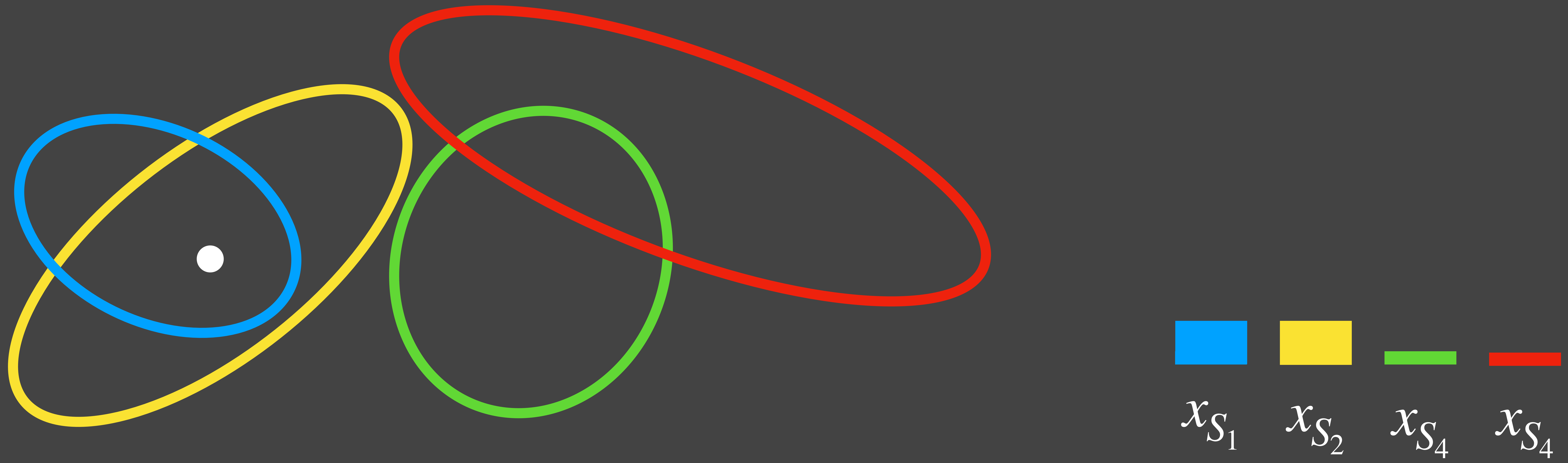

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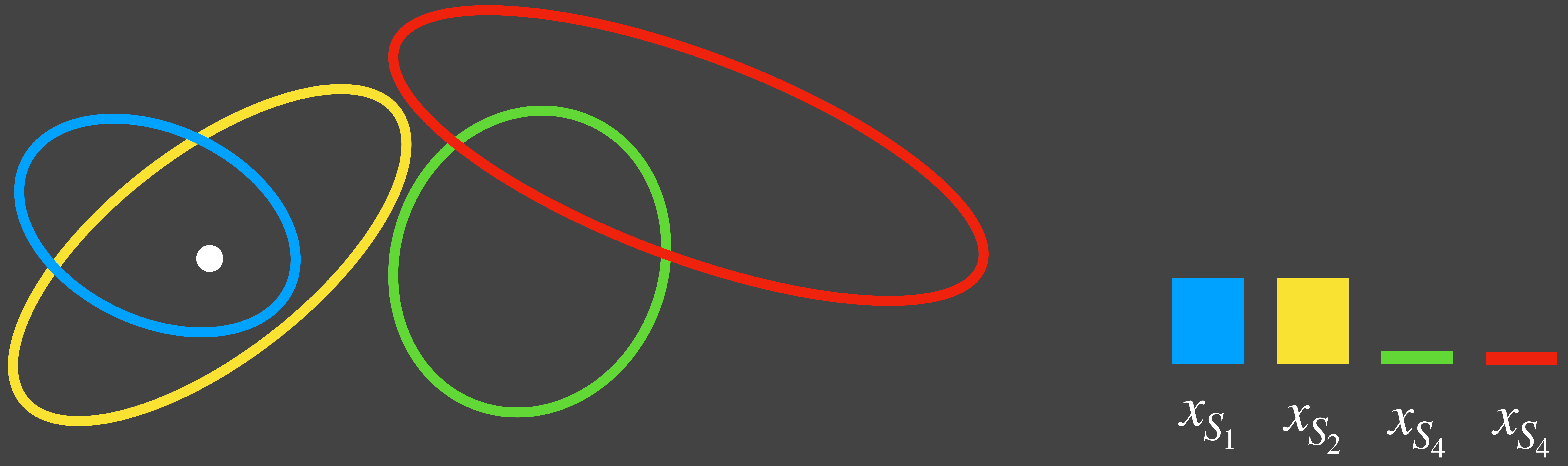

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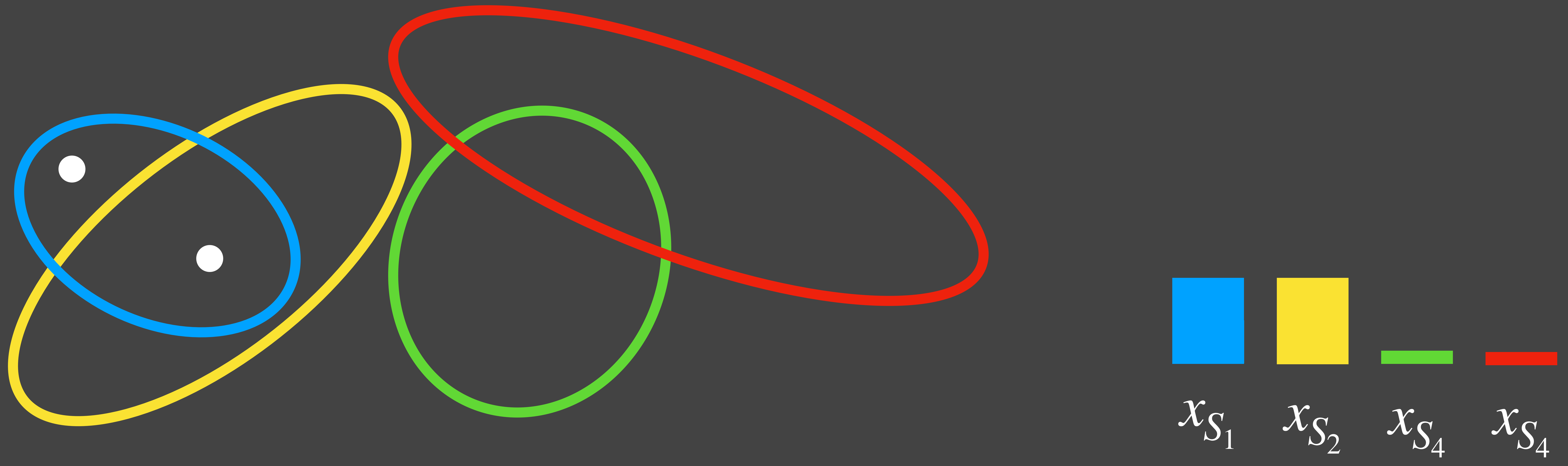

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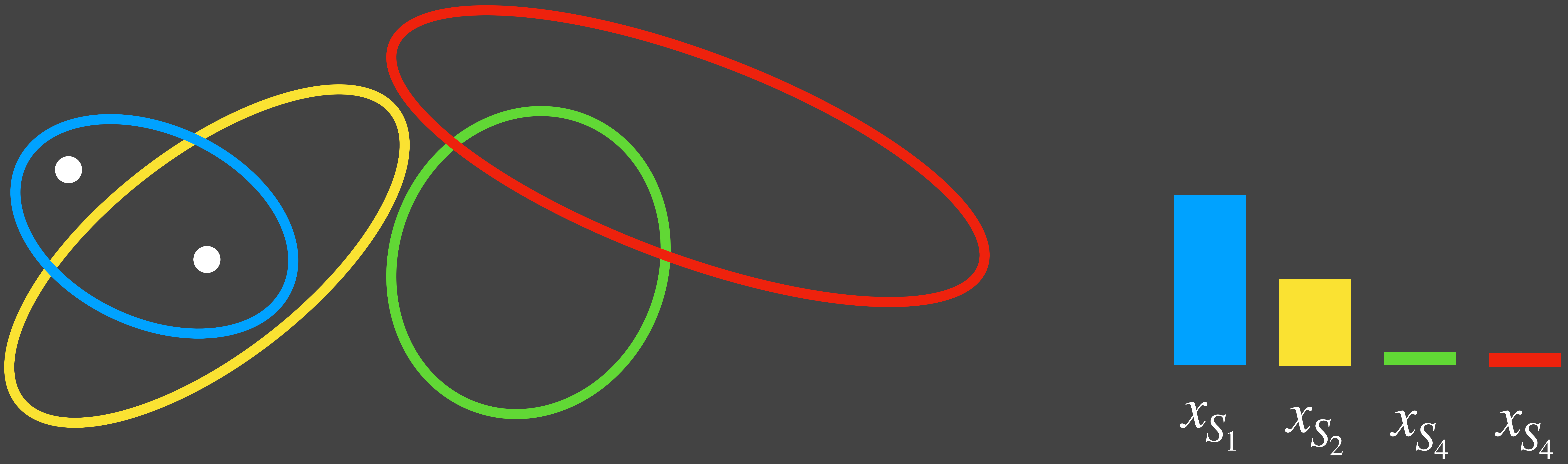

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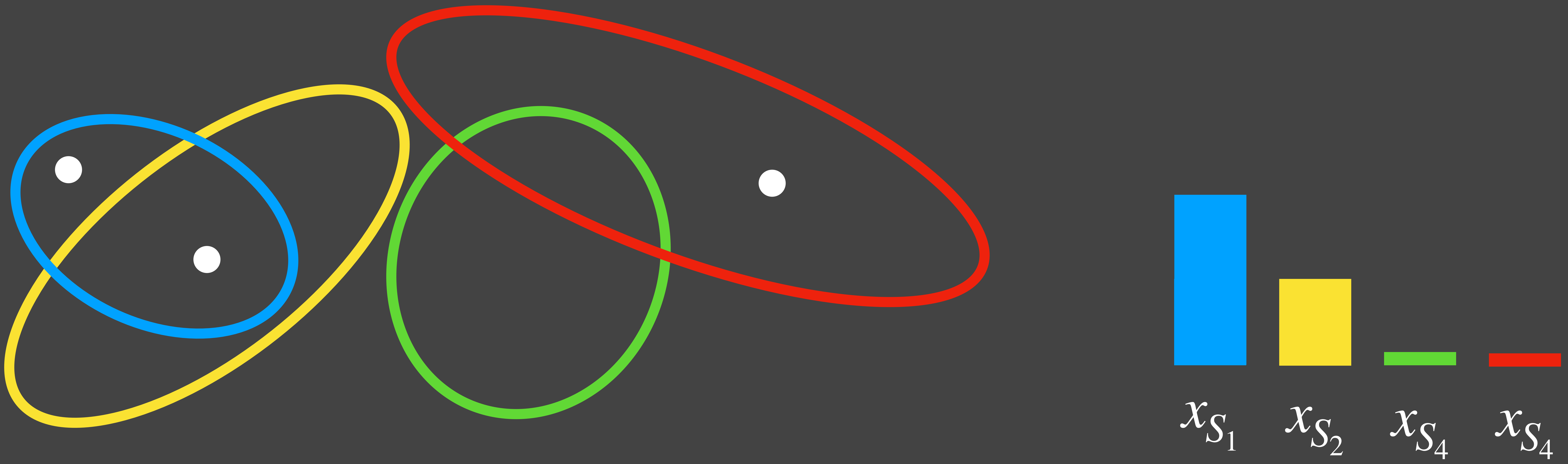

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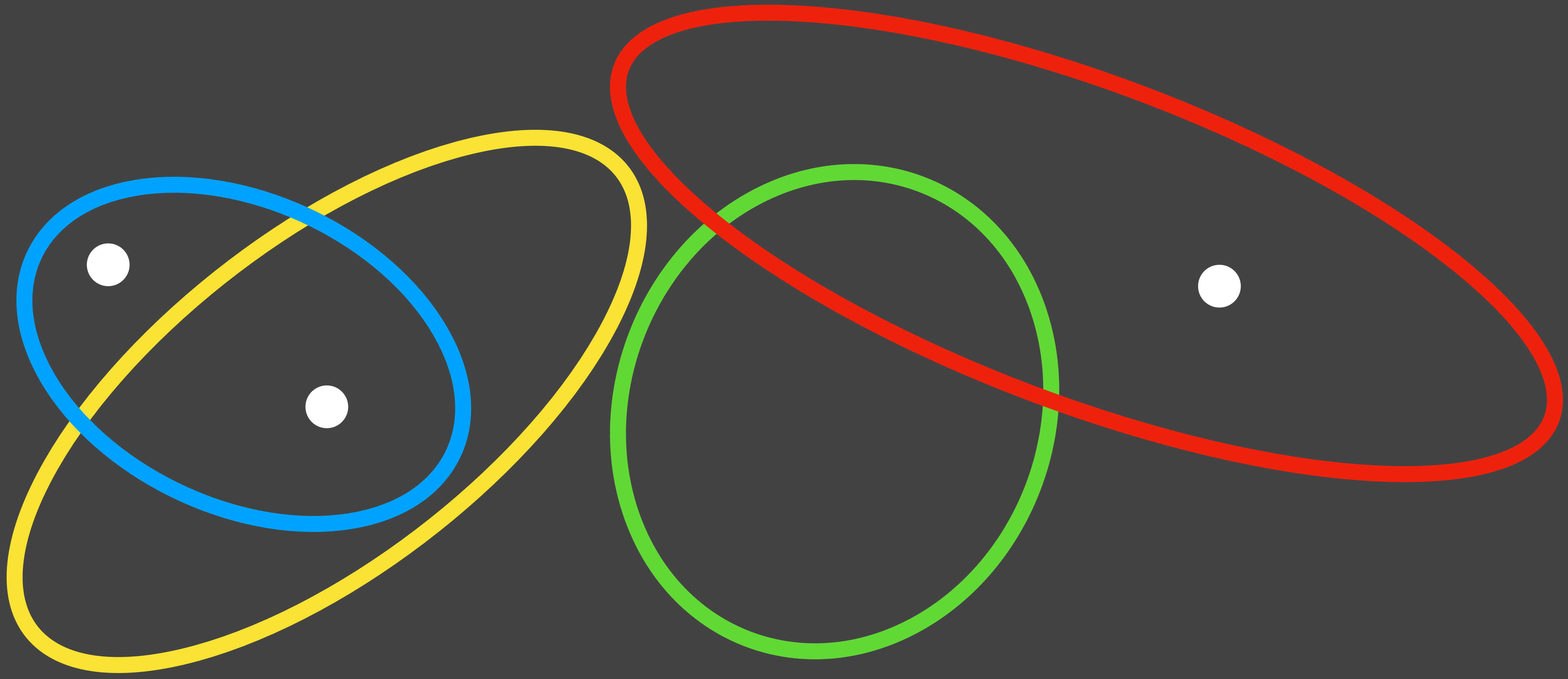

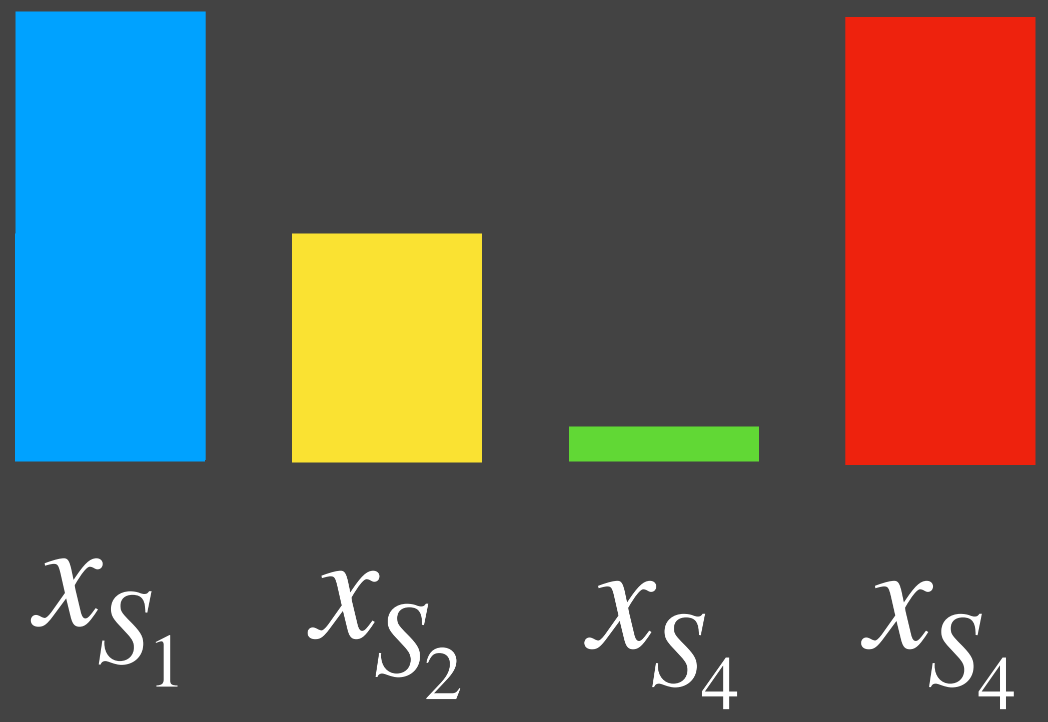

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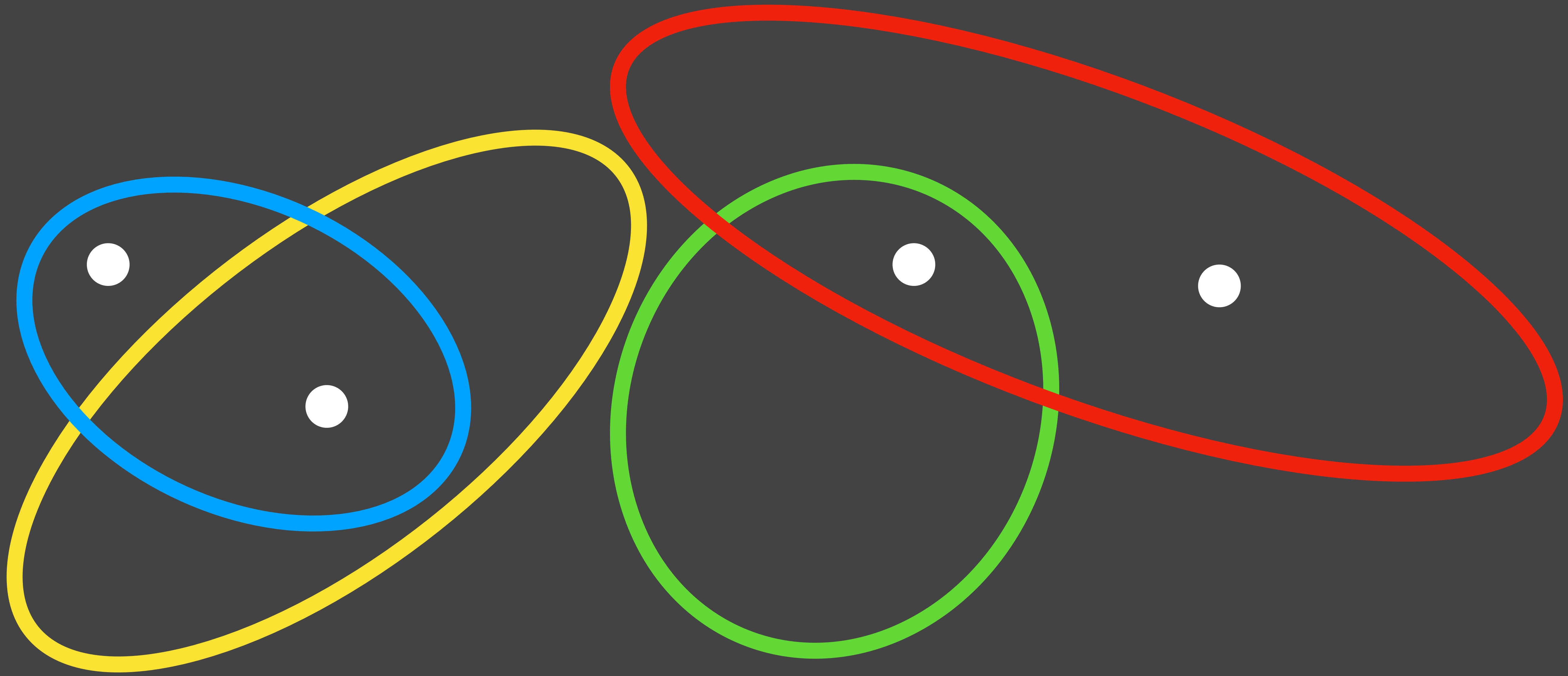

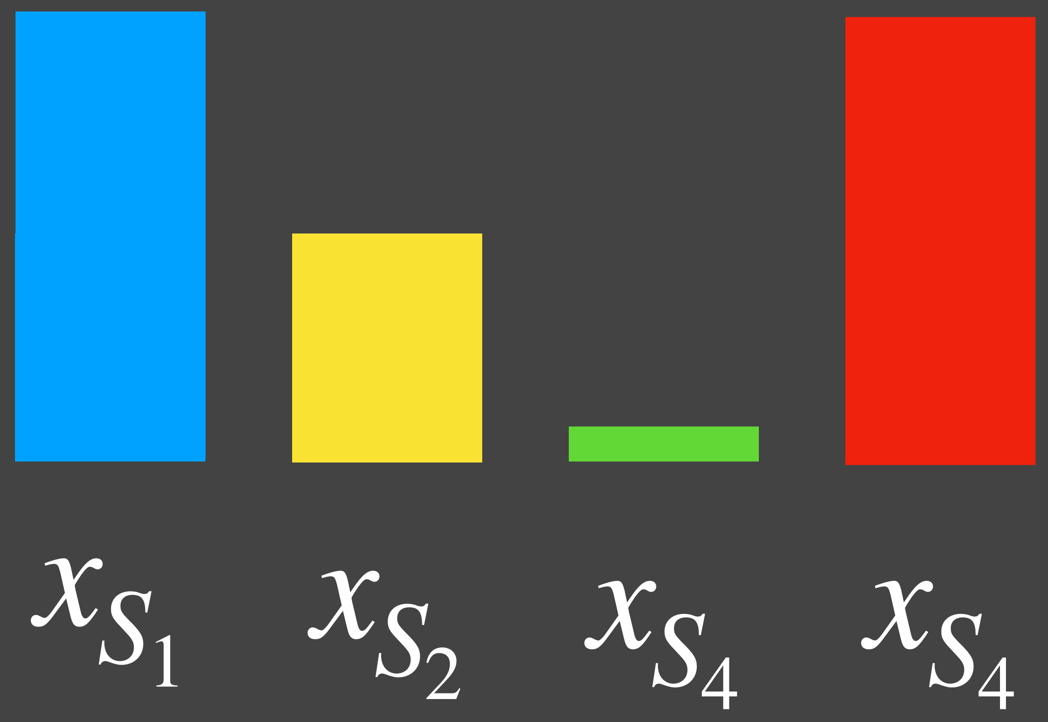

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__Claim 1:__ x feasible for (**P**).

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Claim 2: $c(x) \leq c(y)$

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Claim 3: $y/\log m$ feasible for (**D**).

Neither stage of [Alon+ 03] can be improved!

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<u>**Theorem**</u> [Gupta Kehne **L.**]: $\Omega(\log m)$ for <u>fractional</u> algos in RO.

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__Theorem__ [Gupta Kehne **L.**]: $\Omega(\log m)$ for __fractional__ algos in RO.

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New algorithm needed!

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New algorithm needed!

We maintain coarse solution x, neither feasible nor monotone,
but round x anyway…

Talk Outline

Intro

Previous Work

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

Extensions & Lower Bounds

Talk Outline

- Intro
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- Extensions & Lower Bounds

LearnOrCover

(Unit cost, exp time warmup)

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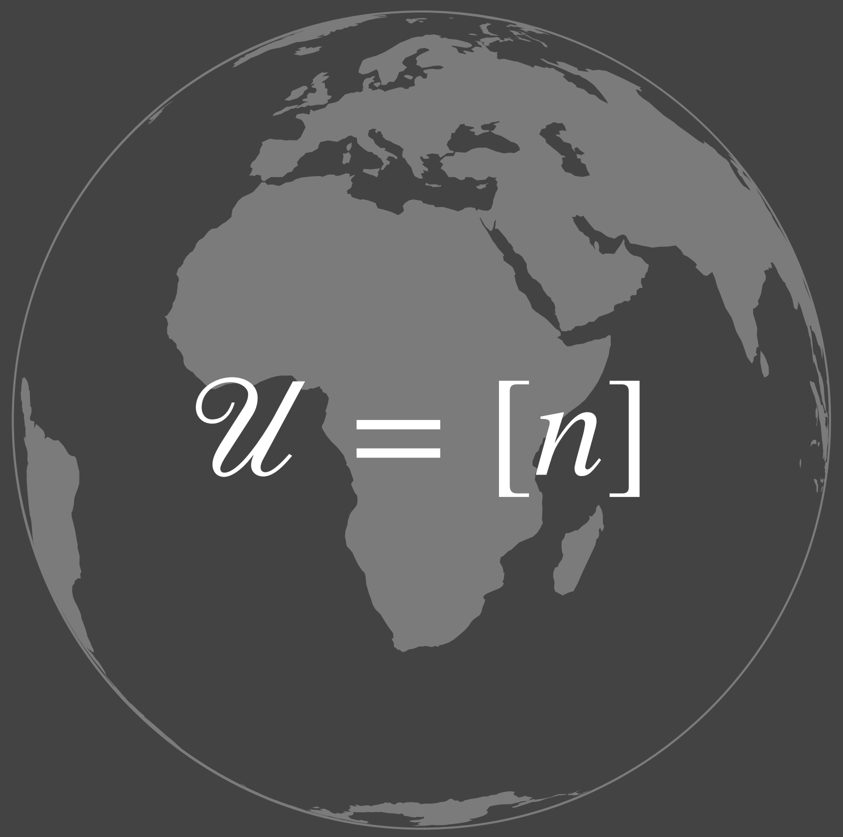
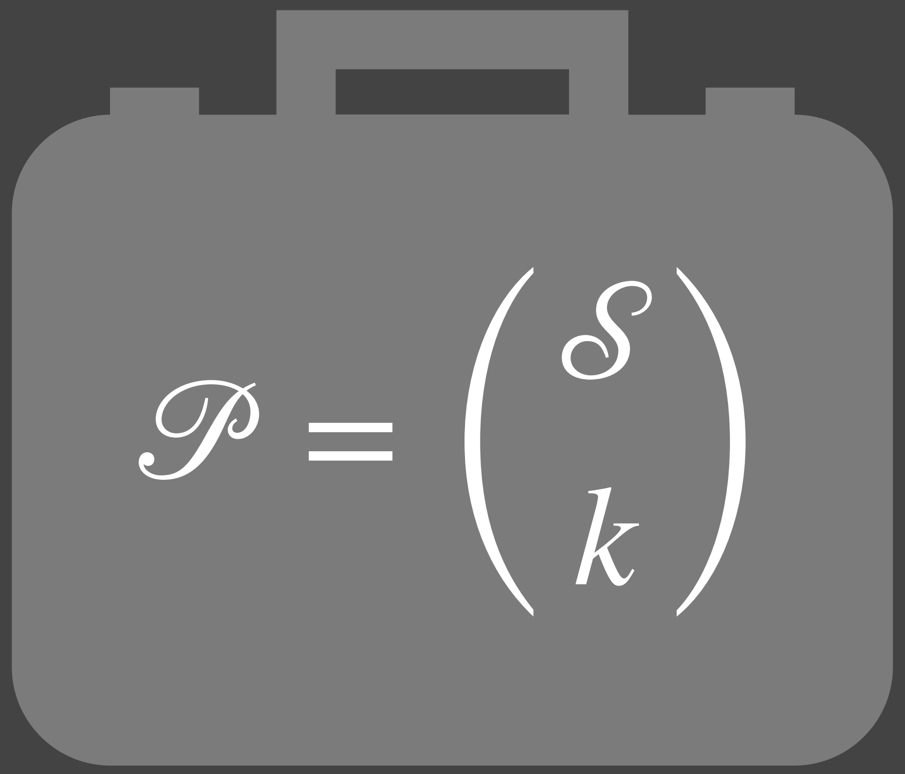


$$\mathscr{U} = [n]$$



$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$$k := |OPT|$$

LearnOrCover

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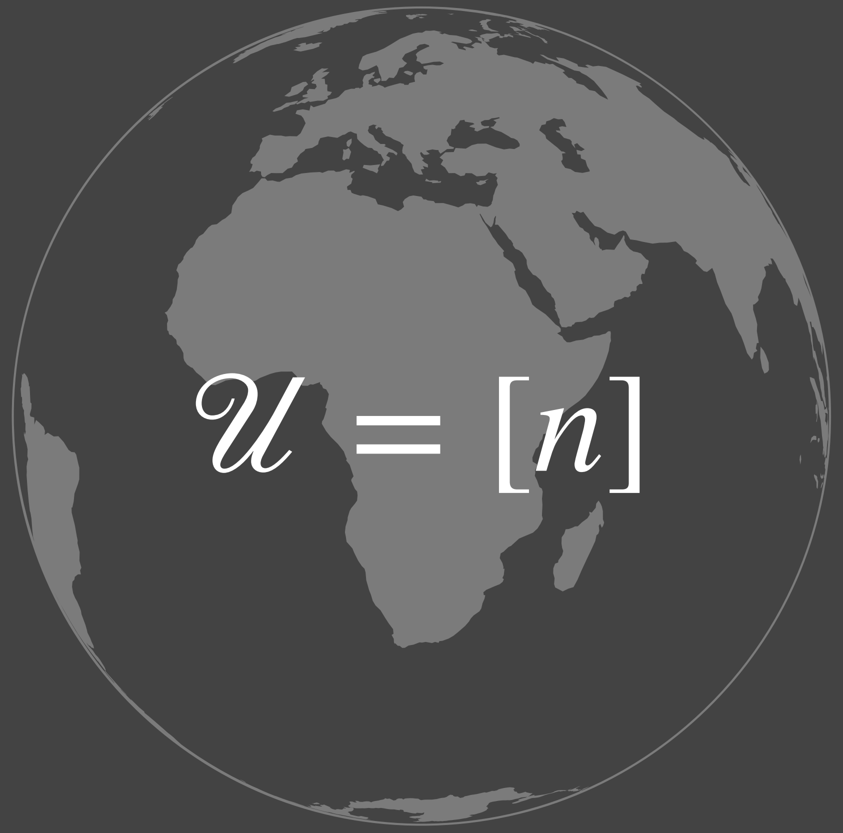

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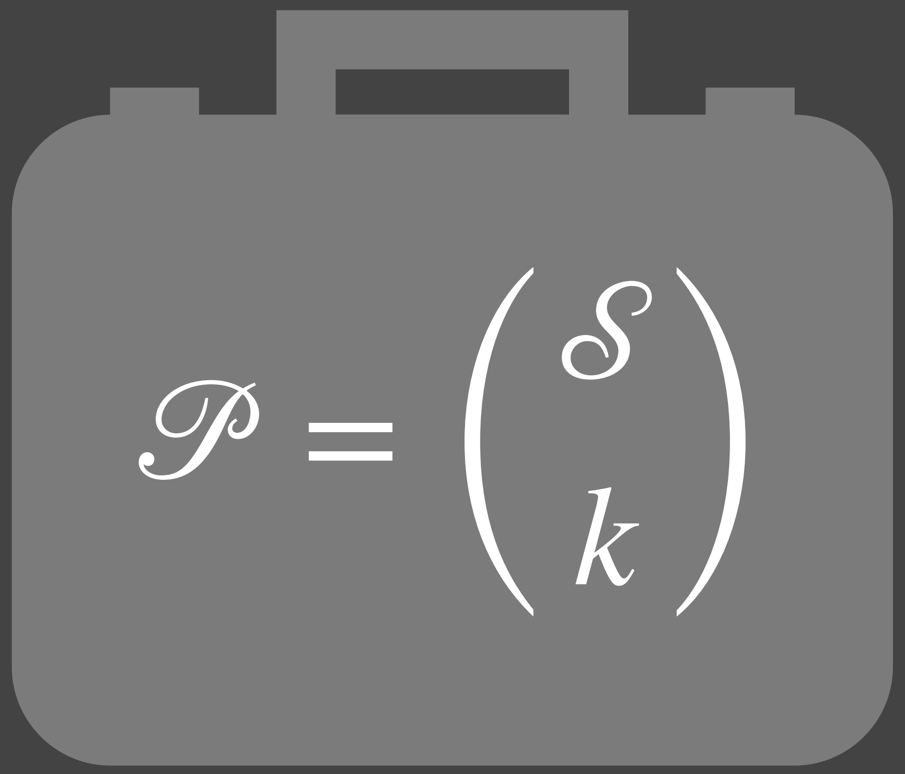

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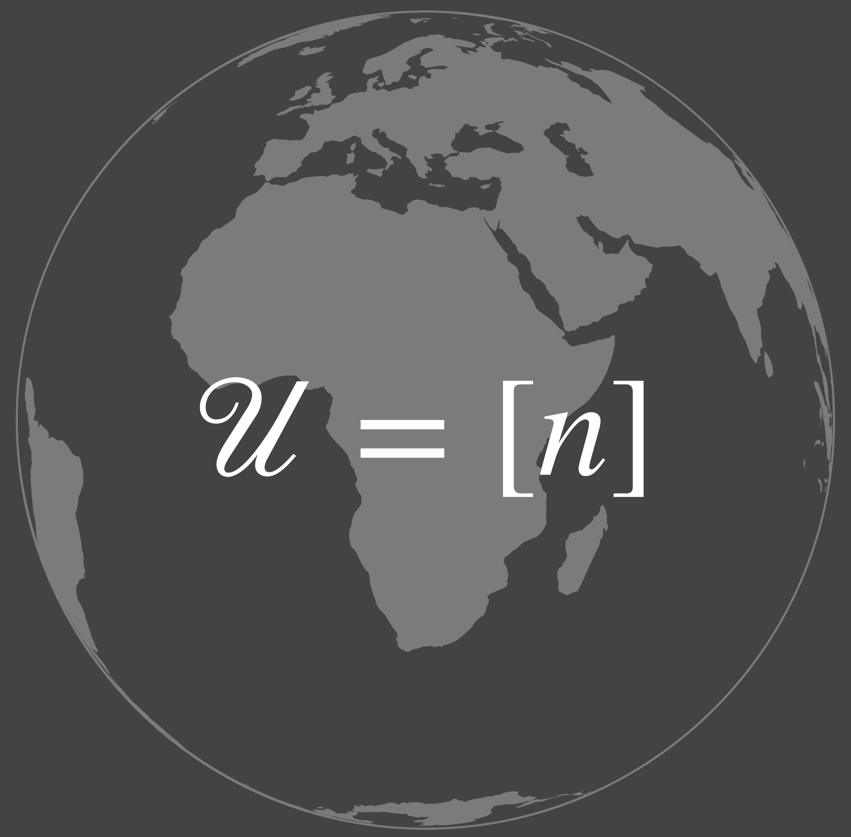

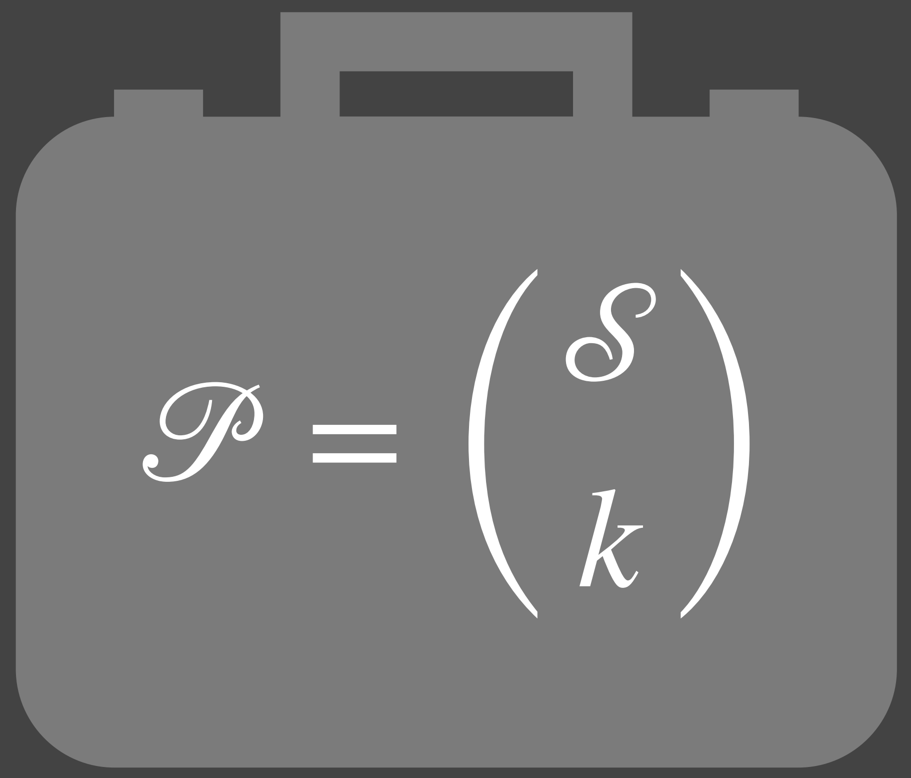

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LearnOrCover
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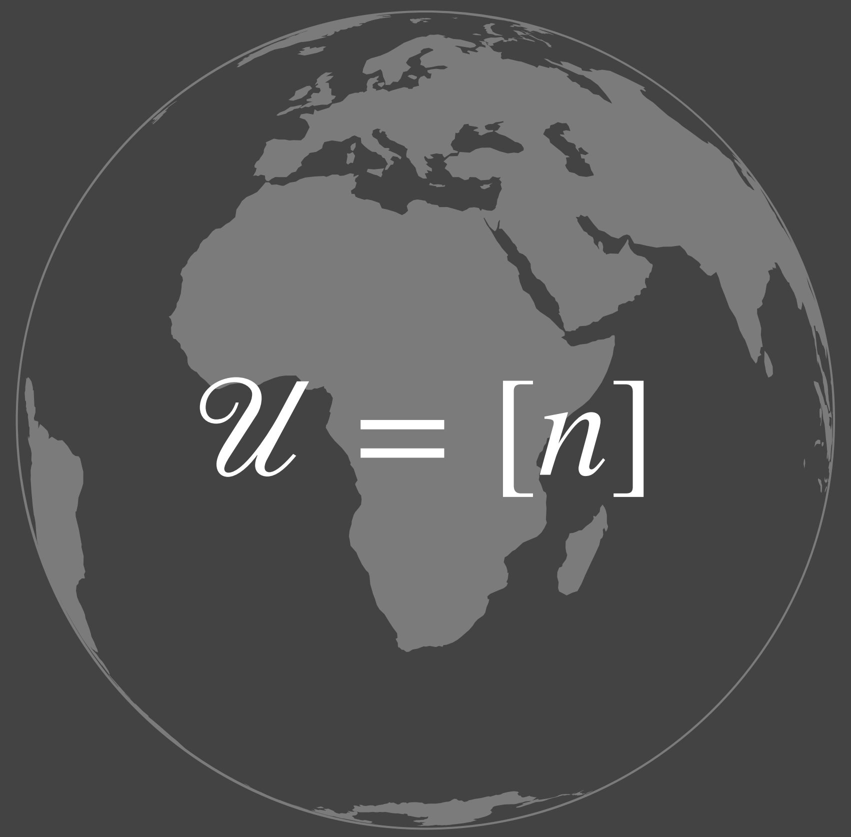

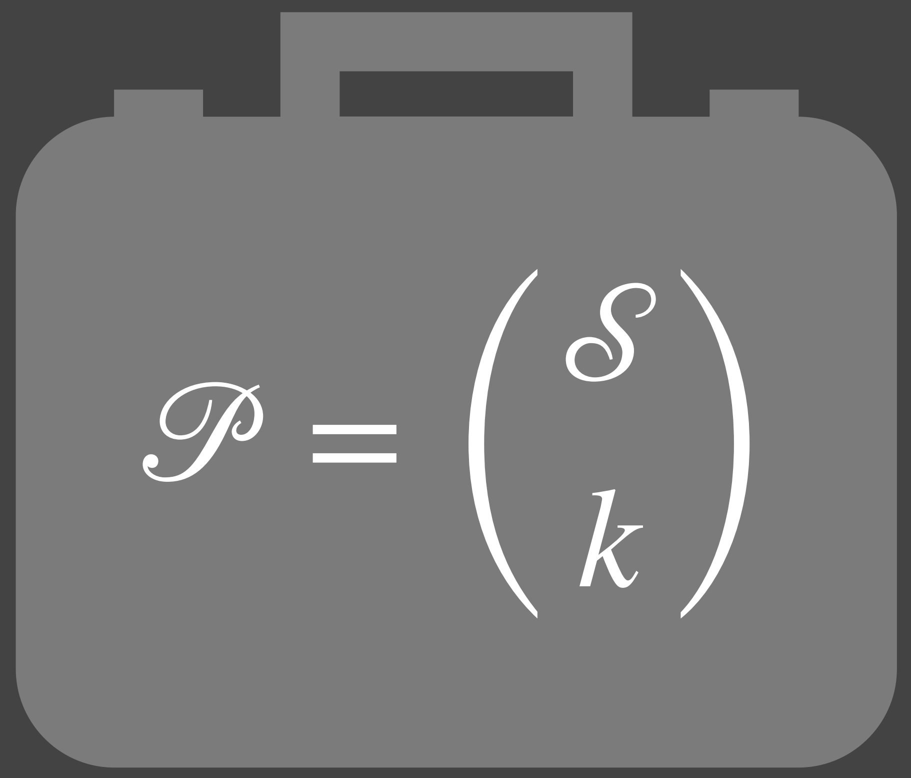

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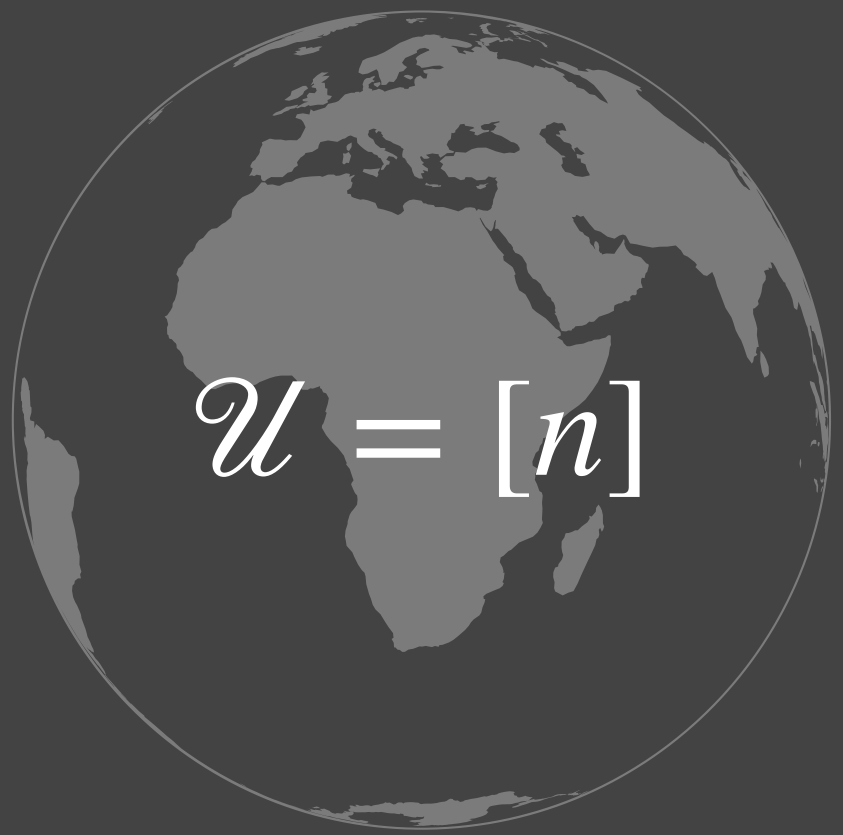

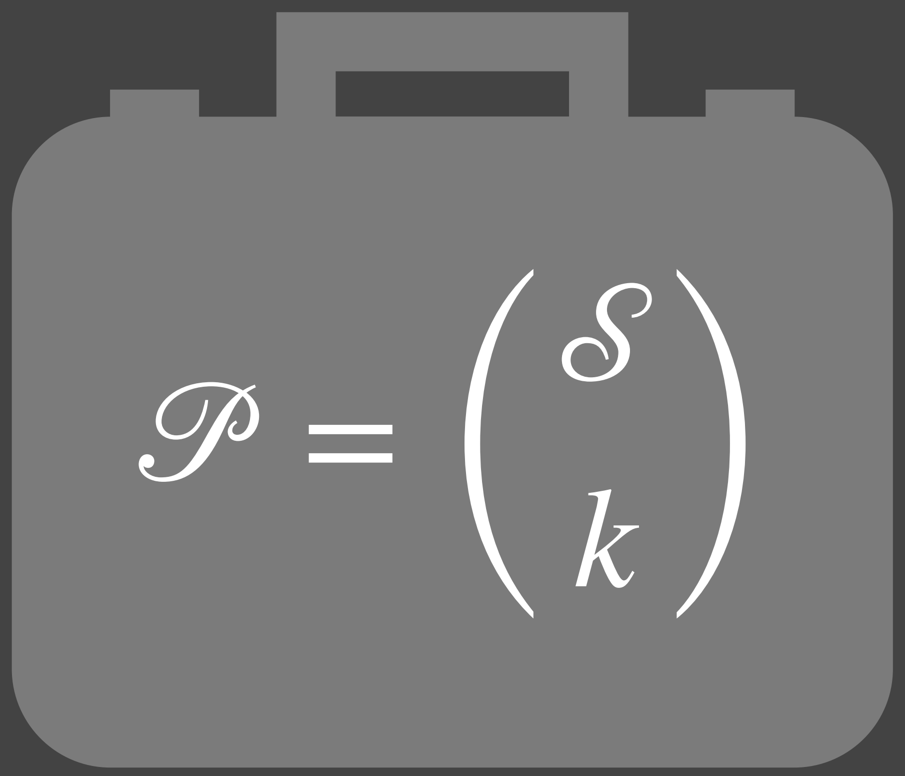

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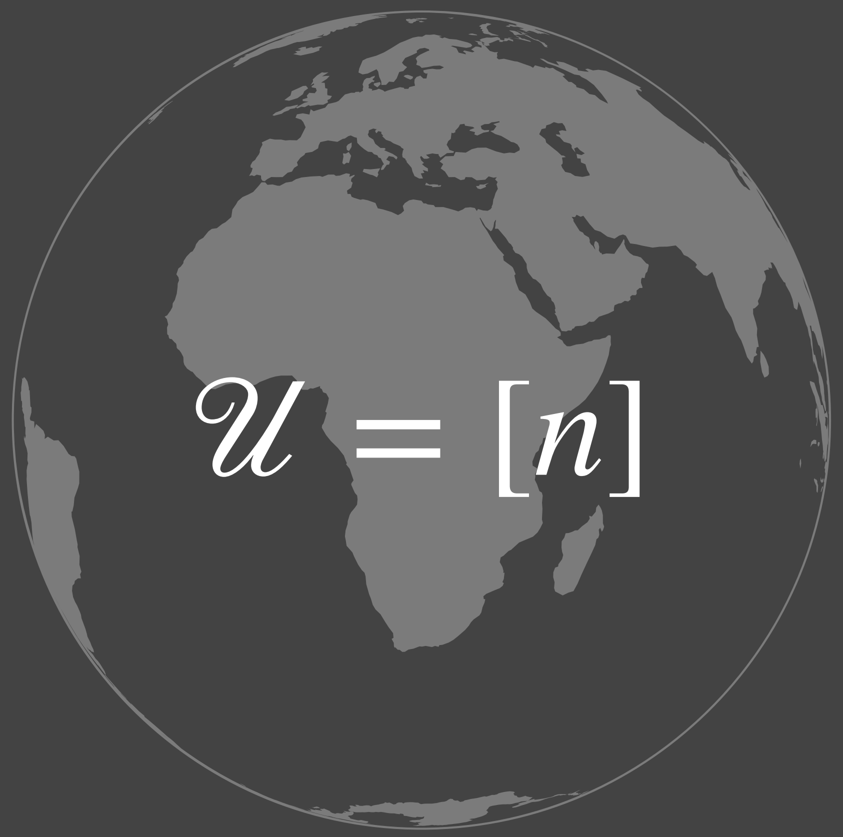

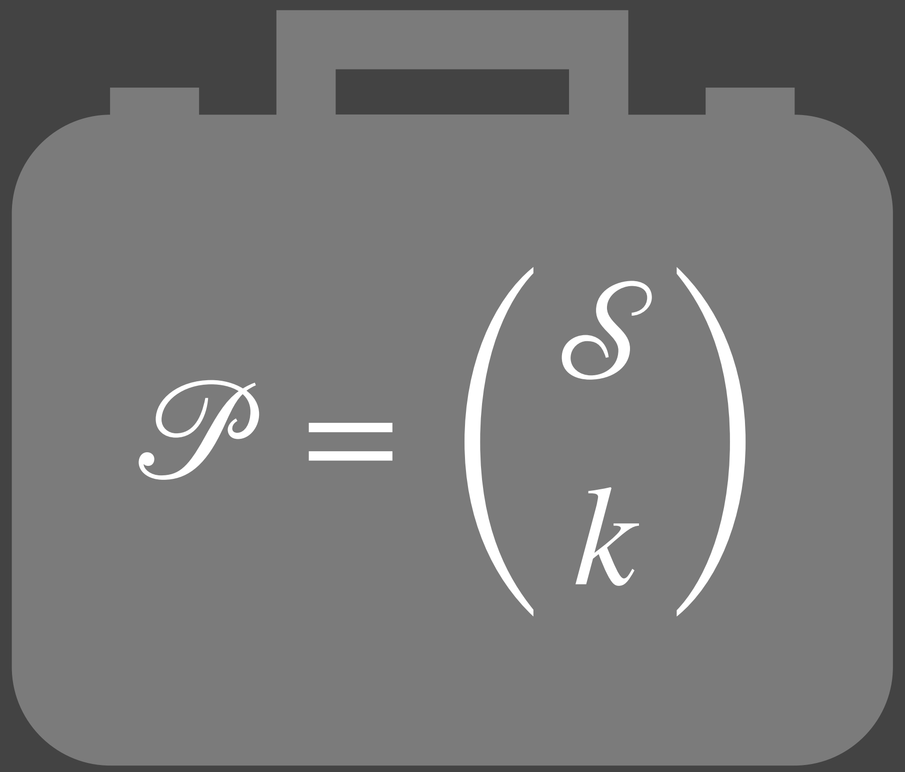

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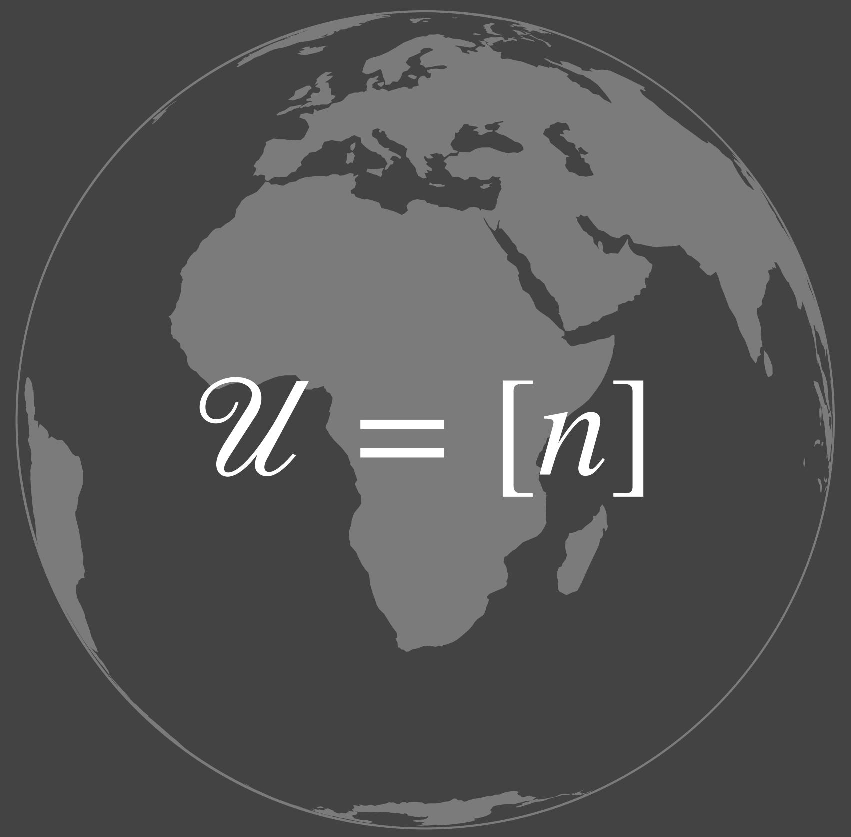

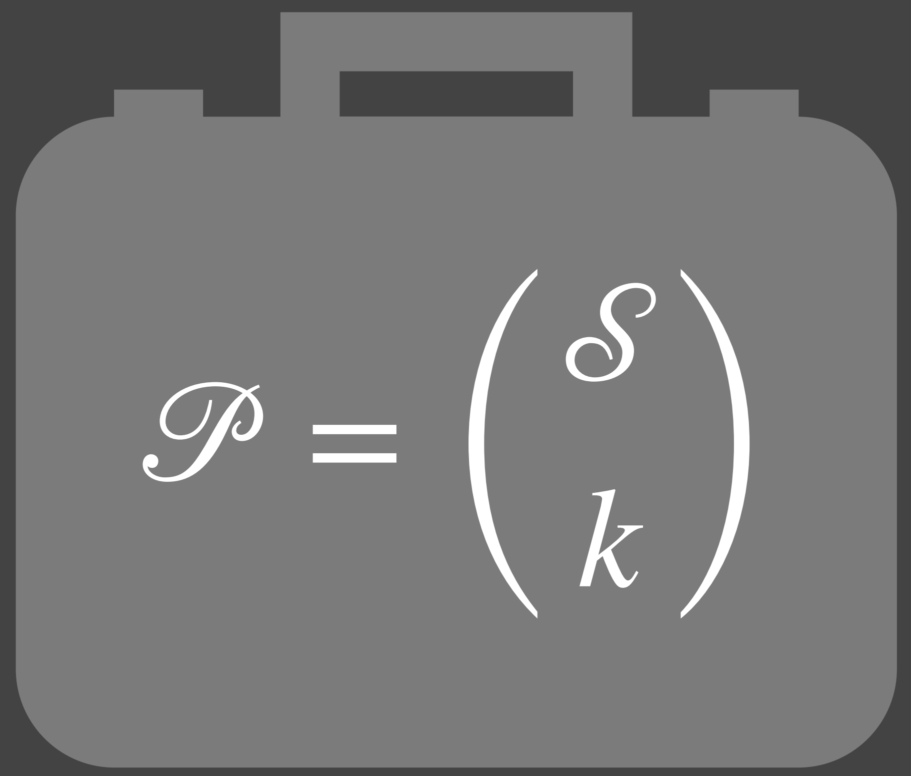

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R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

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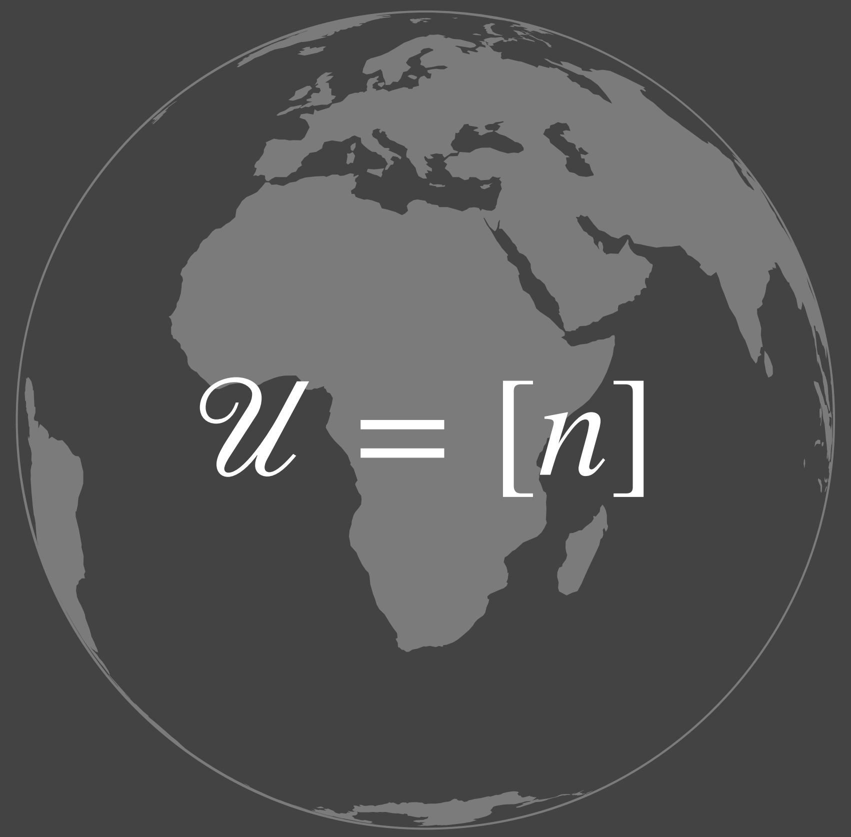
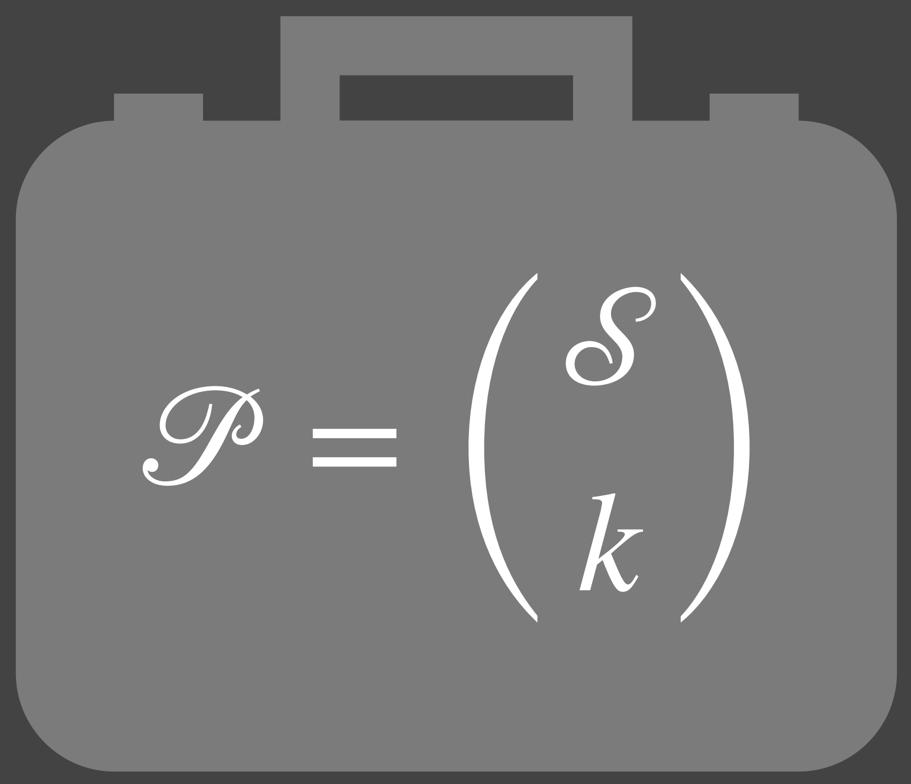
LearnOrCover

(Unit cost, exp time warmup)





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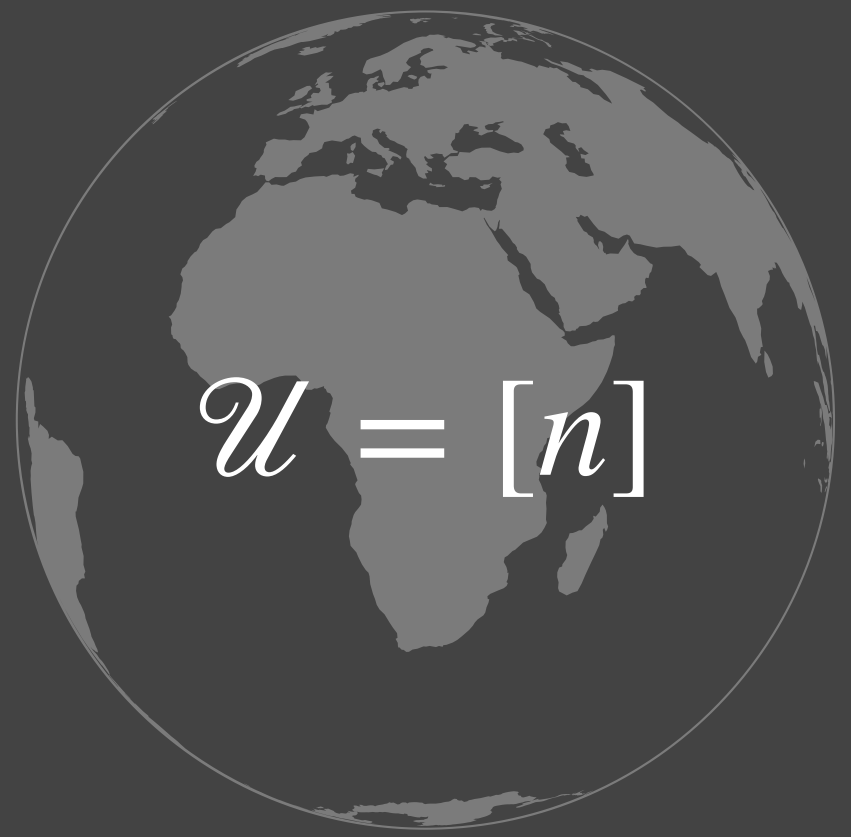

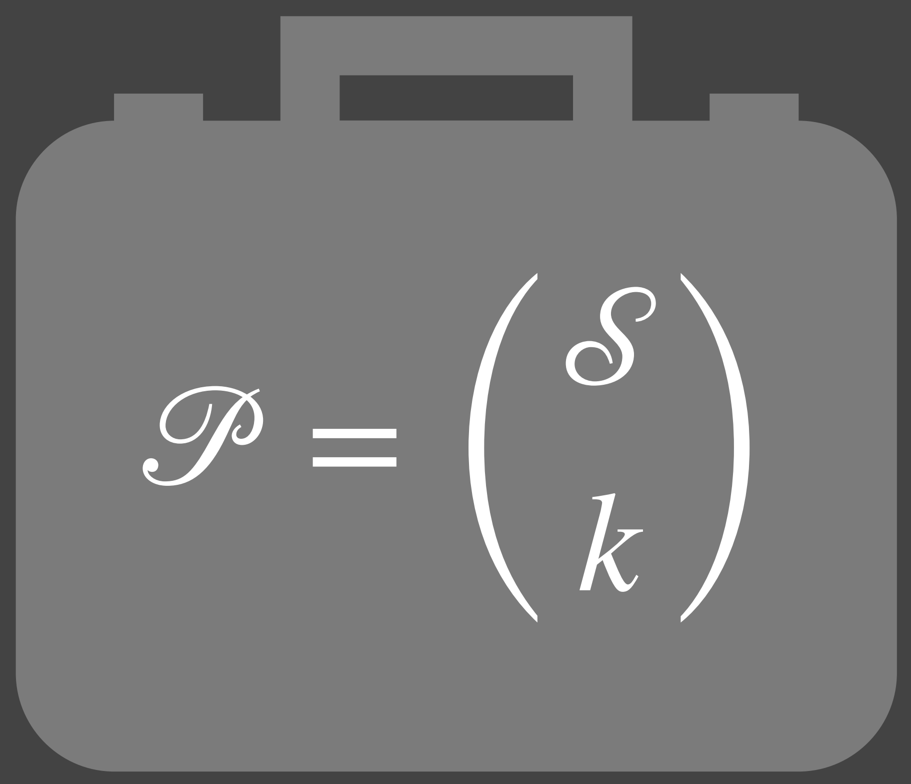

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LearnOrCover

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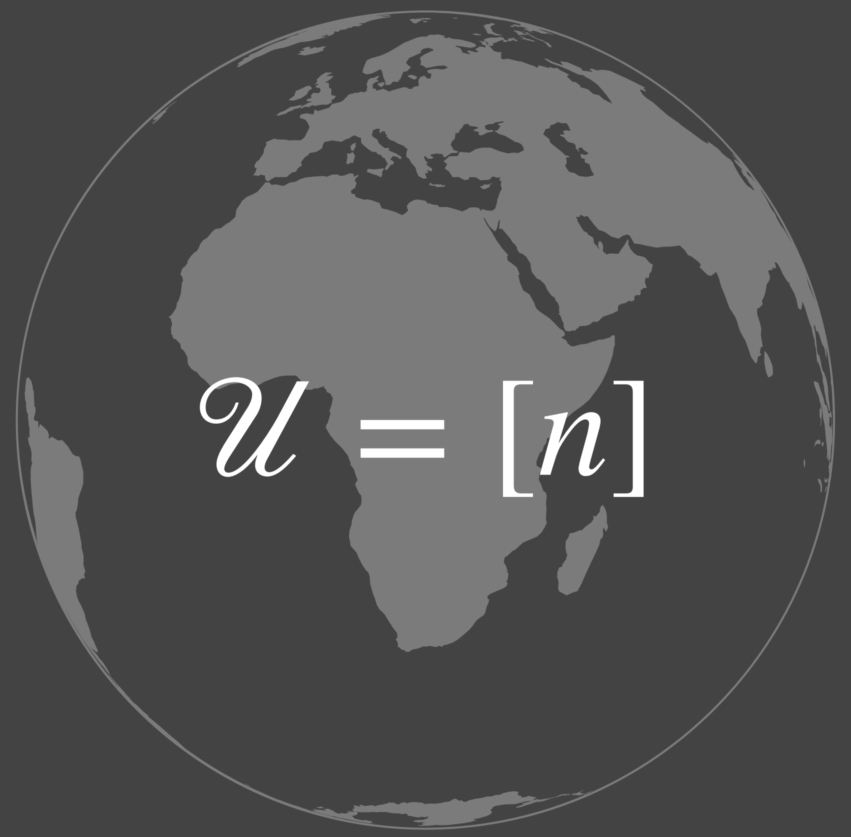

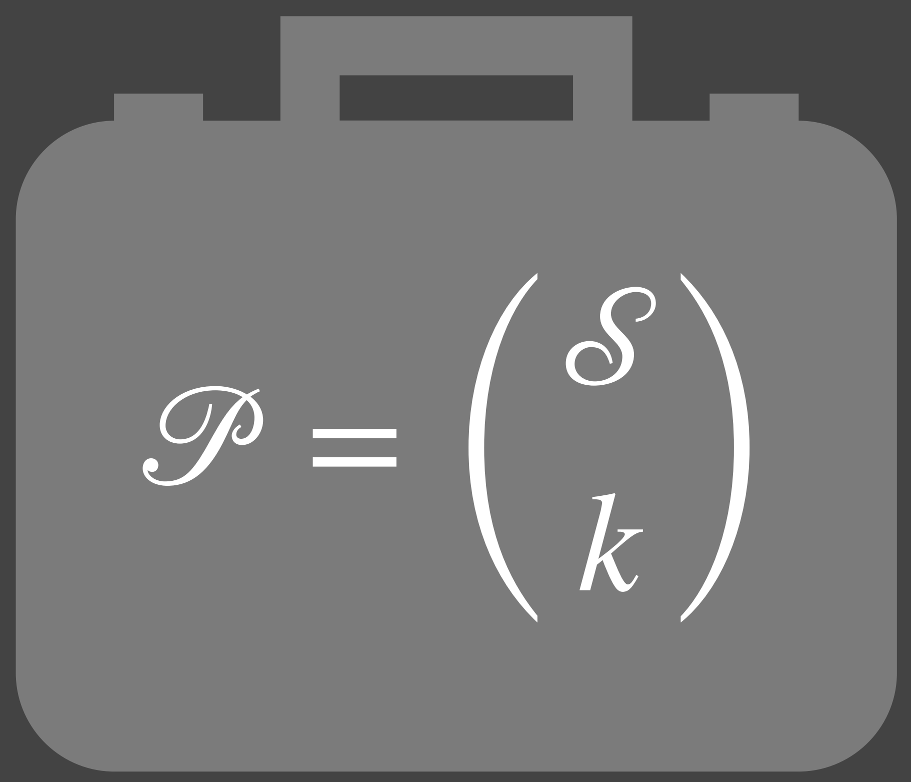

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LearnOrCover

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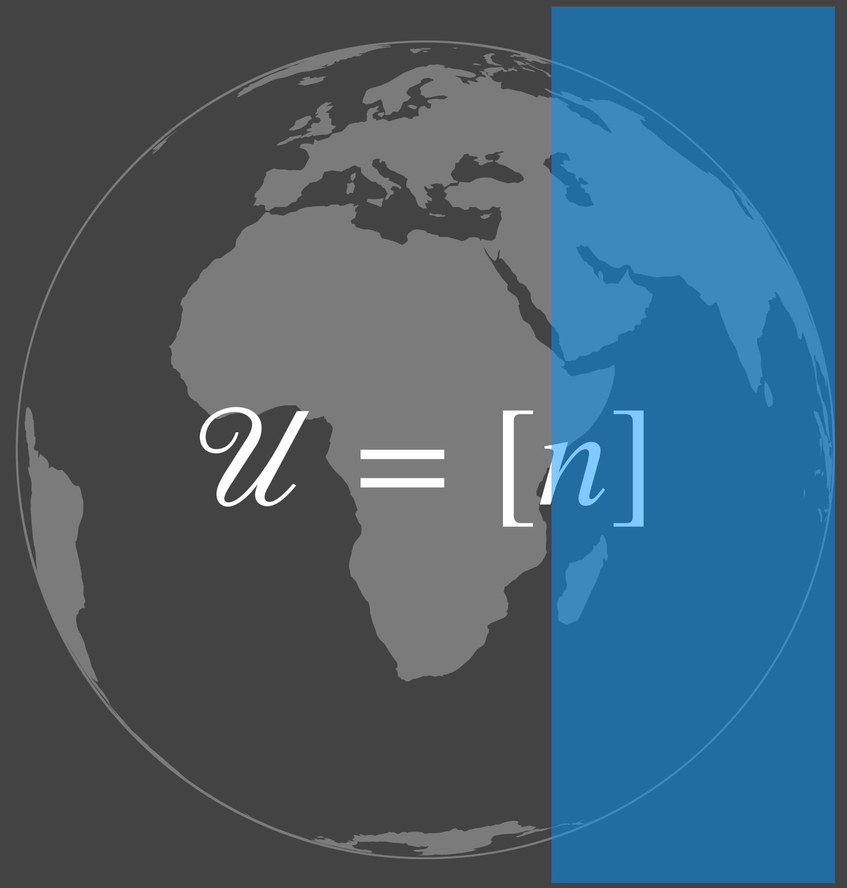

$\mathscr{U} = [n]$

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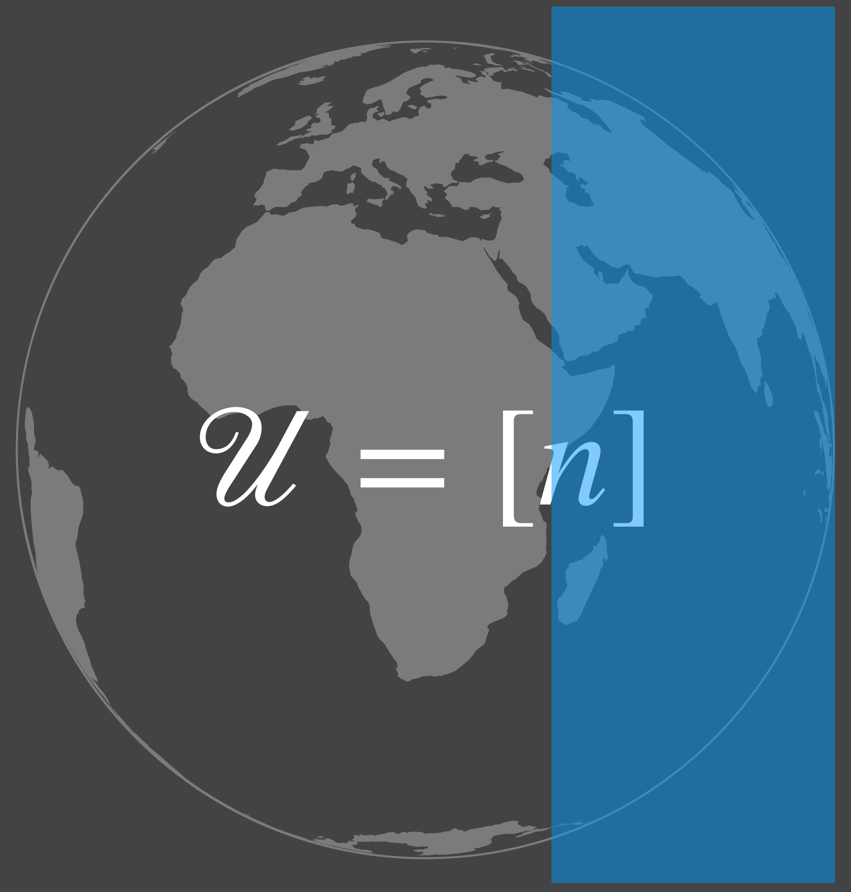
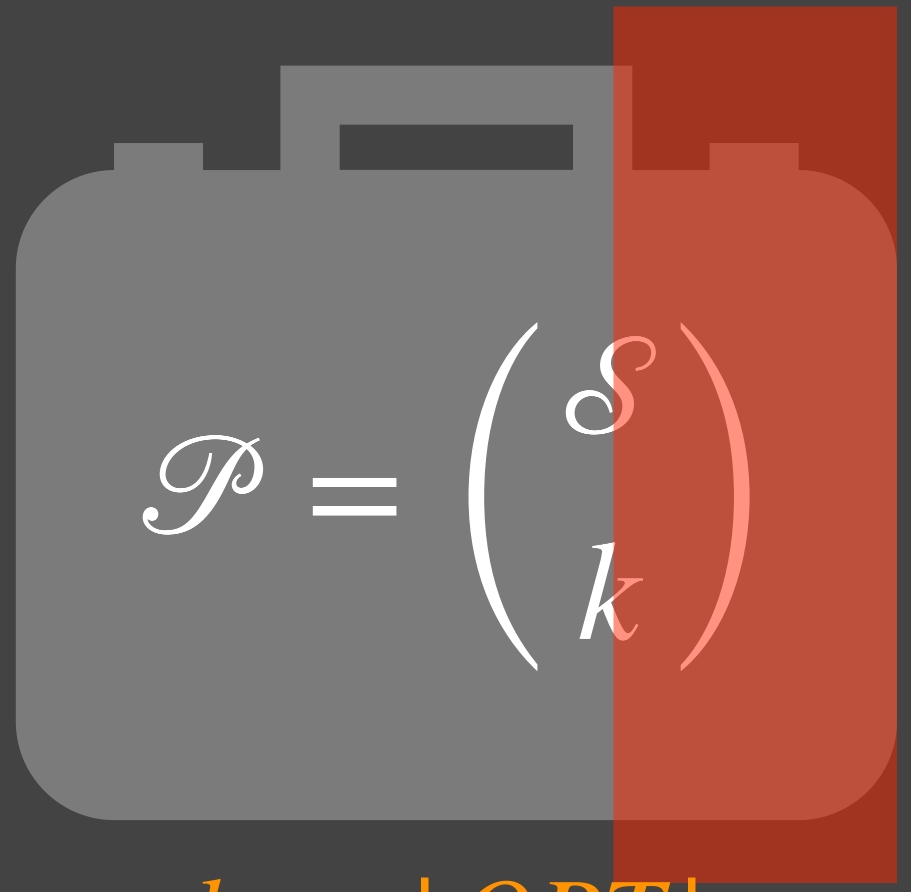
LearnOrCover
(Unit cost, exp time warmup)





$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.

(II) "Prune" $T \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ of $T \in \mathscr{P}$ cover $\geq 1/2$ of $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ of $T \in \mathscr{P}$ cover $< 1/2$ of $\mathscr{U}$.

$\geq 1/2$ of $T \in \mathscr{P}$ pruned w.p. $1/2$.

$\mathscr{P}$ shrinks by $3/4$ in expectation.

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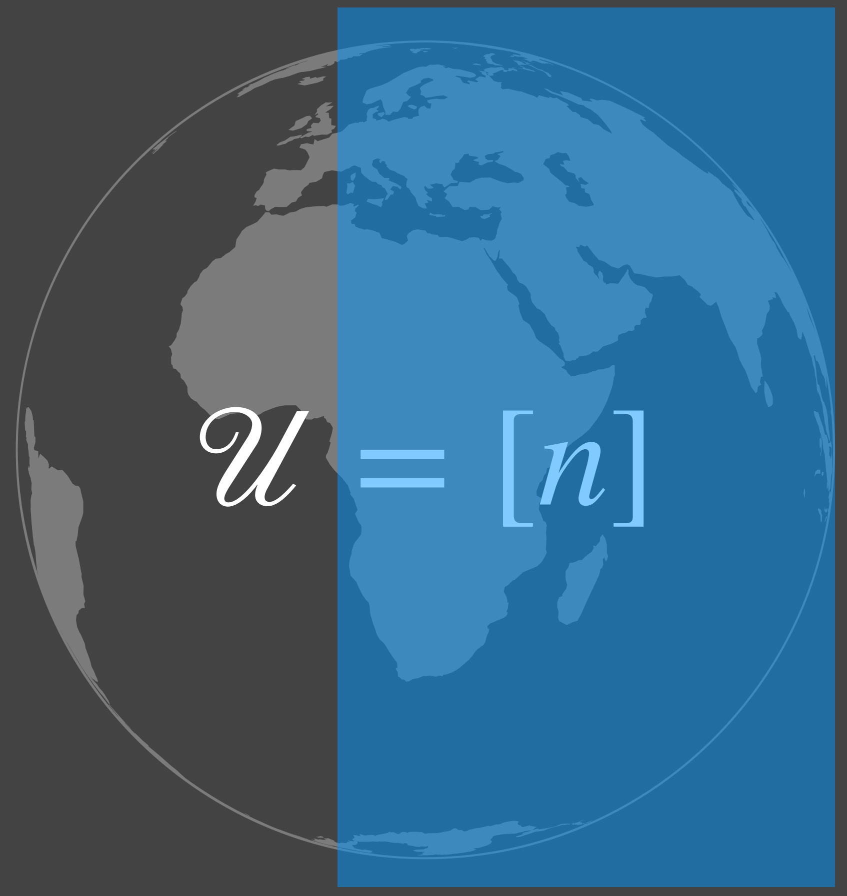

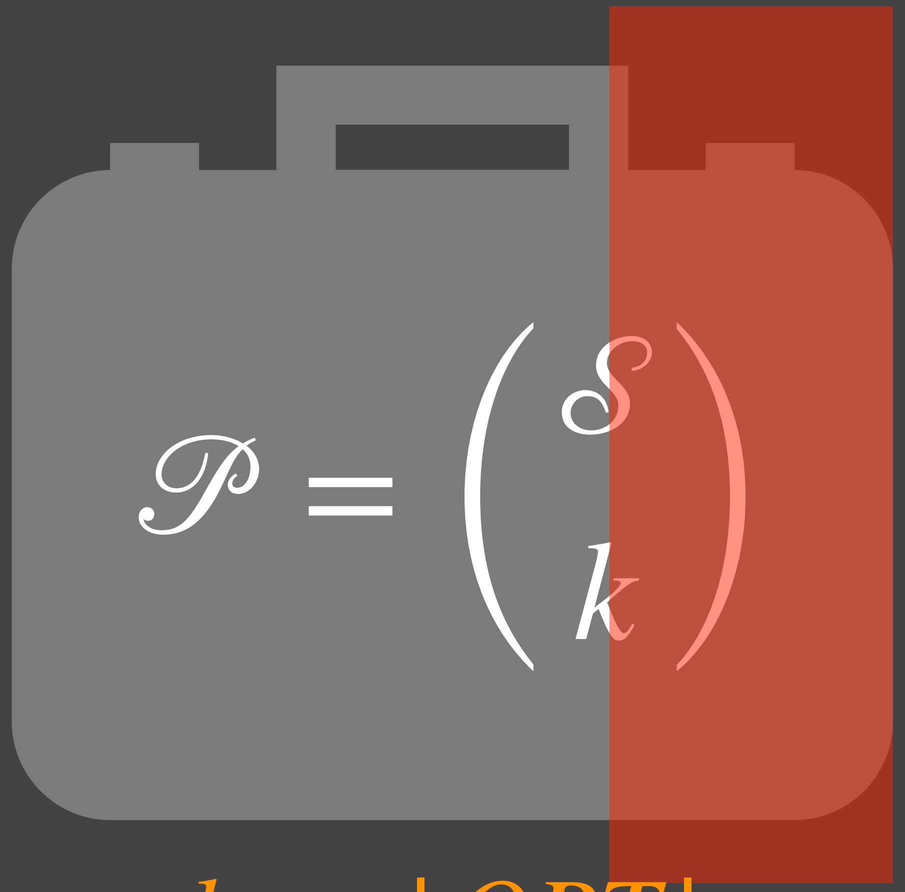

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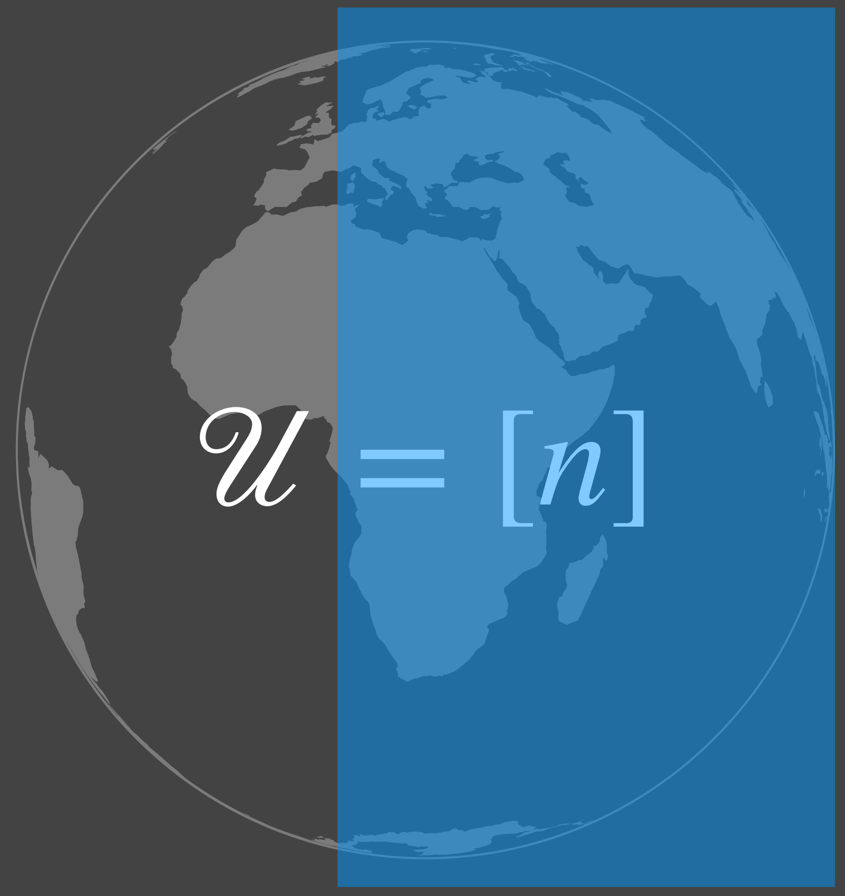

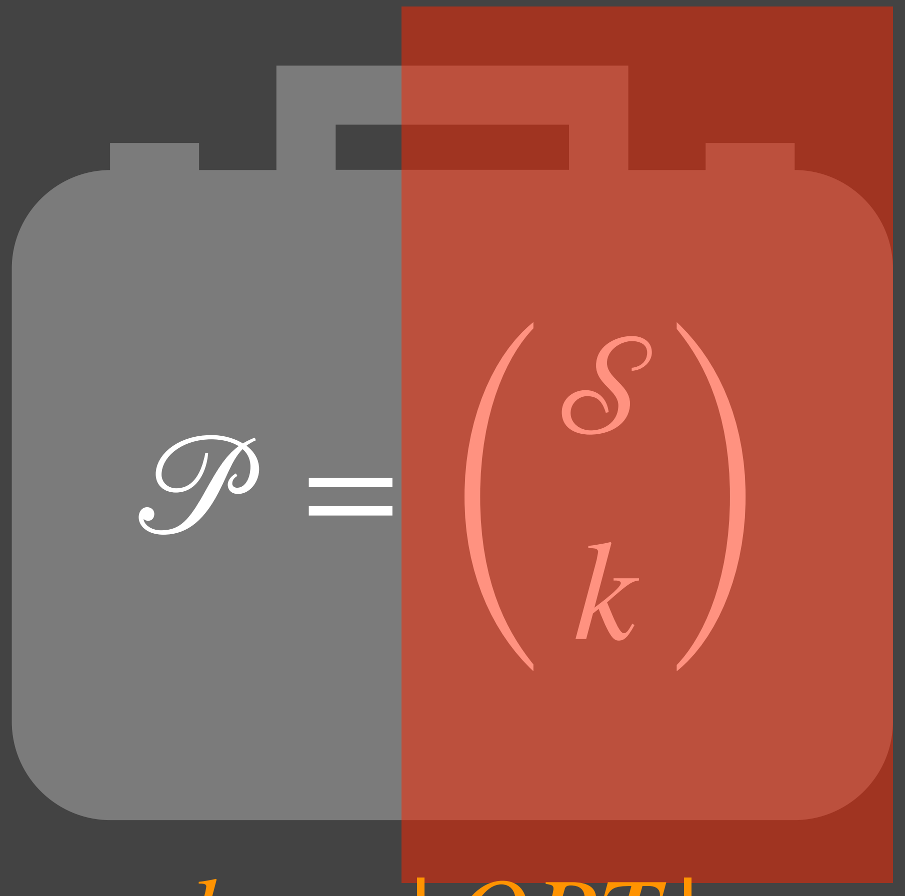

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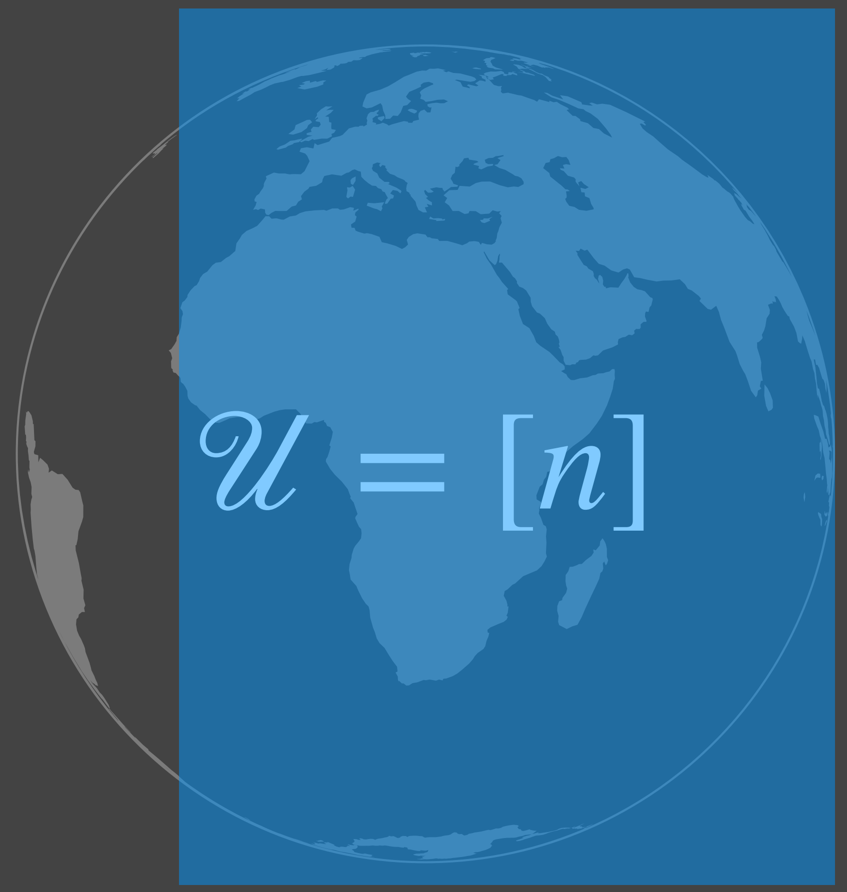

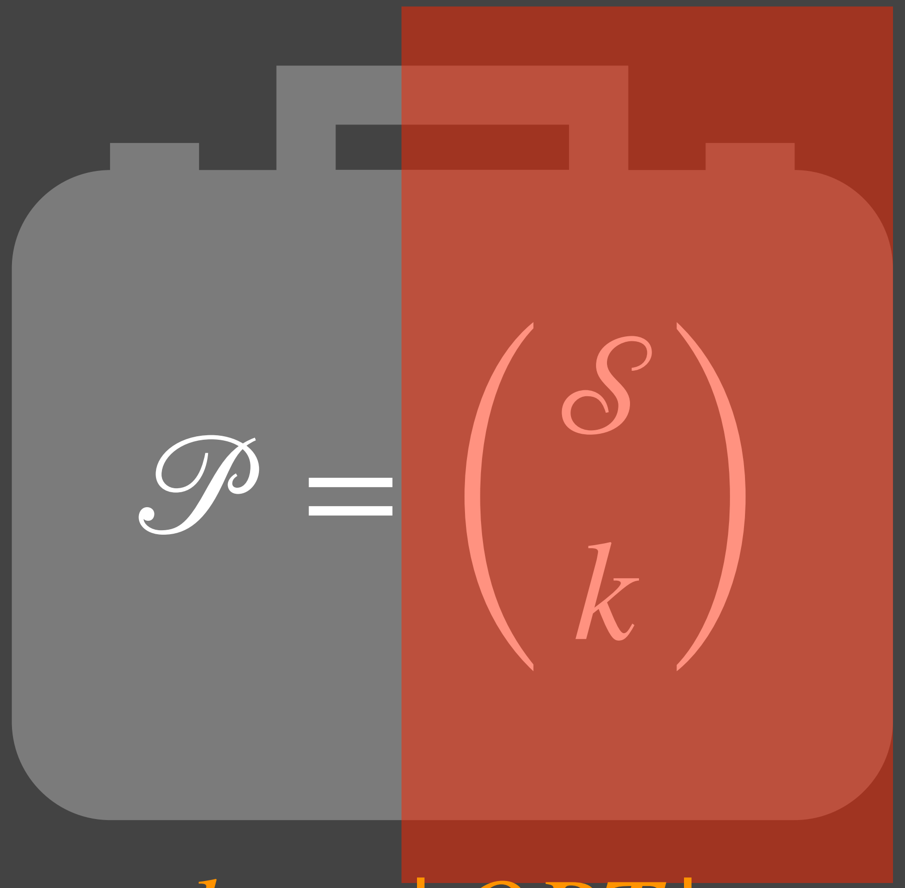

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RO Set Cover
(Exponential Time Warmup)

Case 1: (COVER)

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Case 2: (LEARN)

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$|\mathscr{U}|$ initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

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$|\mathscr{P}|$ initially $\binom{m}{k} \approx m^k$, $\Rightarrow$ $O(k \log m)$ LEARN steps suffice.

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$\Rightarrow O(k \log mn)$ steps suffice.

But how to make polytime?

Can we reuse LEARN/COVER intuition?

Talk Outline

Intro

Previous Work

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

Extensions & Lower Bounds

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Extensions & Lower Bounds

LearnOrCover
(Unit cost)

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Renormalize $x \leftarrow x/\|x\|_1$.

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Idea! Measure convergence with potential function:

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$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) + c_2 \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

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Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

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(Recall $k = |OPT|$)

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$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$

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Bound $E_R[\Delta \log |\mathcal{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v.

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Bound $E_R[\Delta \log |\mathscr{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v. $\longleftarrow$ This is where we use RO!

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

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Proof:

$$\mathbf{KL}(x^* \,||\, x^t) - \mathbf{KL}(x^* \,||\, x^{t-1})$$

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Proof:

$$\sum_X x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_X x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

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$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

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Claim 2a: If v^t uncovered,

$$\textcolor{red}{E_v[\Delta \mathrm{KL}]} \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

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$$= \sum_S x_S^* \log Z - \sum_{S \ni v} x_S^* \log e$$

Claim 2b: If v^t uncovered,

$$\textcolor{blue}{E_R[\Delta \log |\mathcal{U}^t|]} \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

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$$\leq \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

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Use $\log(1+z) \le z$, take expectation over v, $\blacksquare$.

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$$\leq \log\left(1+\sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \leq z$, take expectation over v, $\blacksquare$.

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \leq -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathcal{U}^t| - \log|\mathcal{U}^{t-1}|$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_X x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_X x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

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$$E_R[\Delta \log|\mathscr{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

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$$\log|\mathscr{U}^t| - \log|\mathscr{U}^{t-1}|$$

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Proof:

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$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathcal{U}^t|] \le -\frac{1}{k}$$

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$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1)\cdot E_R[\Delta \log|\mathcal{U}^t|] \leq -\frac{1}{k}$$

Since $\Phi(0) = O(\log(mn))$, expected total cost is $k\log(mn)$.

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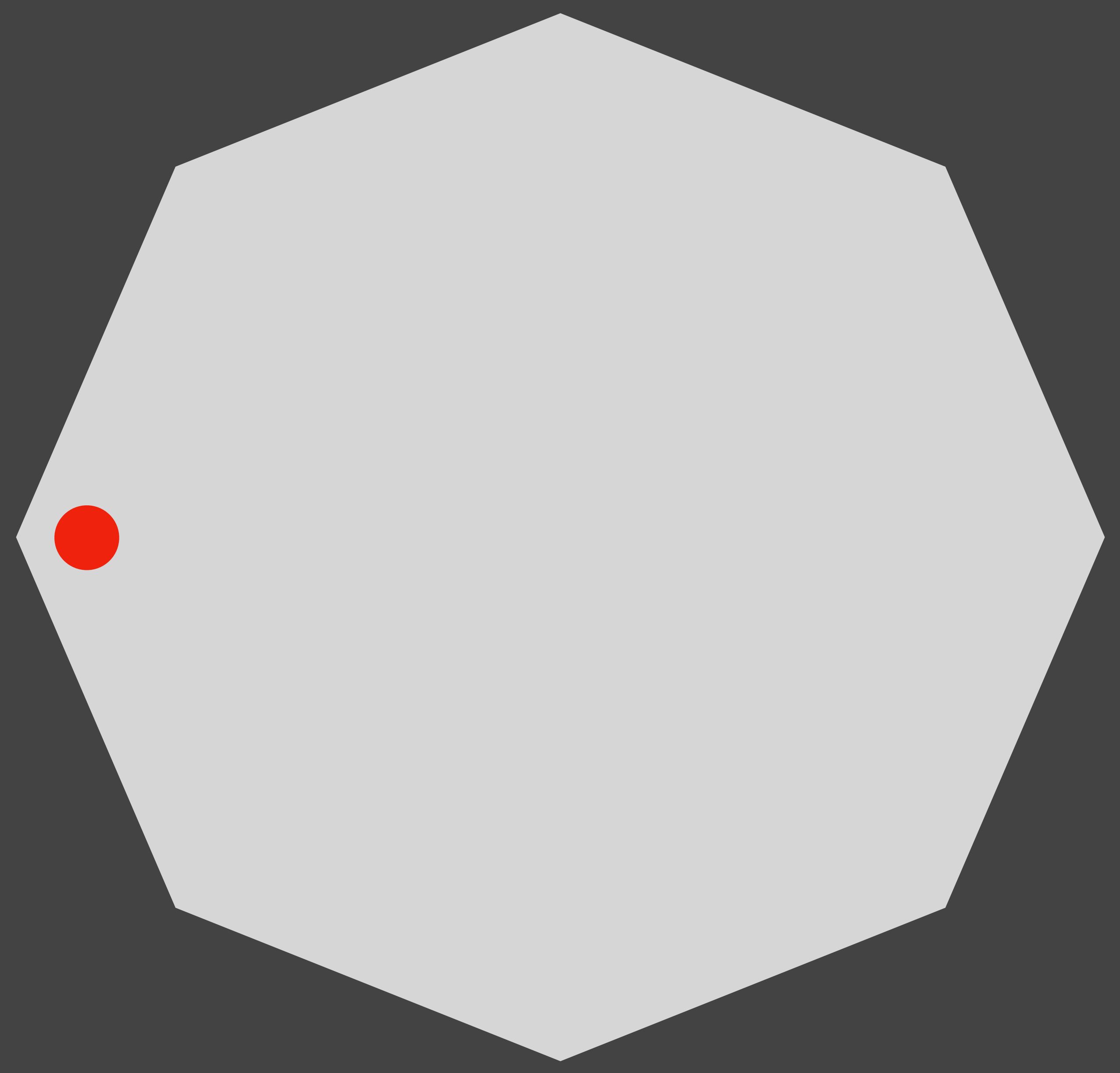
Perspective 1:



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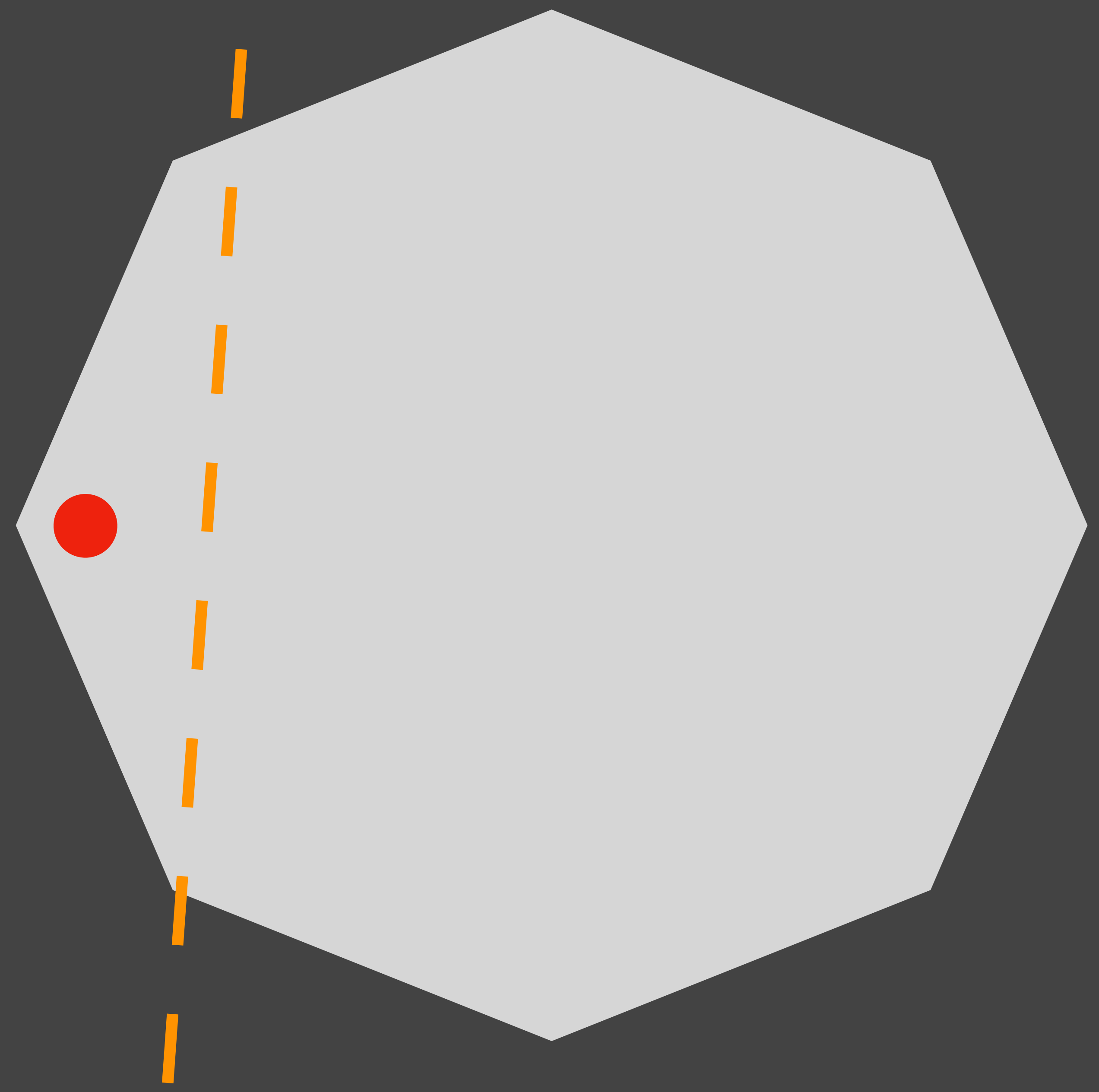

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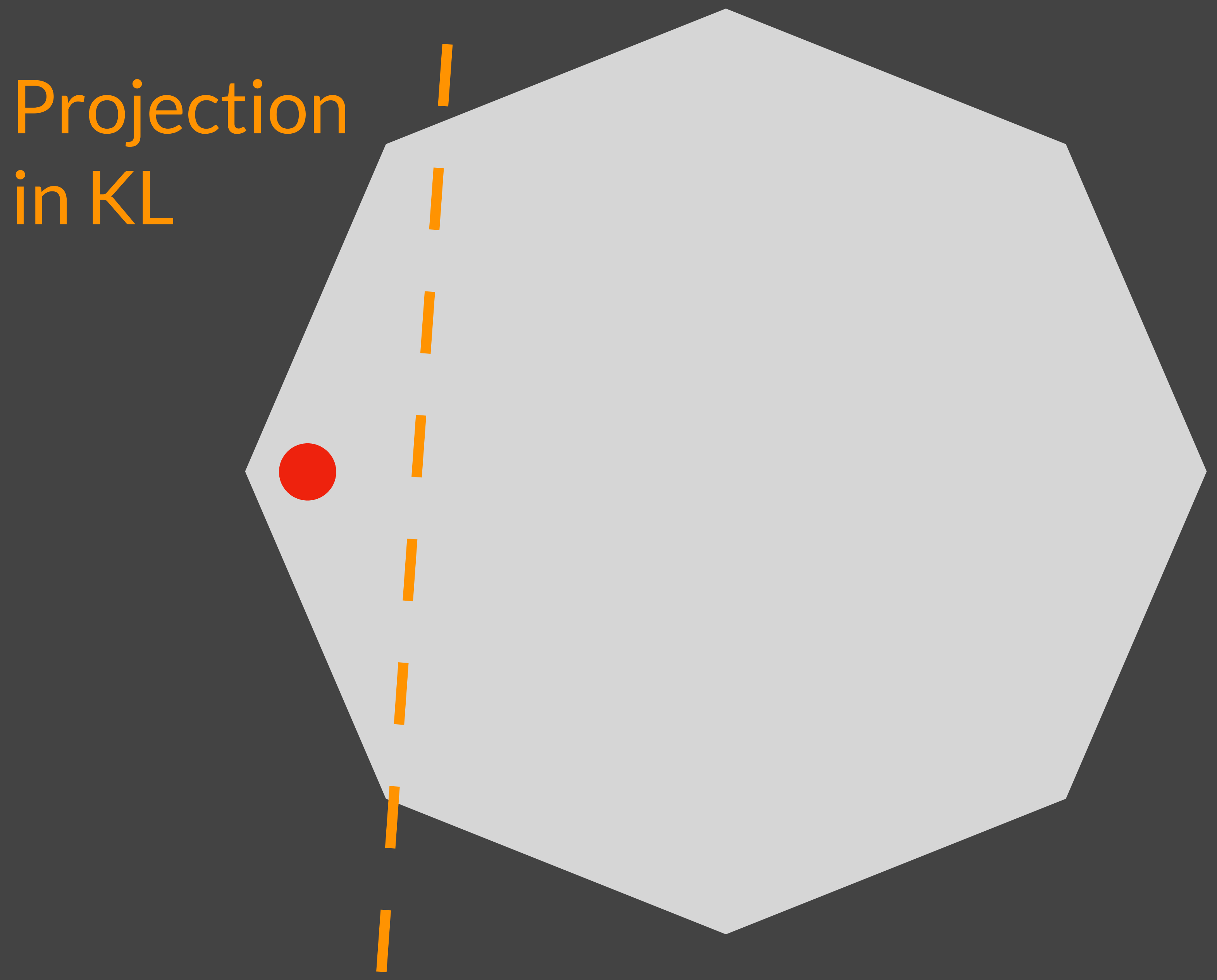

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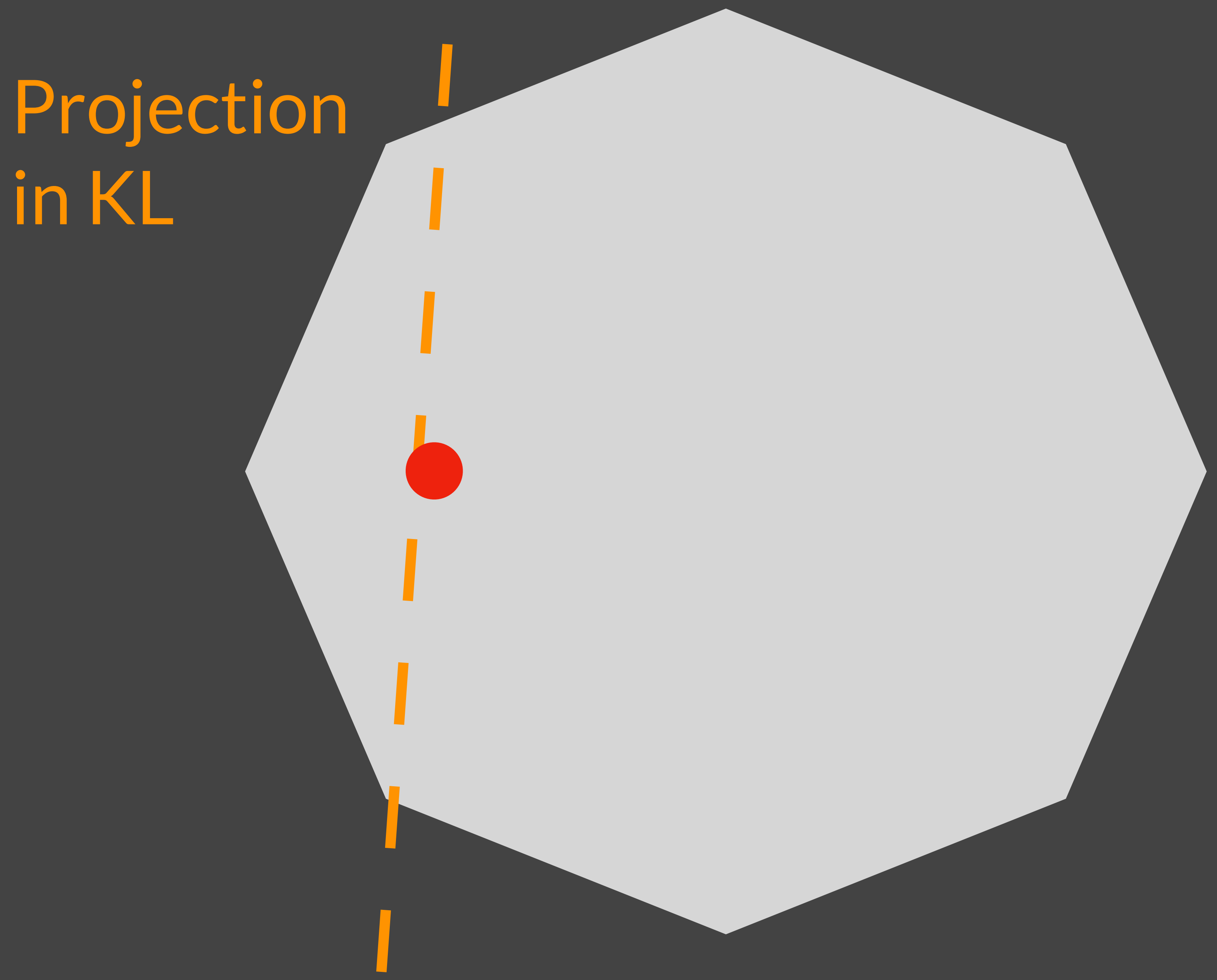

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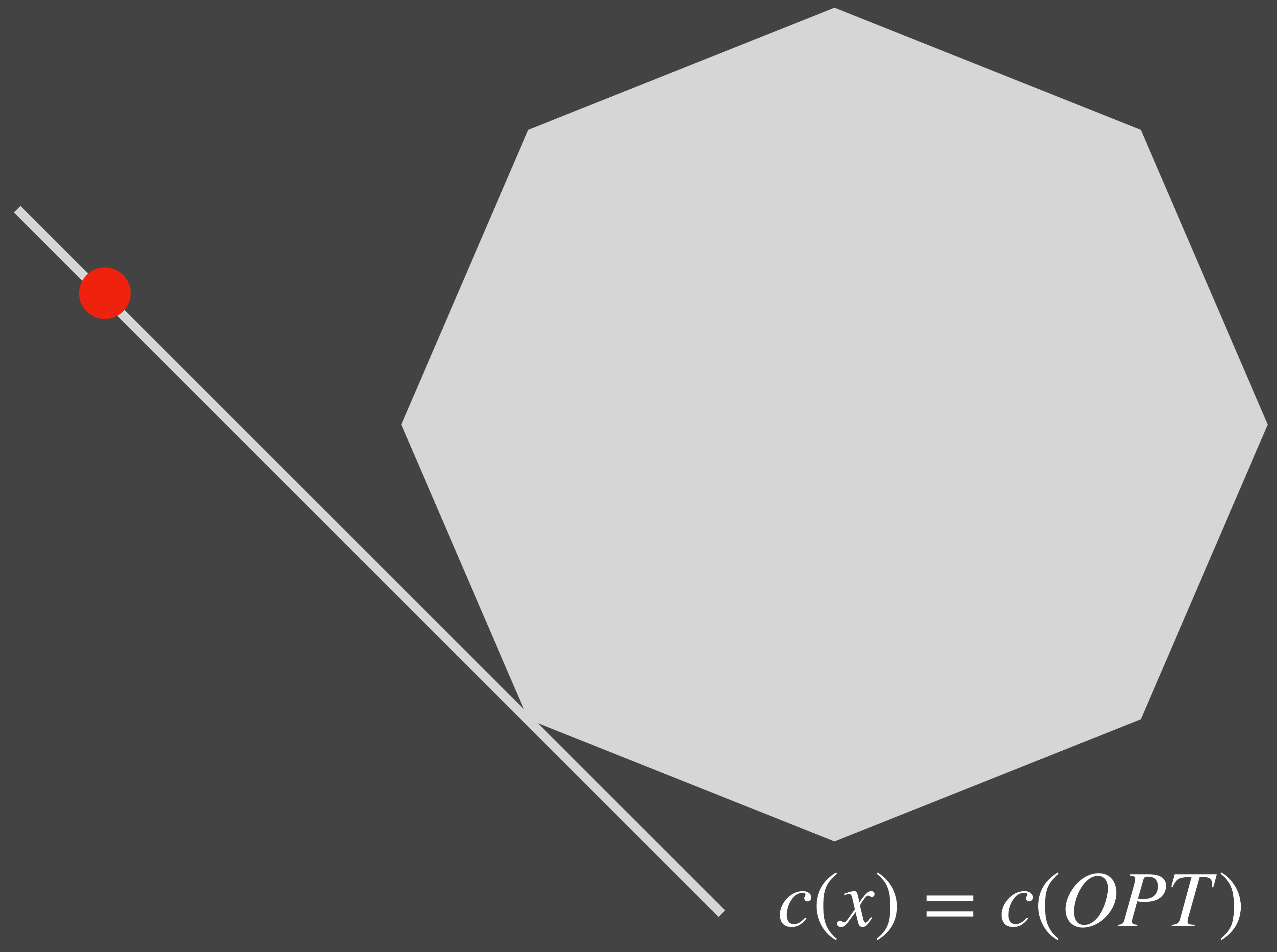

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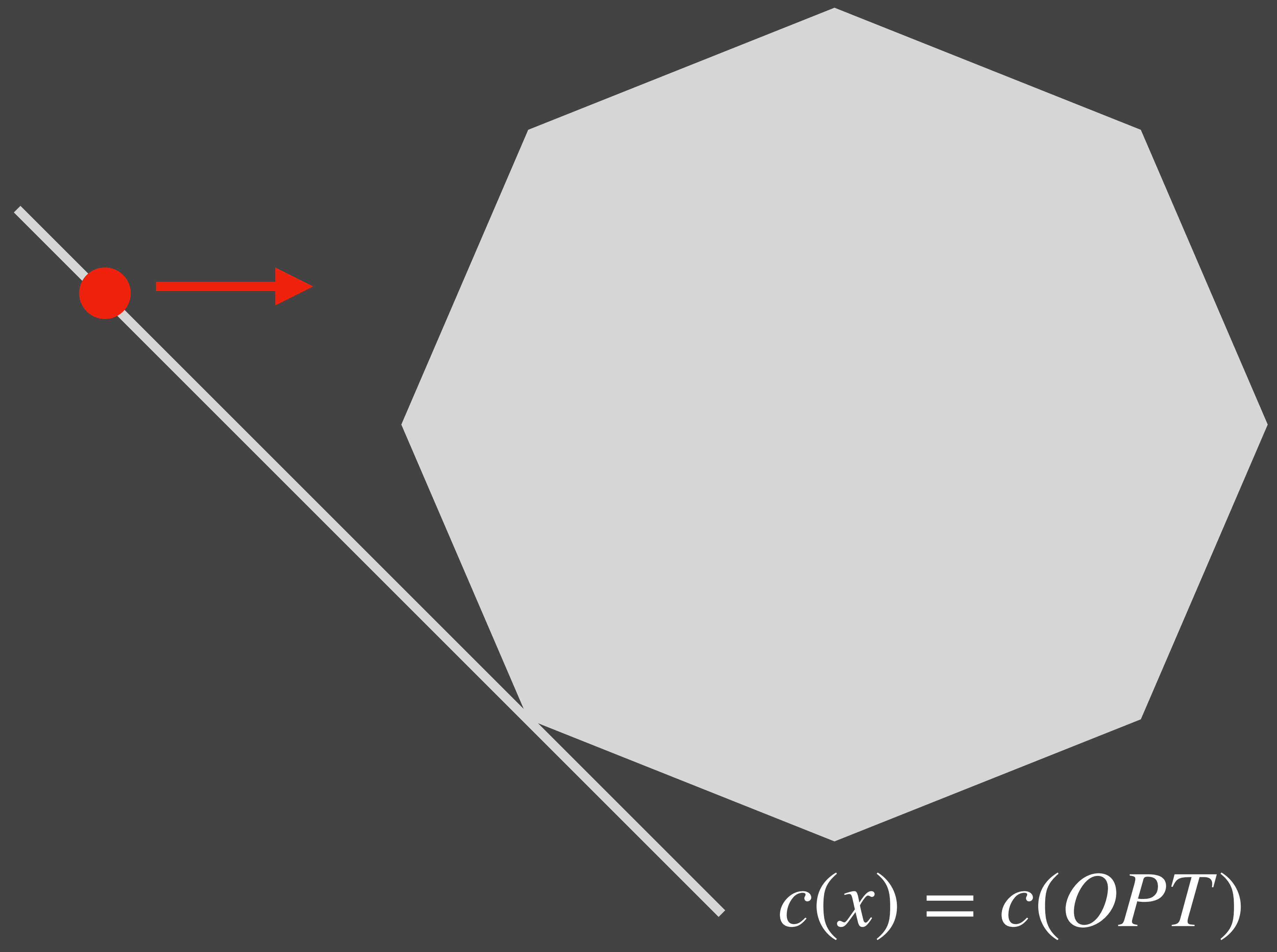

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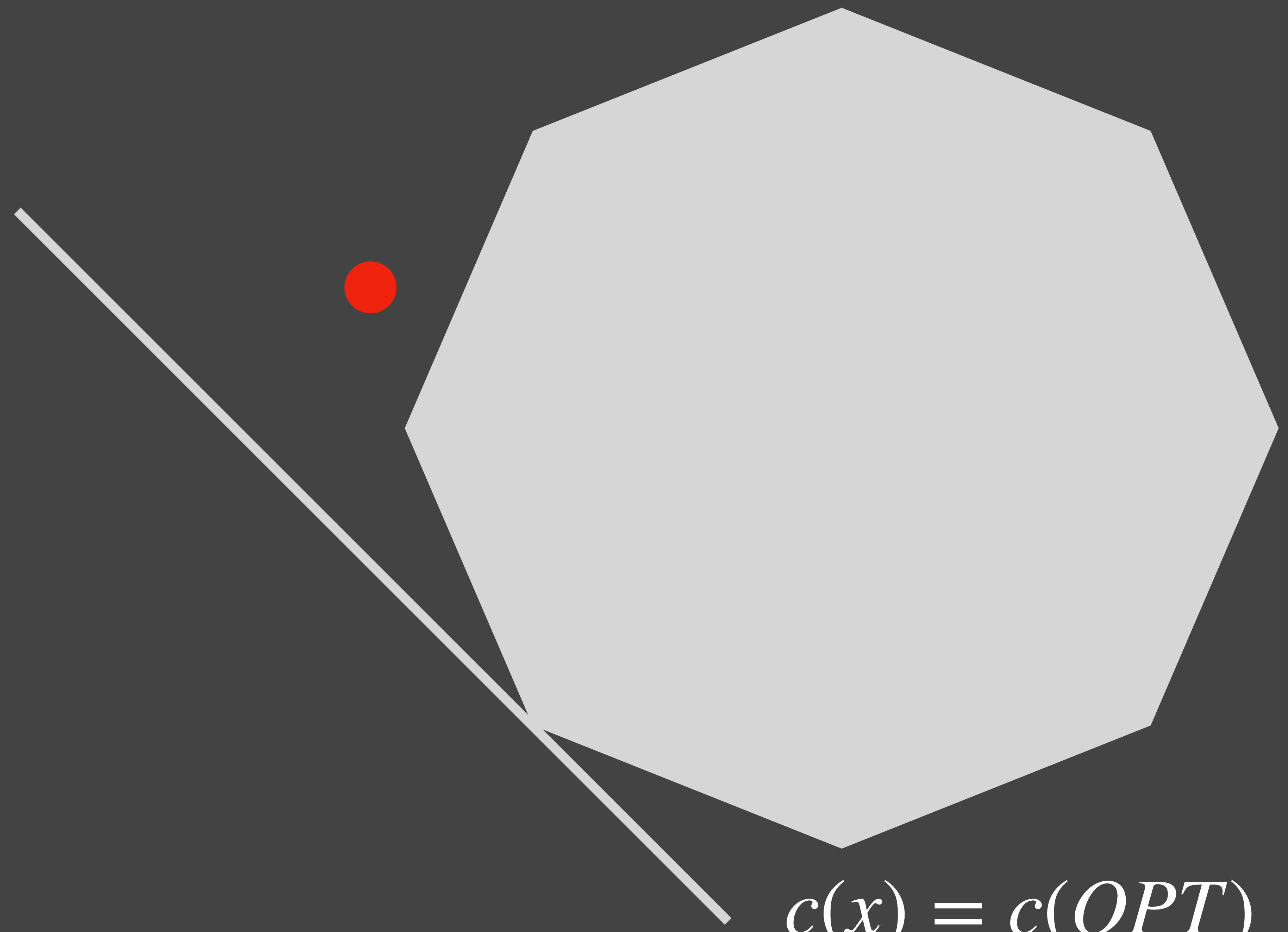

$c(x) = c(OPT)$

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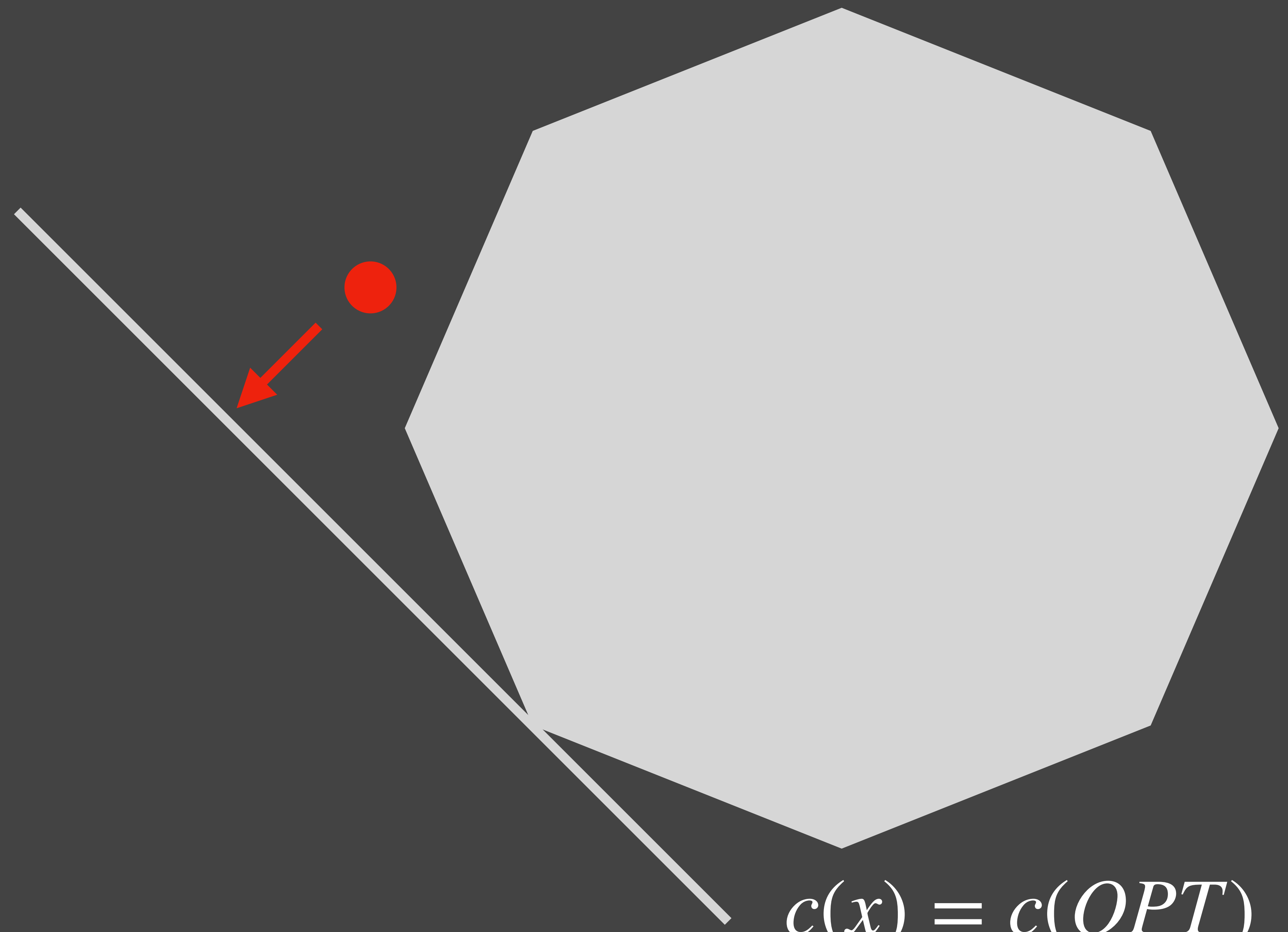

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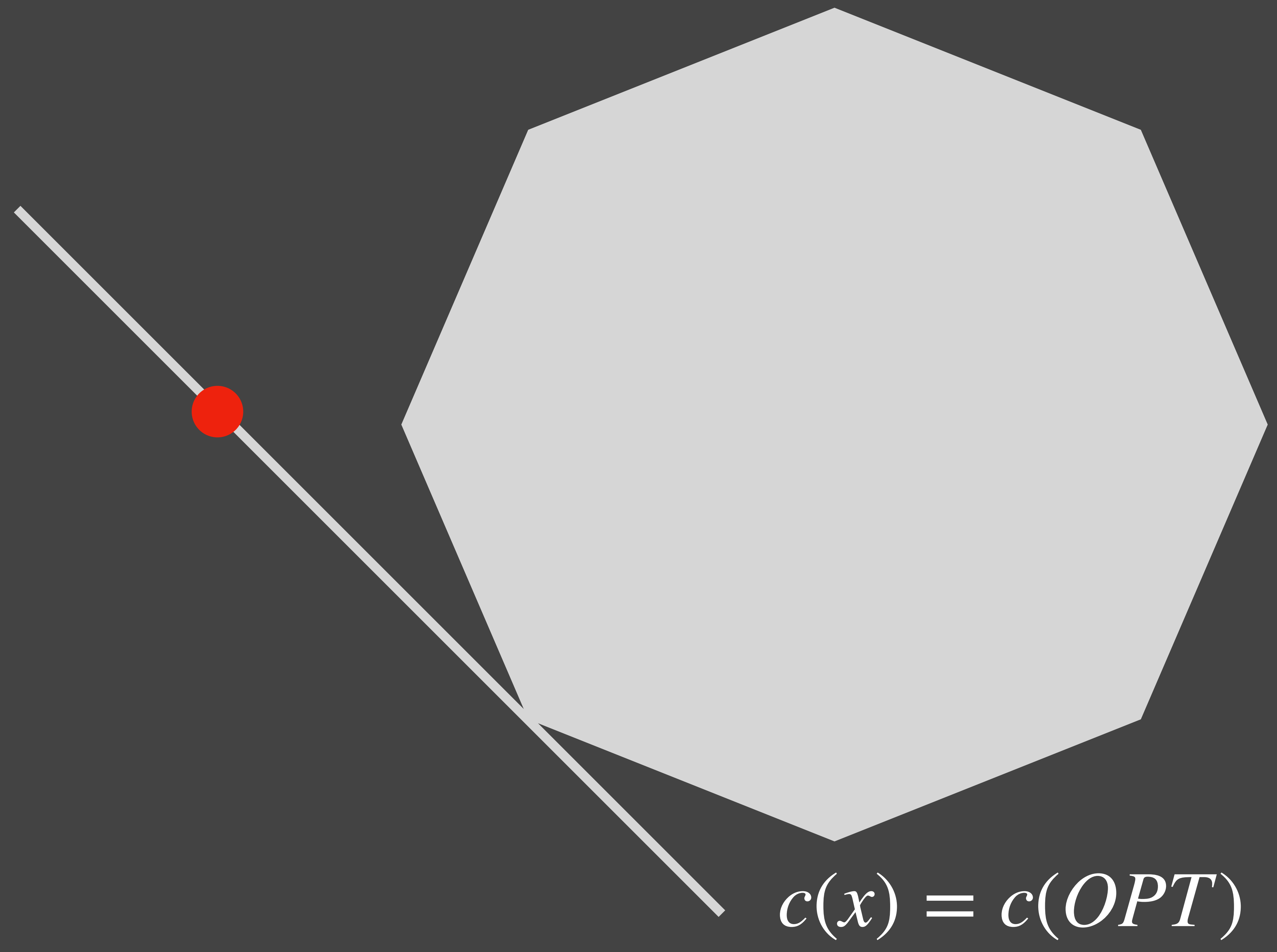

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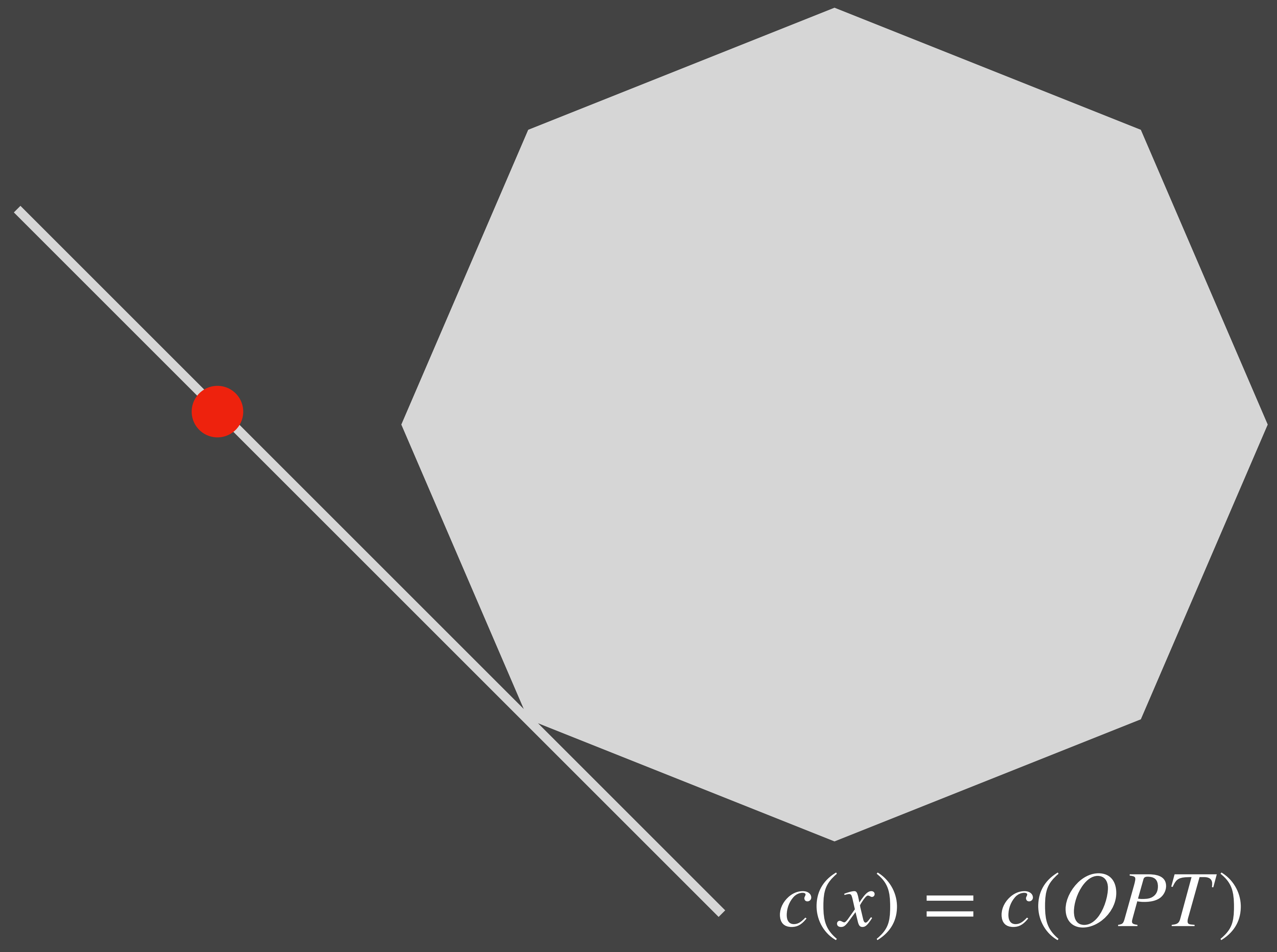

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Perspective 2:

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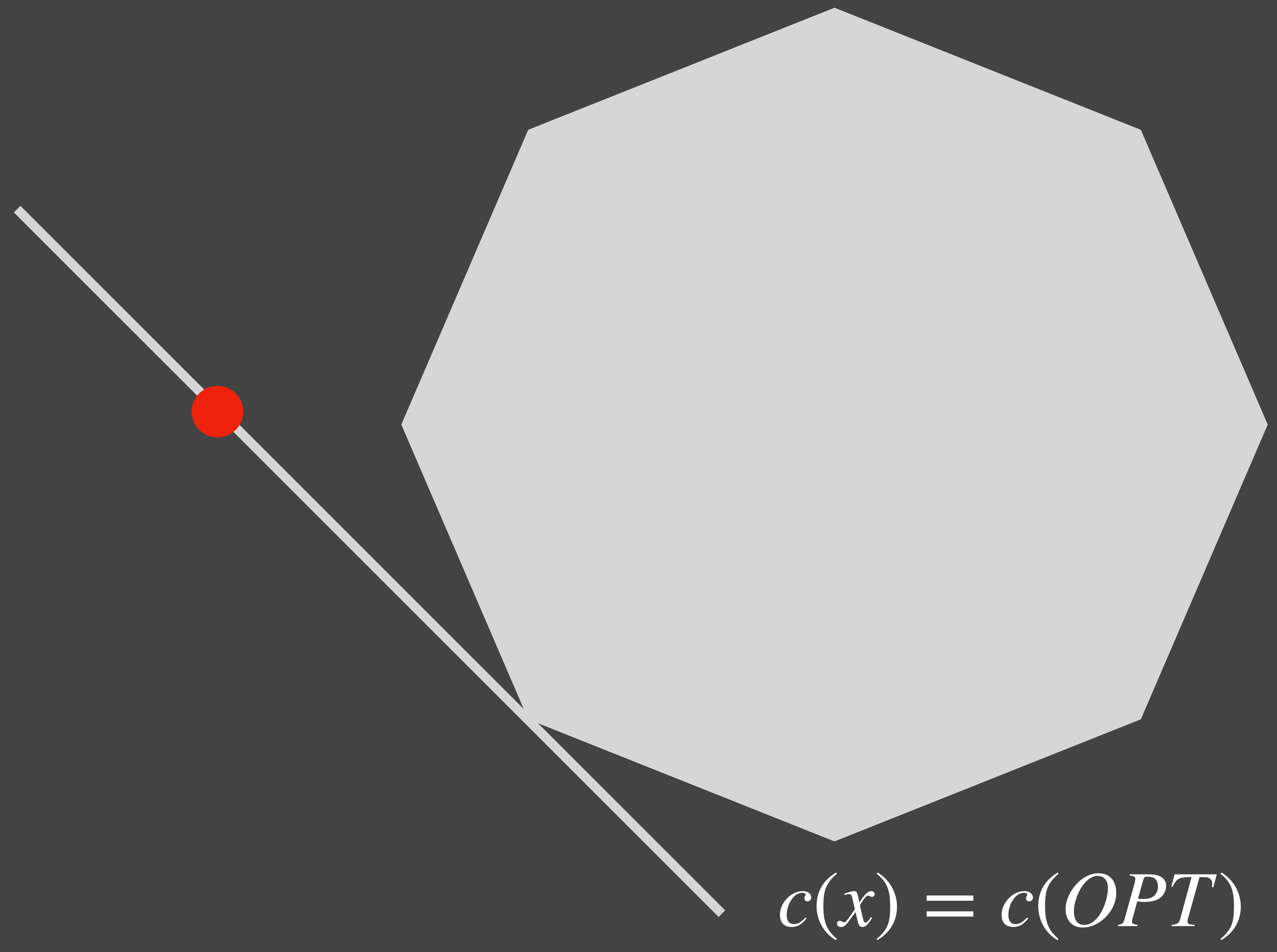

Perspective 2:

Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

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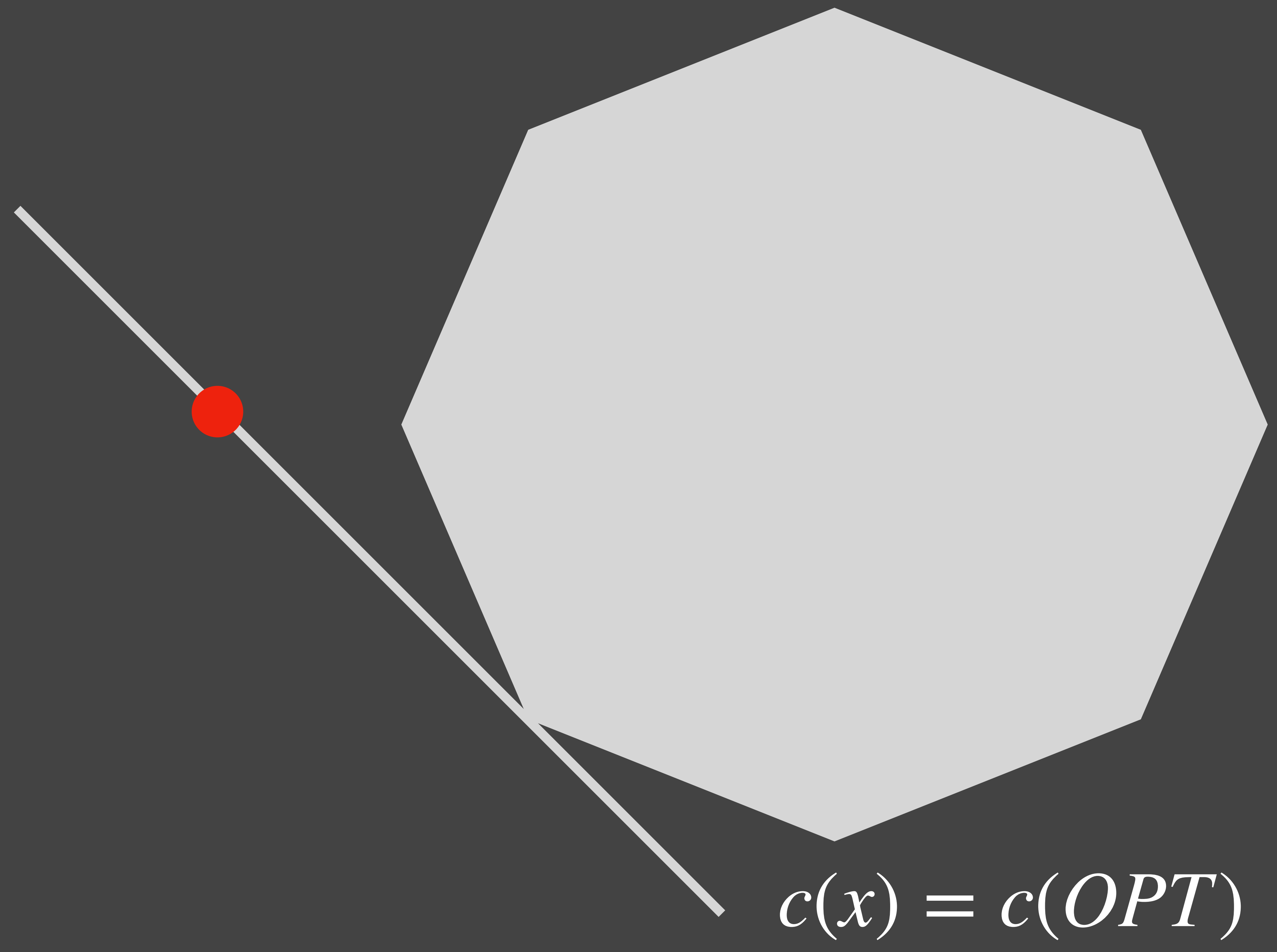

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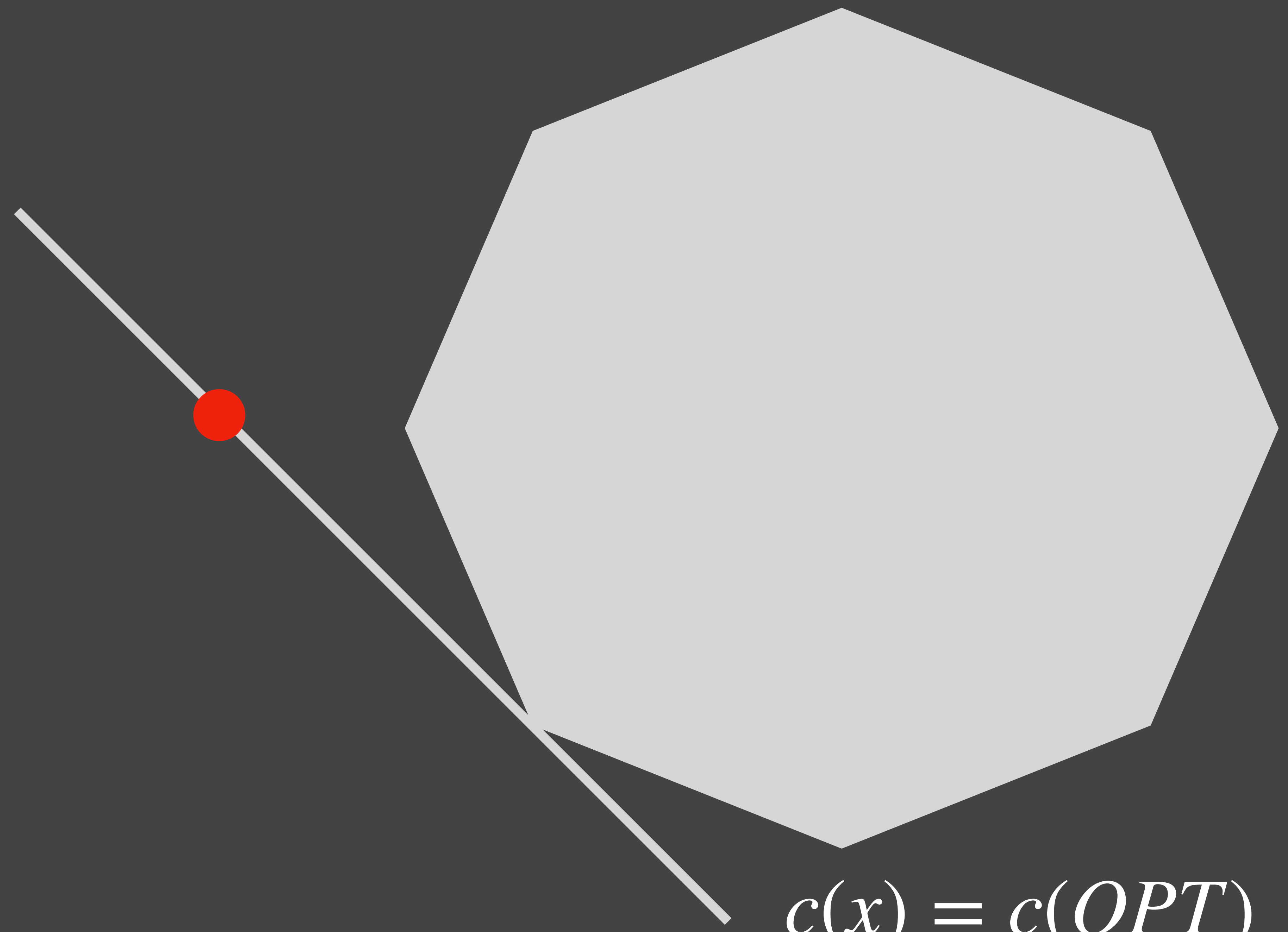

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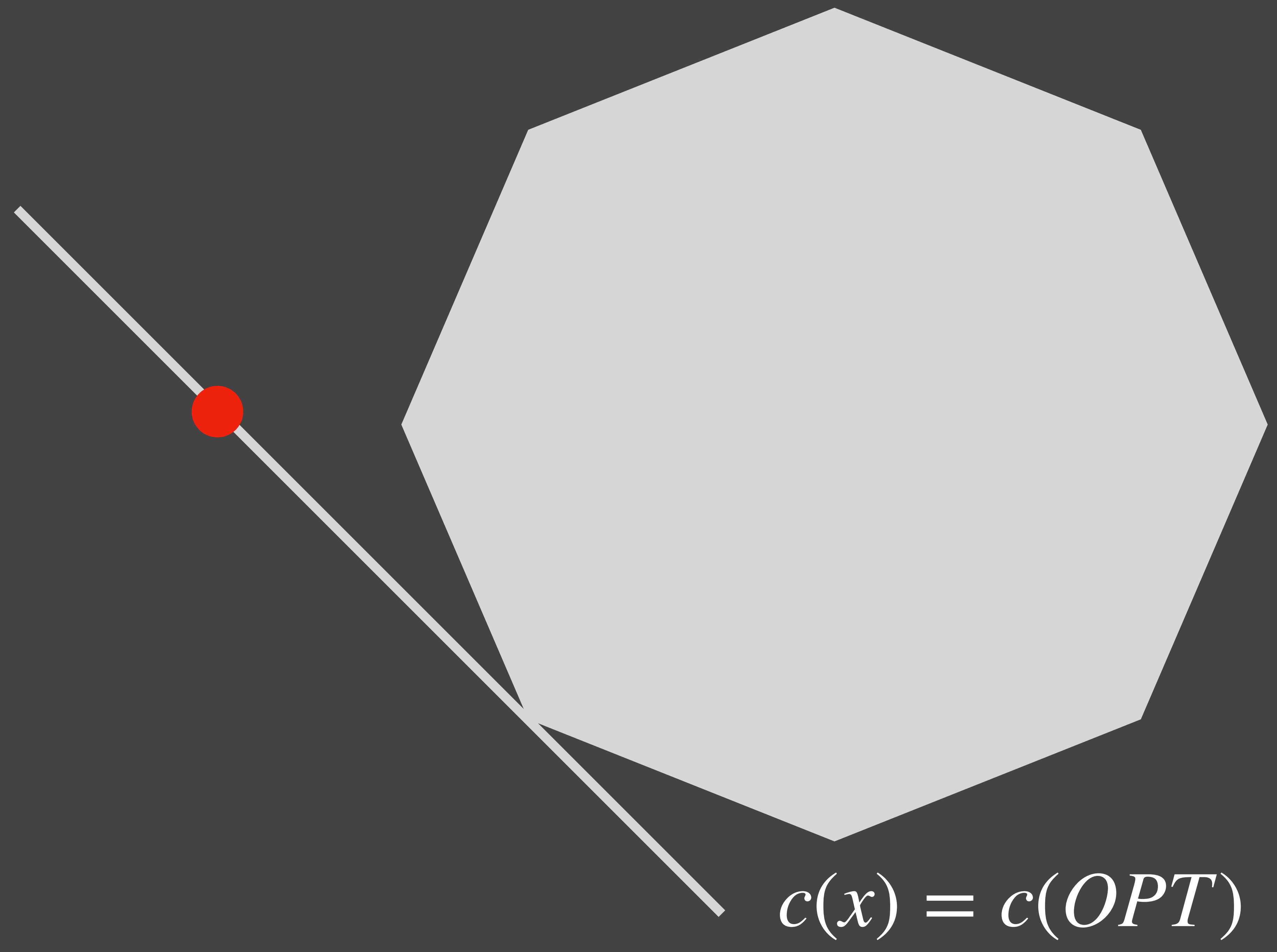

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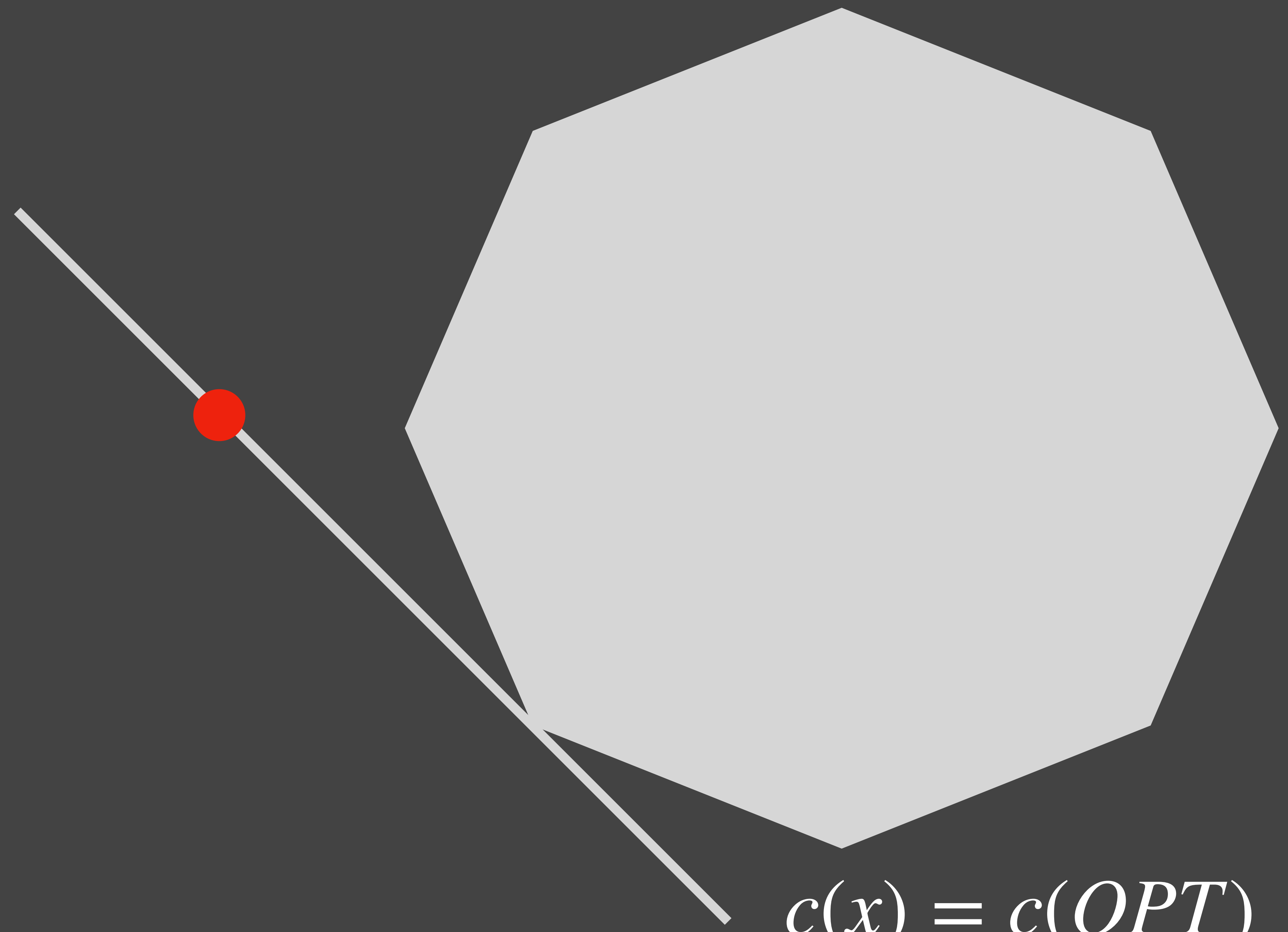

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RO reveals stochastic gradient…

LearnOrCover for non-unit costs

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Main issue: # uncovered elements not good proxy for cost.

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Init. $x_S \leftarrow \beta/(c_S \cdot m)$.

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If v covered, do nothing.

Else:

(I) Buy every set R w.p. $\kappa_v x_R/\beta$.

(II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.

Renormalize $x = \beta x/\langle c, x\rangle$.

Buy cheapest set to cover v.

Generalized potential:

$$\boxed{\Phi(t) = \mathrm{KL}_c(x^* \,||\, x^t) + \beta \log\left(\sum_{v \in \mathcal{U}^t} \frac{\kappa_v}{\beta}\right)}$$

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Main Idea: tune <u>learning</u> & <u>sampling</u> rates as a function of κ_v.

<u>Claim 1</u>: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

$\beta := c(OPT)$

κ_v := cost of cheapest set covering v

LearnOrCover

Init. $x_S \leftarrow \beta/(c_S \cdot m)$.

@ time t, element v arrives:

- If v covered, do nothing.
- Else:
  - (I) Buy every set R w.p. $\kappa_v x_R/\beta$.
  - (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
  - Renormalize $x = \beta x/\langle c, x\rangle$.
  - Buy cheapest set to cover v.

Generalized potential:

$$\boxed{\Phi(t) = \mathrm{KL}_c(x^* \,||\, x^t) + \beta \log\left(\sum_{v \in \mathcal{U}^t} \frac{\kappa_v}{\beta}\right)}$$

Main Idea: tune learning & sampling rates as a function of κ_v.

Claim 1: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

Claim 2: $E[\Delta\text{cost(ALG)}] = O(\kappa_v)$.

Talk Outline

Intro

Previous Work

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

Extensions & Lower Bounds

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➡ Extensions & Lower Bounds

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Nice question if this can be matched… best bound is $O(\log m \log(n \cdot f(\mathcal{N})))$ [Gupta **L.** 20].

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Corollary: $O(\log mn)$ for Online Set Cover with-a-sample.

Proof Idea: Run LearnOrCover on the sampled half, buy cheapest set containing any remaining elements from adversarial half.

Future work

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Unified theory? Reinterpret old results as LearnOrCover?

Thanks!