TEL AVIV UNIVERSITY

Online Covering

Secretaries, Prophets, and Universal Maps

FOCS 2021 + Forthcoming Work
Roie Levin

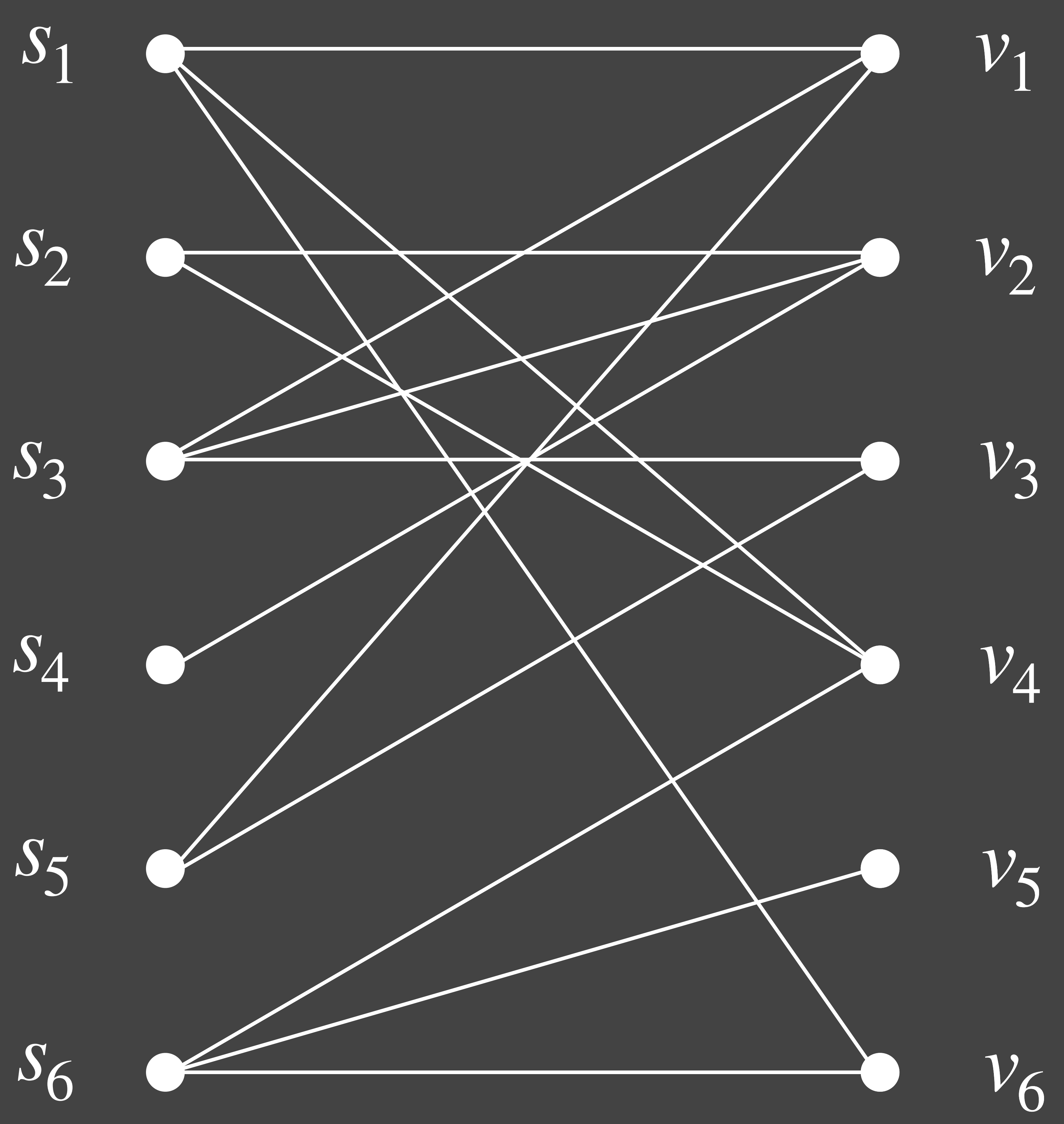
Anupam Gupta (CMU)

Gregory Kehne (Harvard)

Set Cover

$\mathcal{S}$

m sets



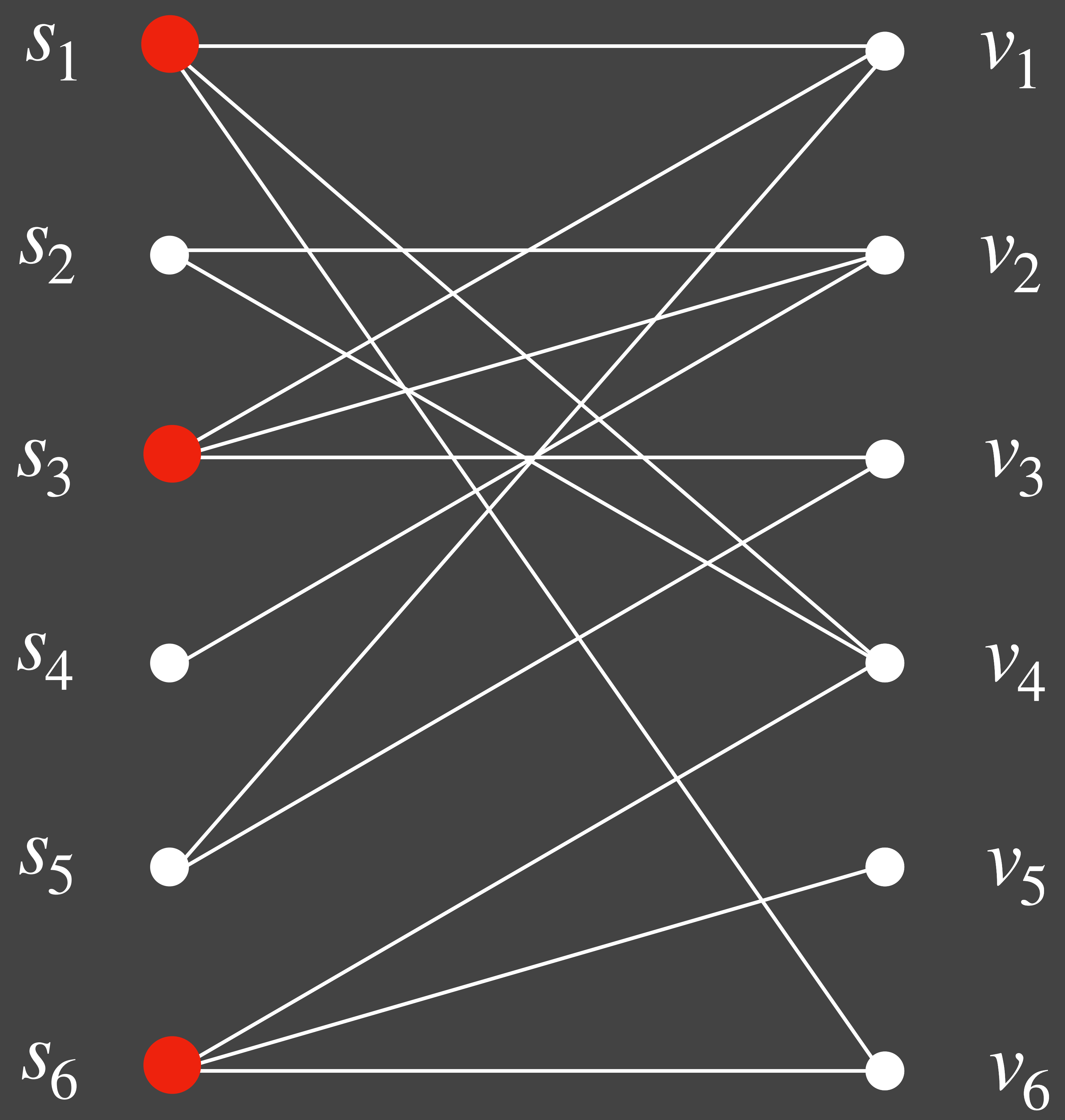
$\mathcal{U}$

n elements

Set Cover

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

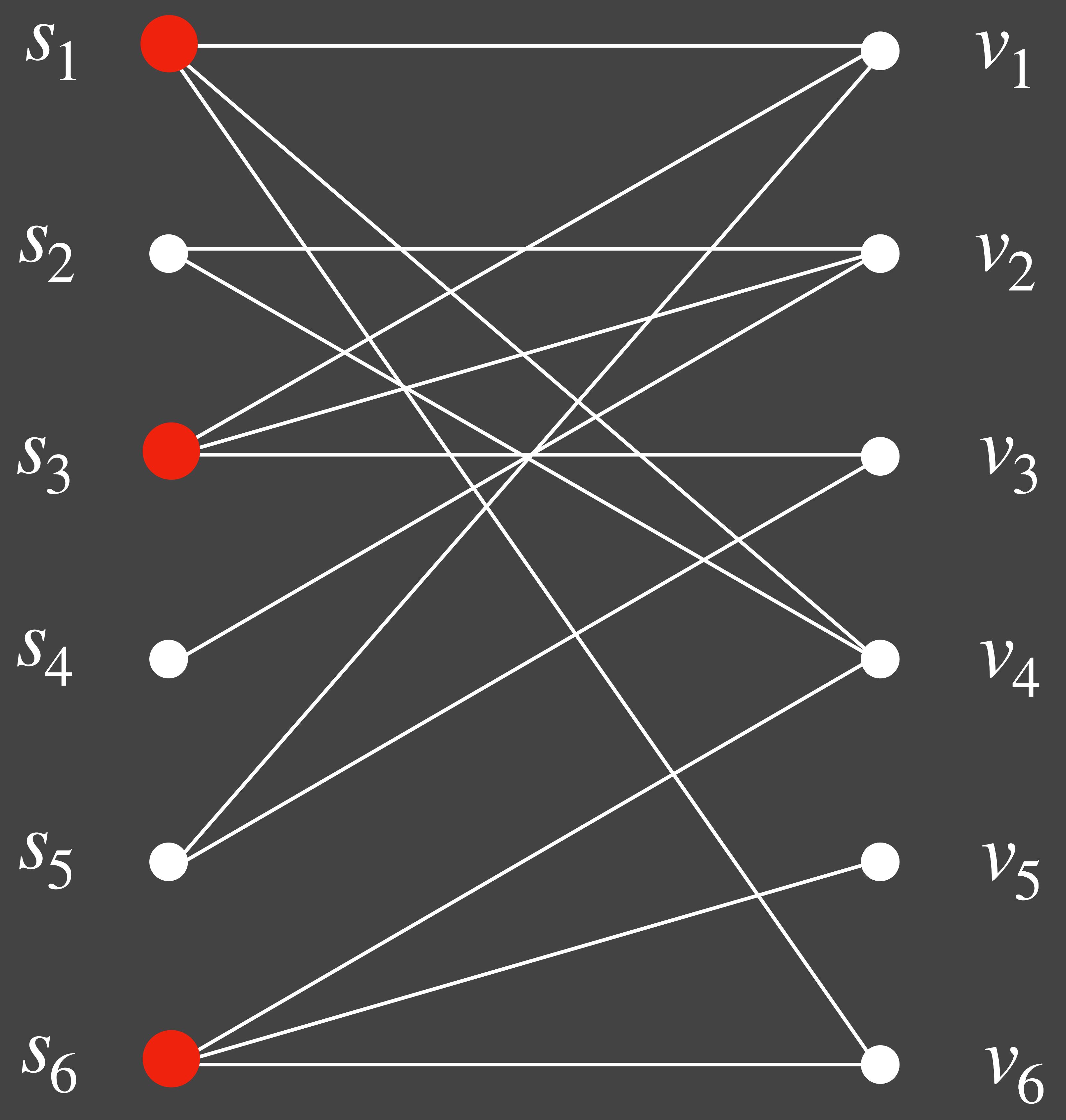
Set Cover

Apx: $\log n + 1$

[Johnson 74],[Lovasz 75],[Chvatal 79]

$\mathcal{S}$

m sets

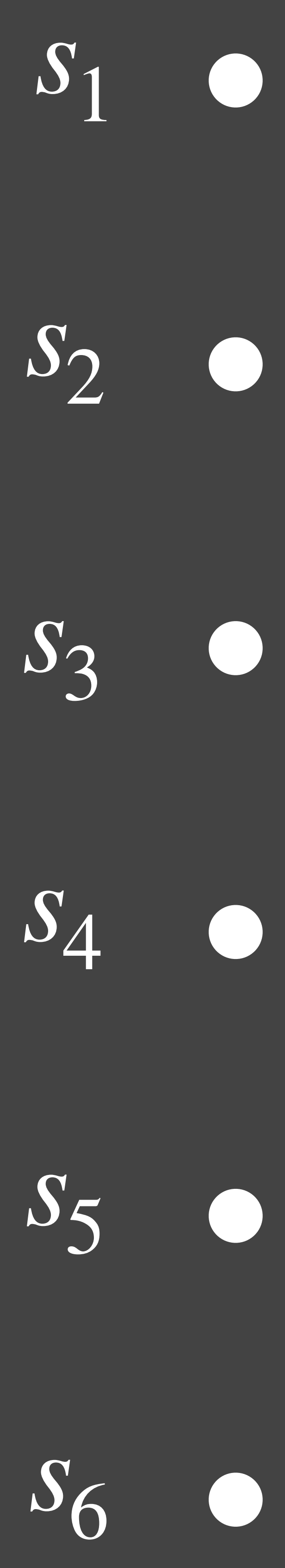


$\mathcal{U}$

n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

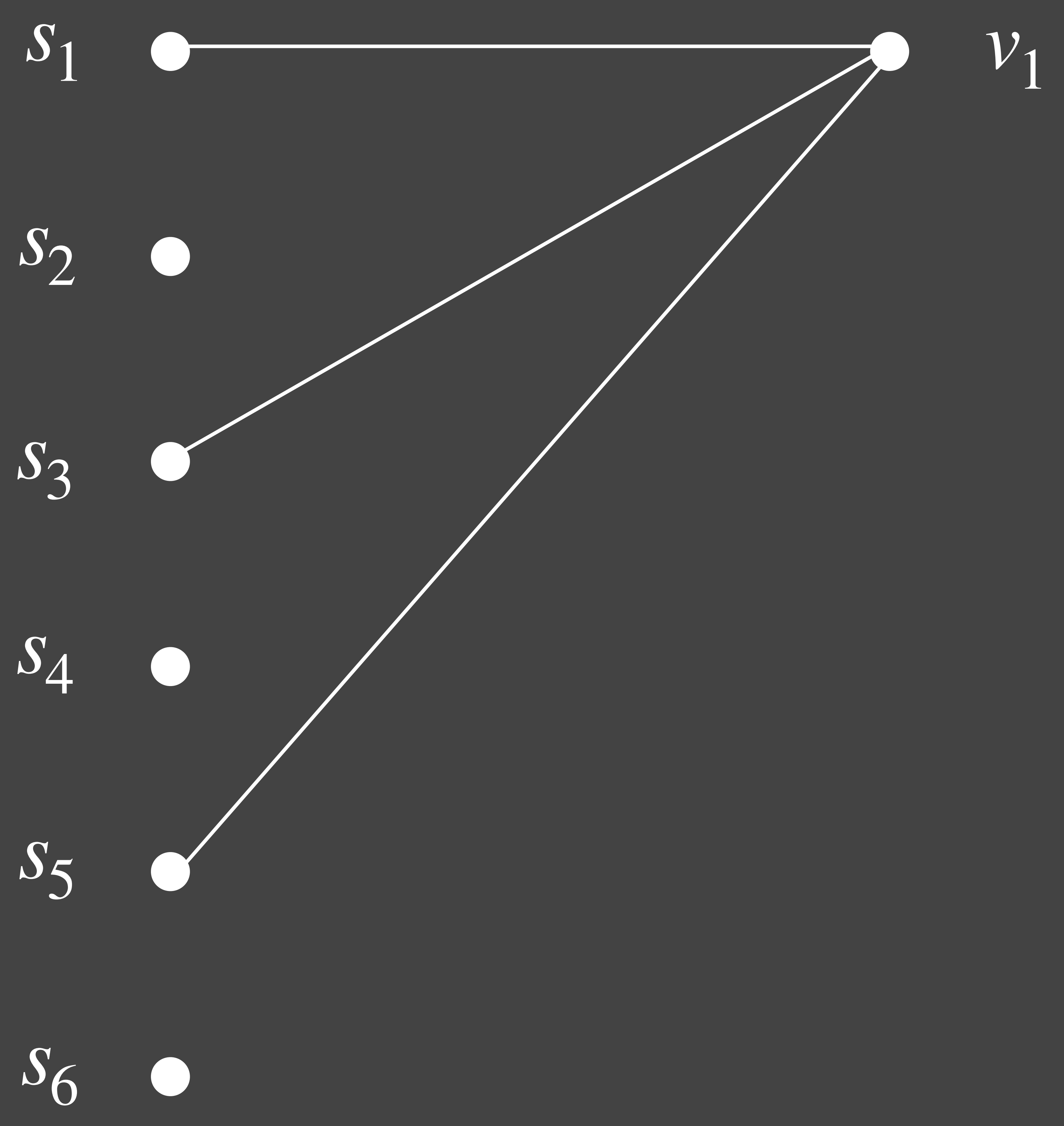


$\mathcal{S}$
m sets

$\mathcal{U}$
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]



$\mathcal{S}$

m sets

$\mathcal{U}$

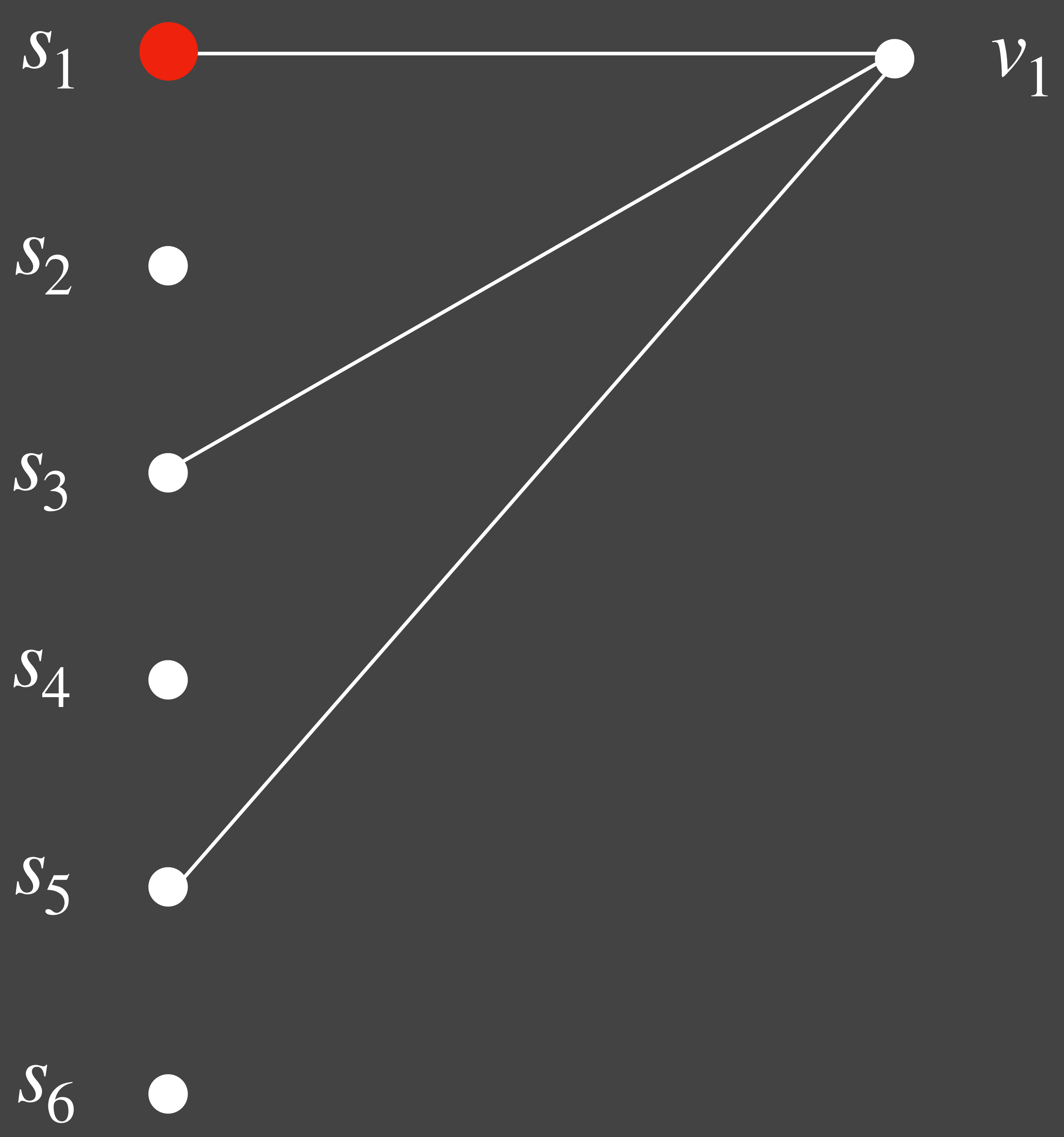
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



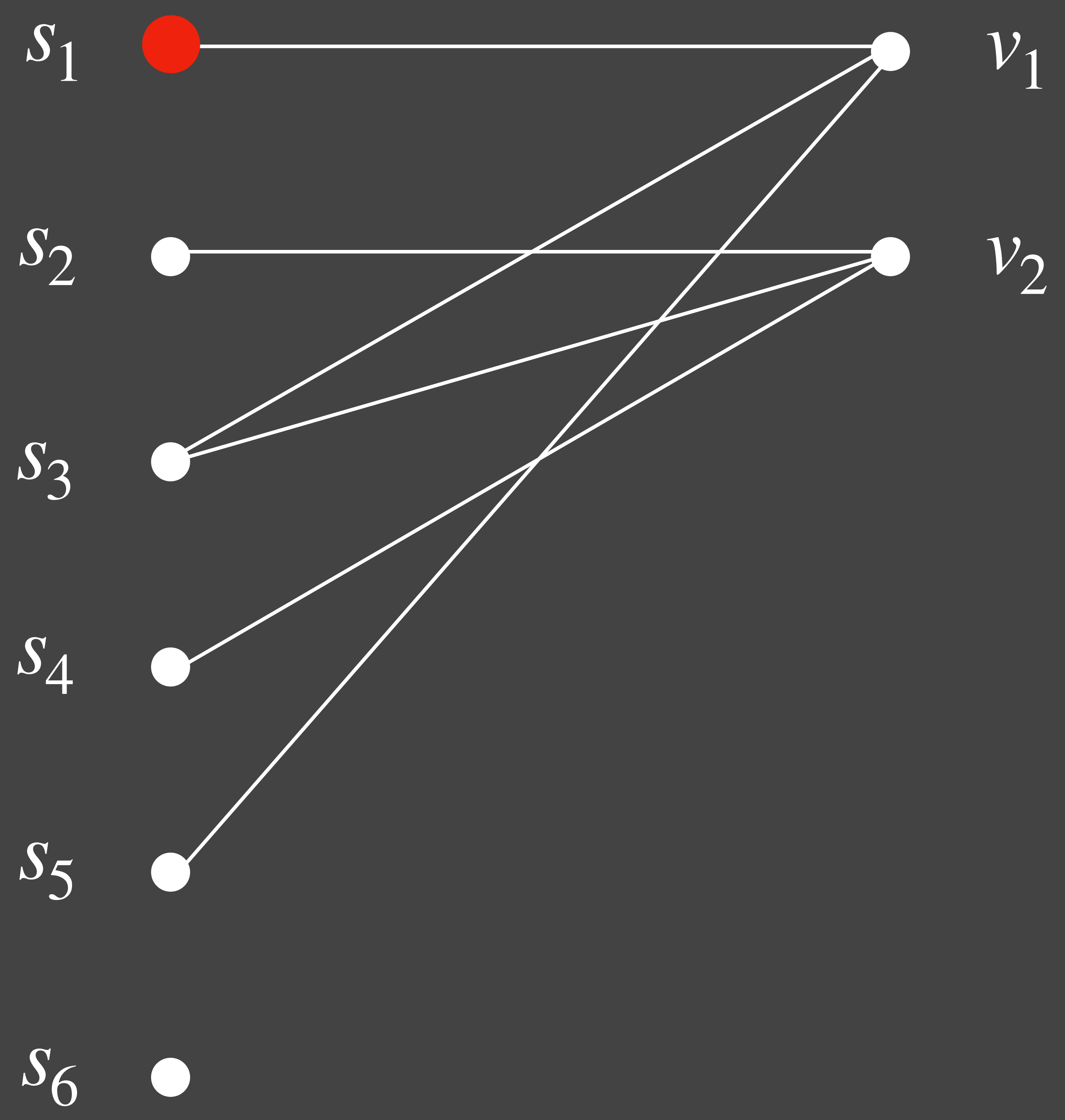
$\mathcal{U}$

n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$
m sets



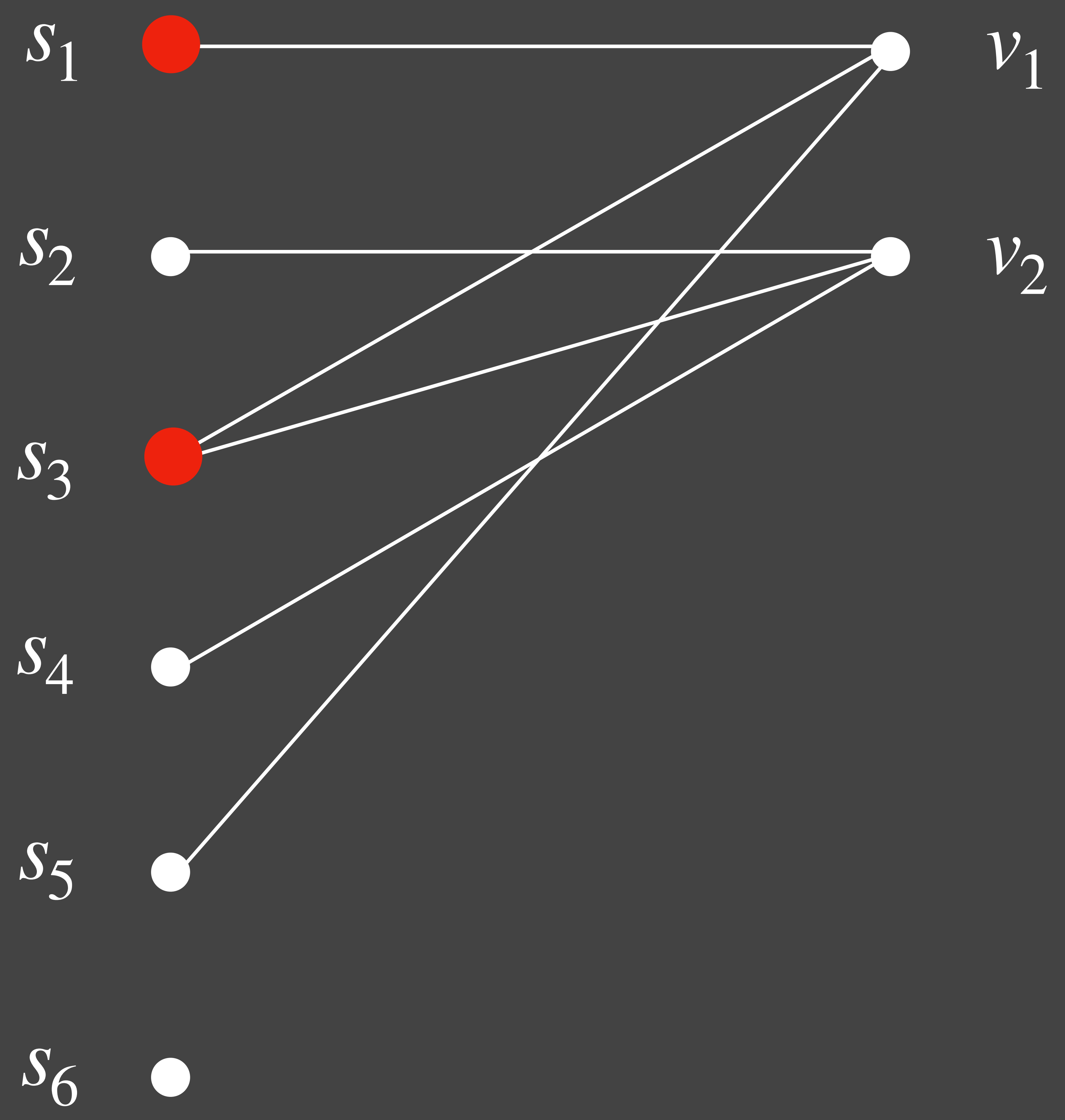
$\mathcal{U}$
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



$\mathcal{U}$

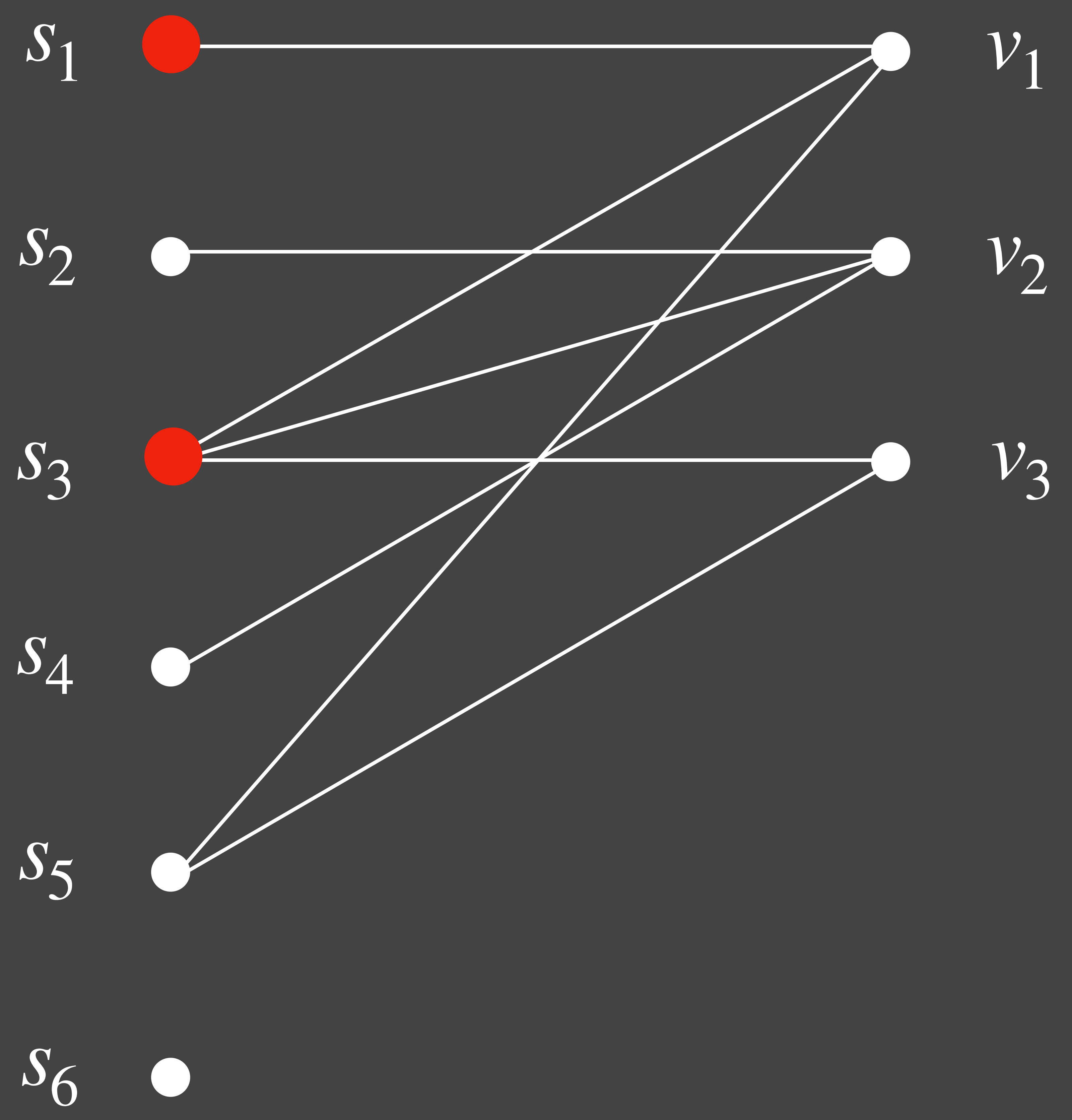
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



$\mathcal{U}$

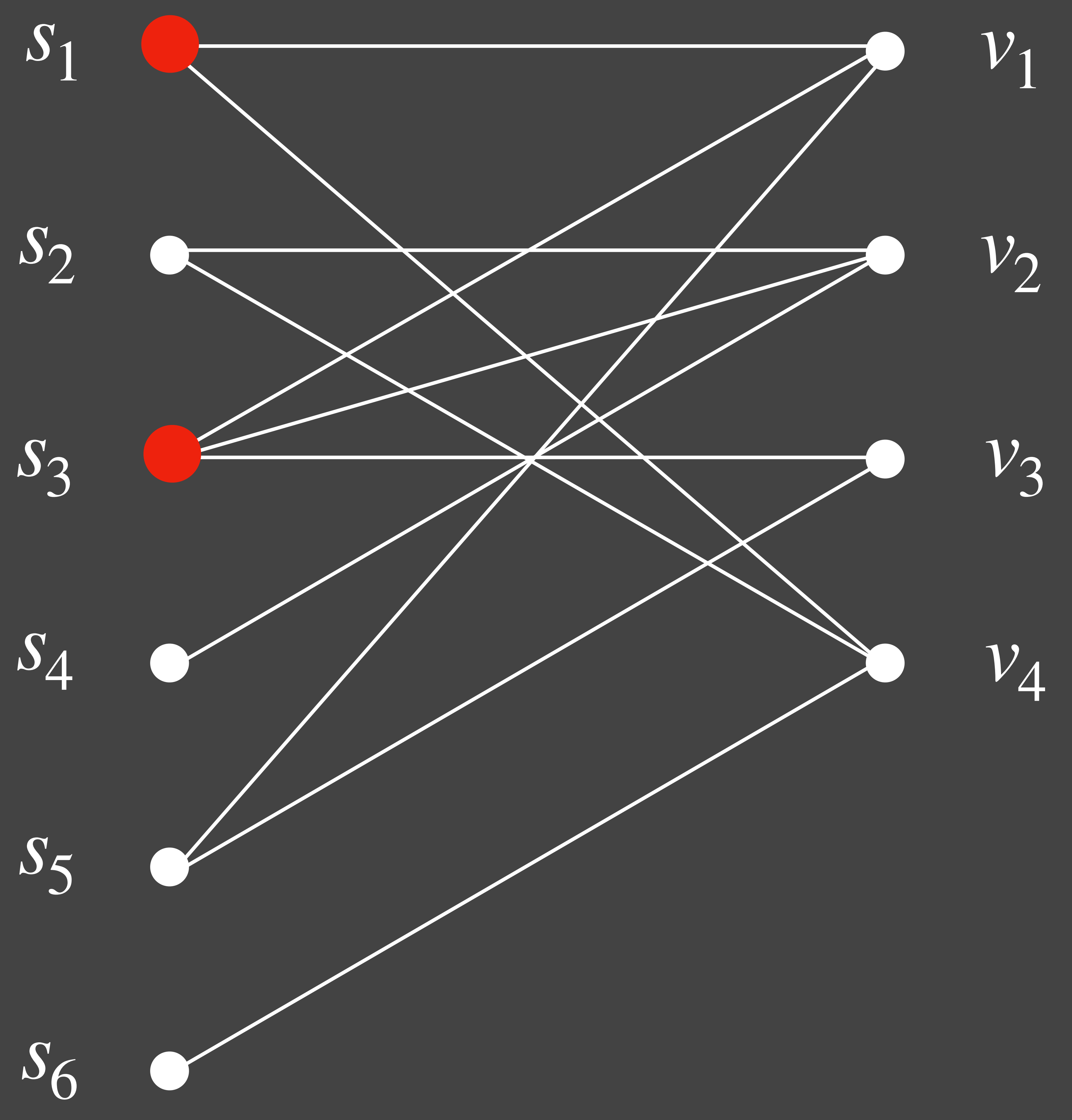
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



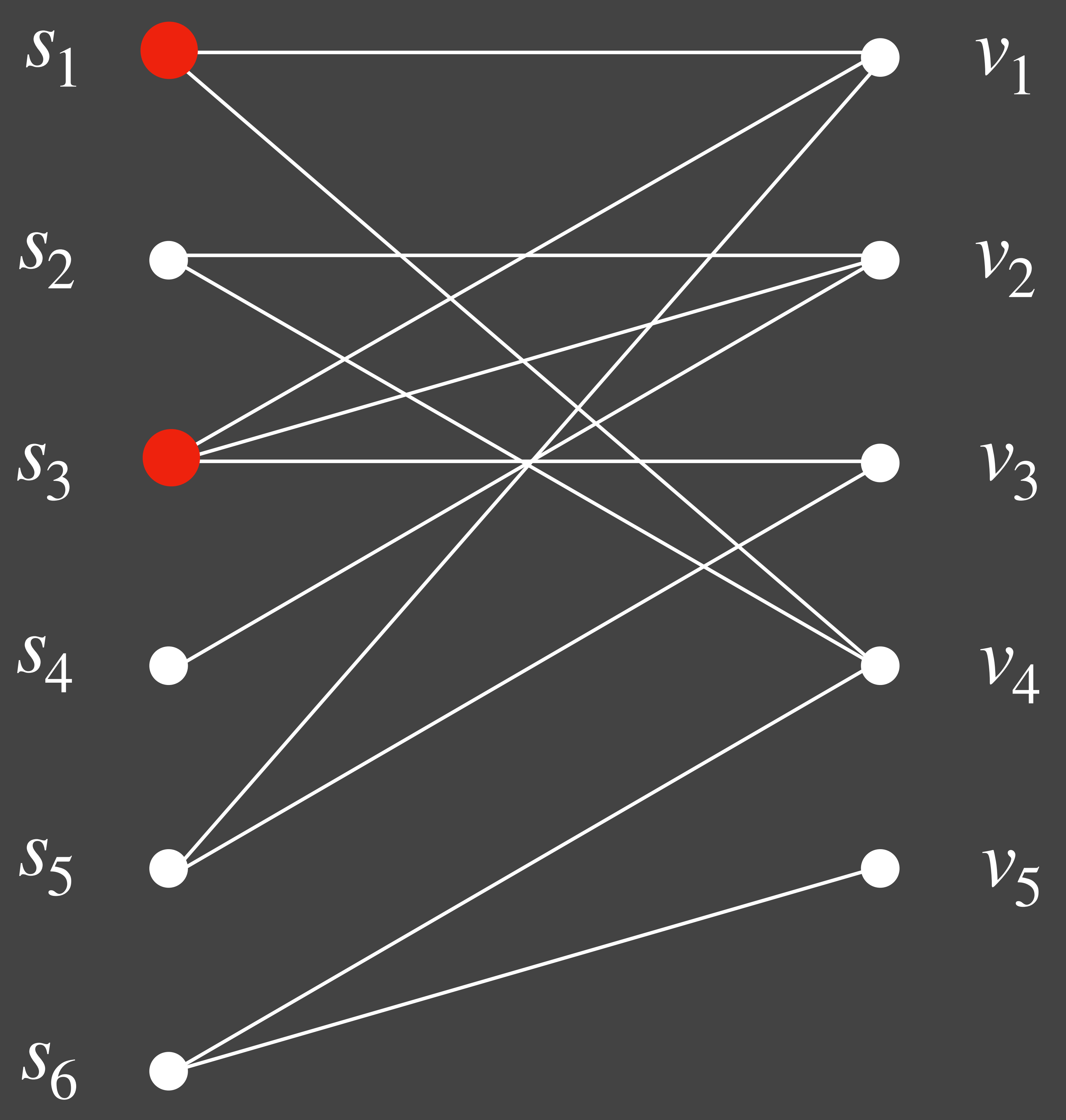
$\mathcal{U}$

n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$
m sets



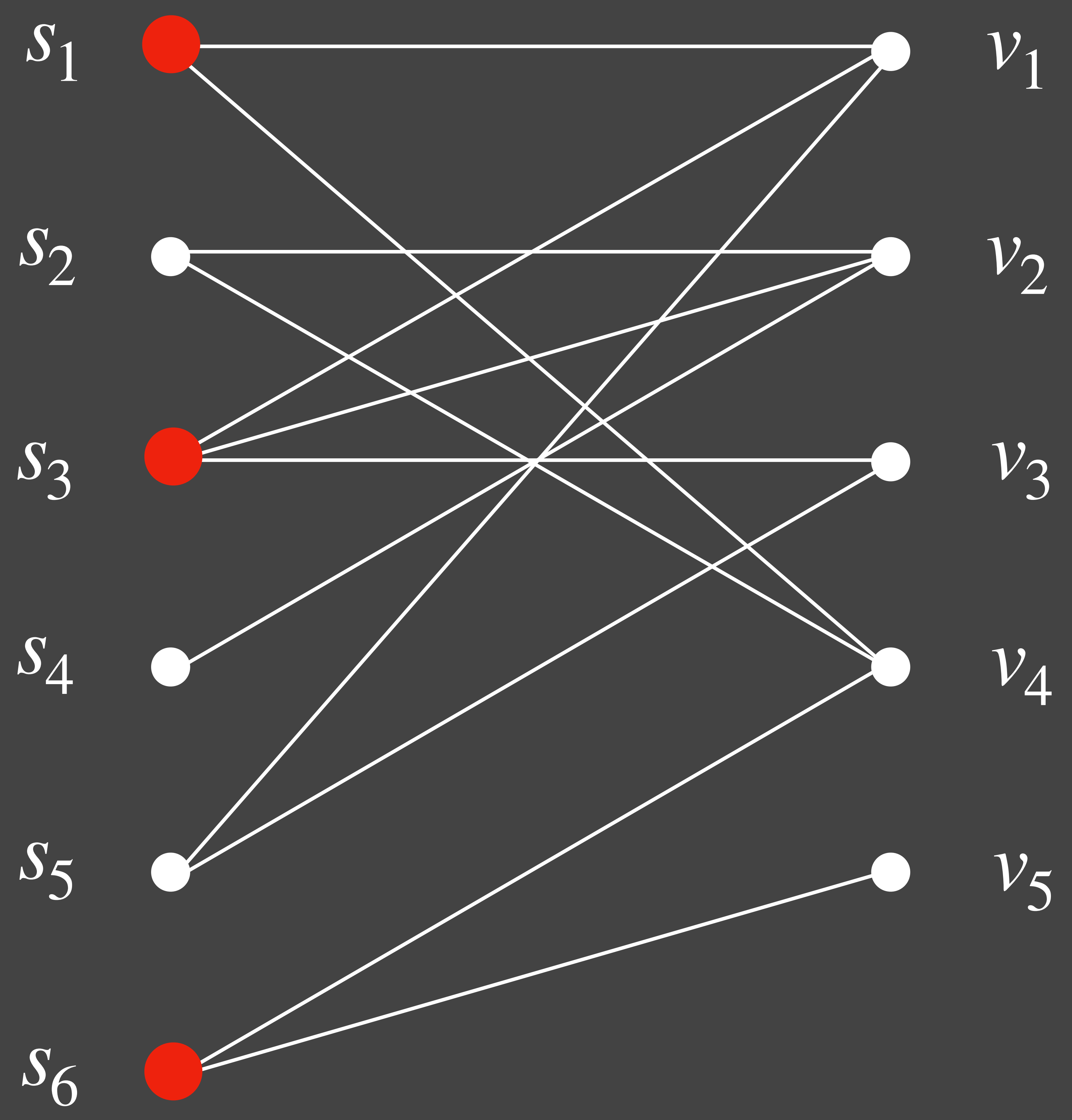
$\mathcal{U}$
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



$\mathcal{U}$

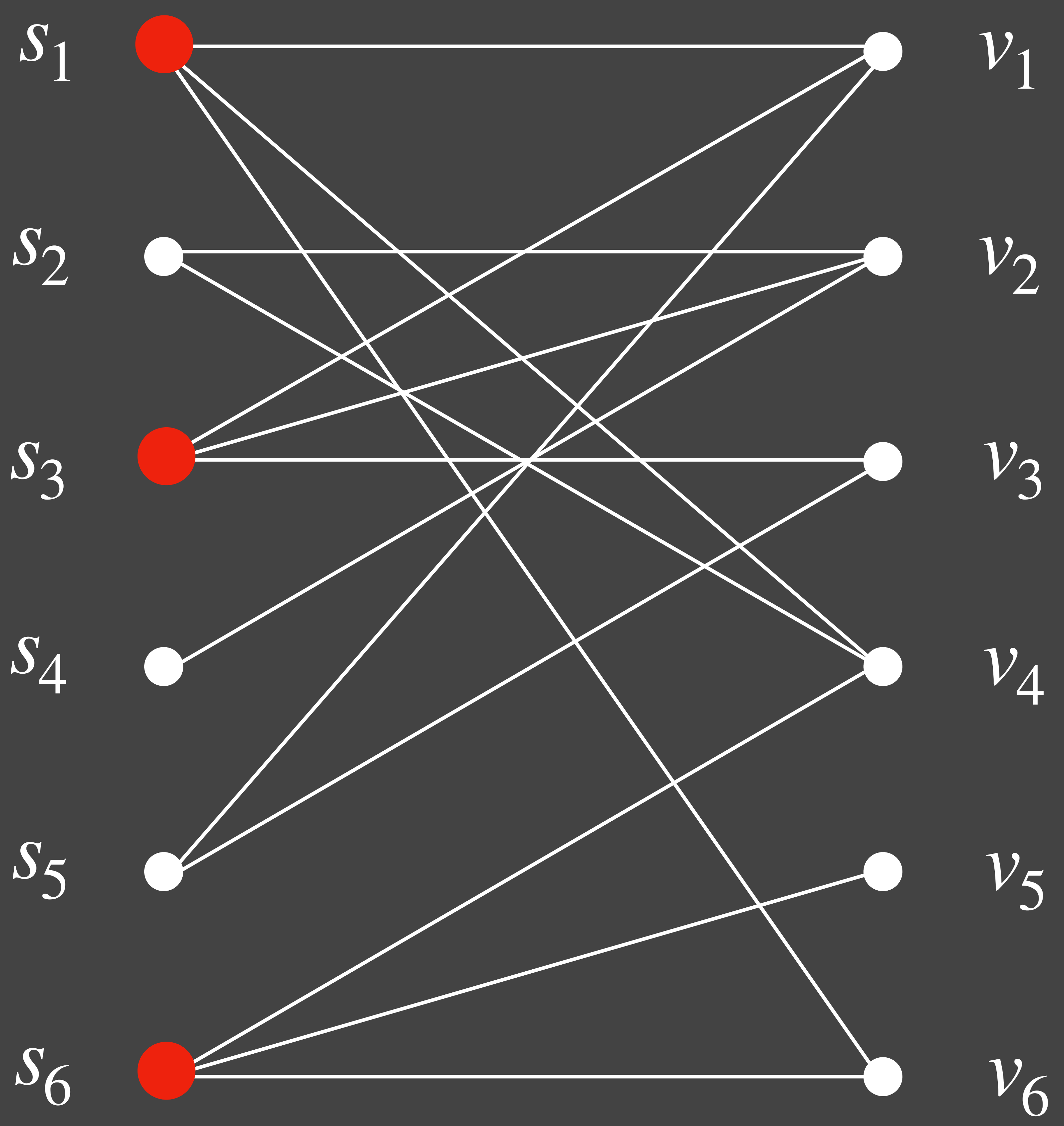
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

CR:

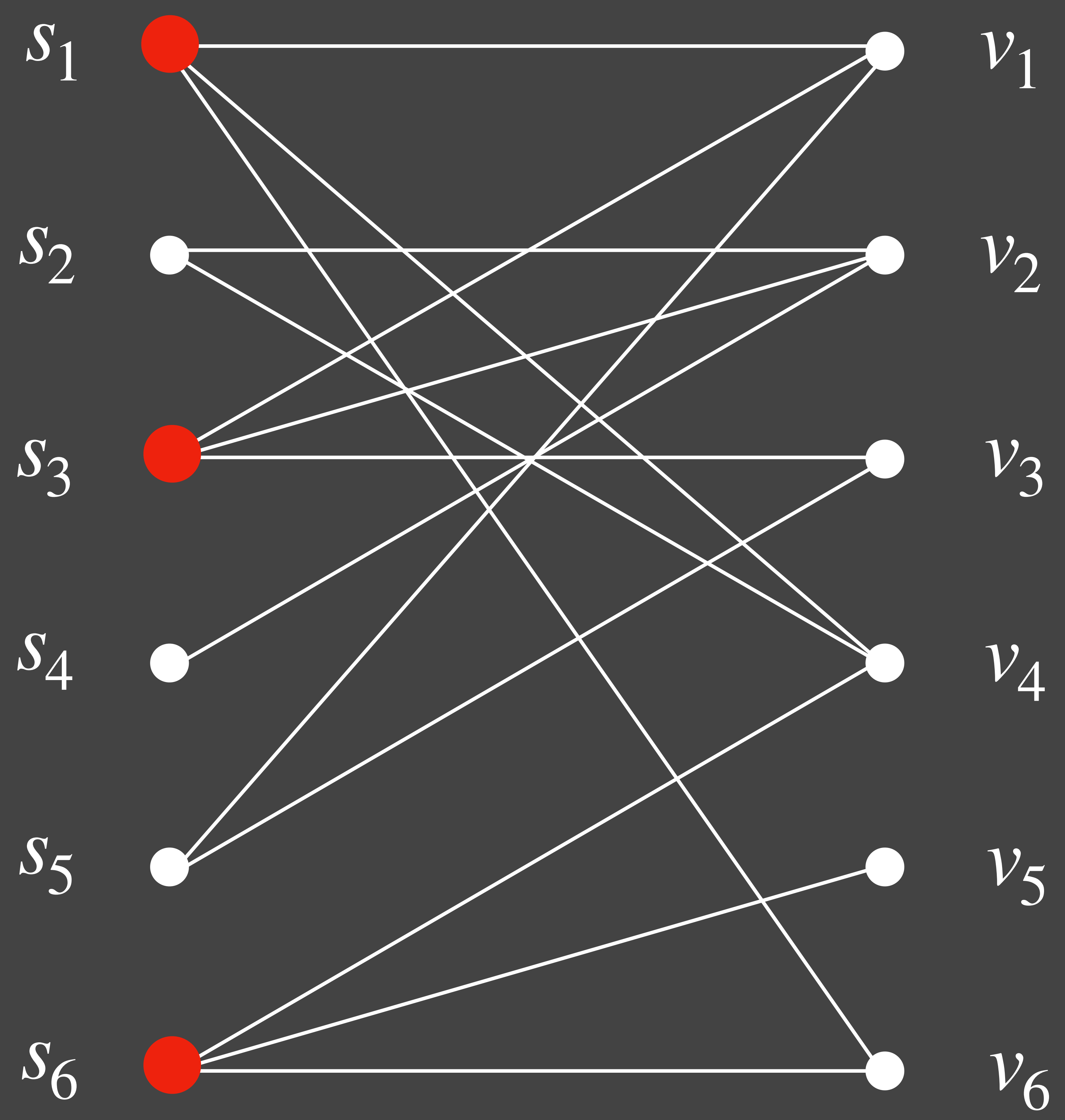
$O(\log n \log m)$

[Alon+ 03]

[Buchbinder Naor 09]

$\mathcal{S}$

m sets



$\mathcal{U}$

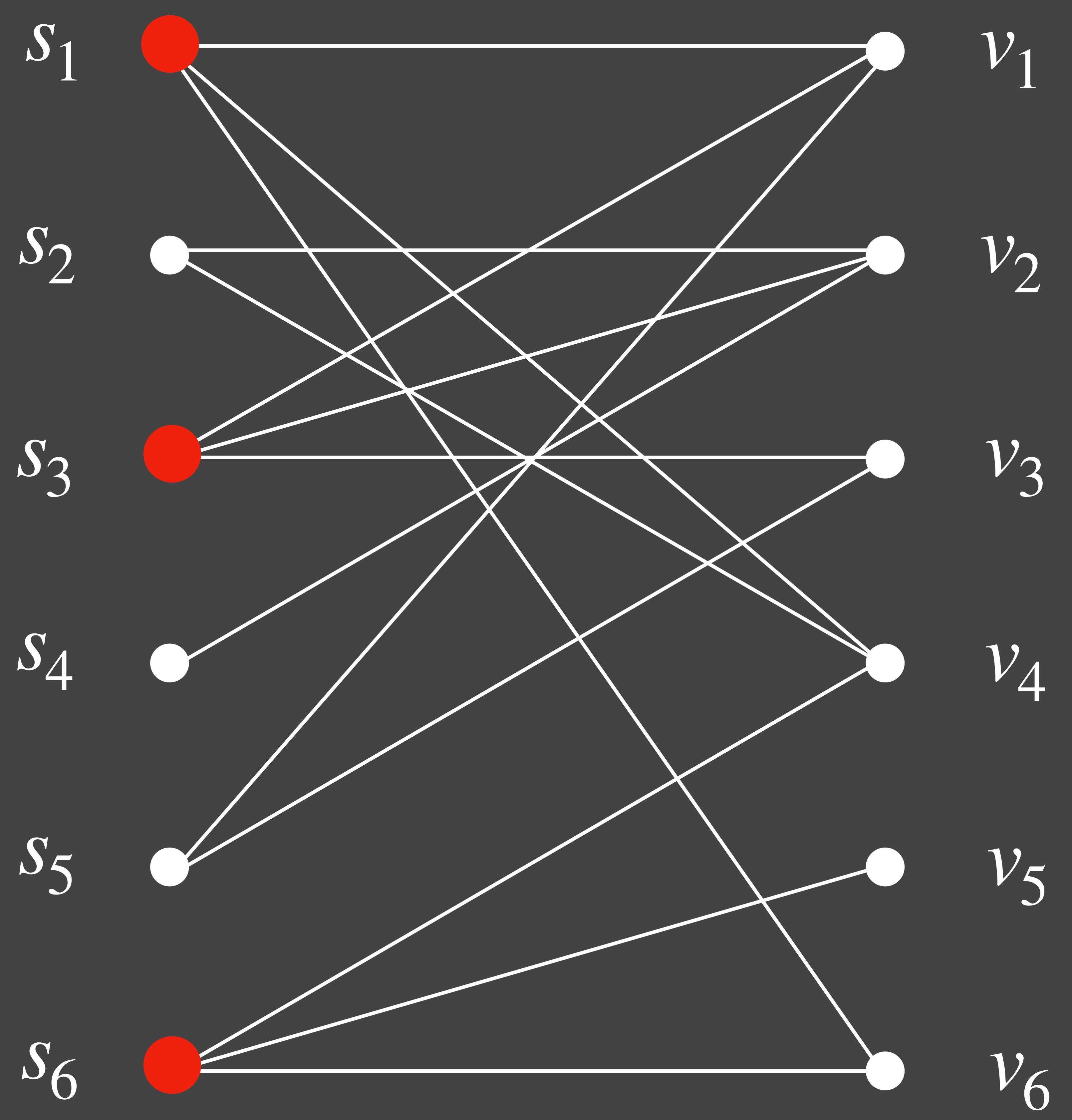
n elements

Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

CR:
$O(\log n \log m)$
[Alon+ 03]
[Buchbinder Naor 09]

$\mathcal{S}$
m sets



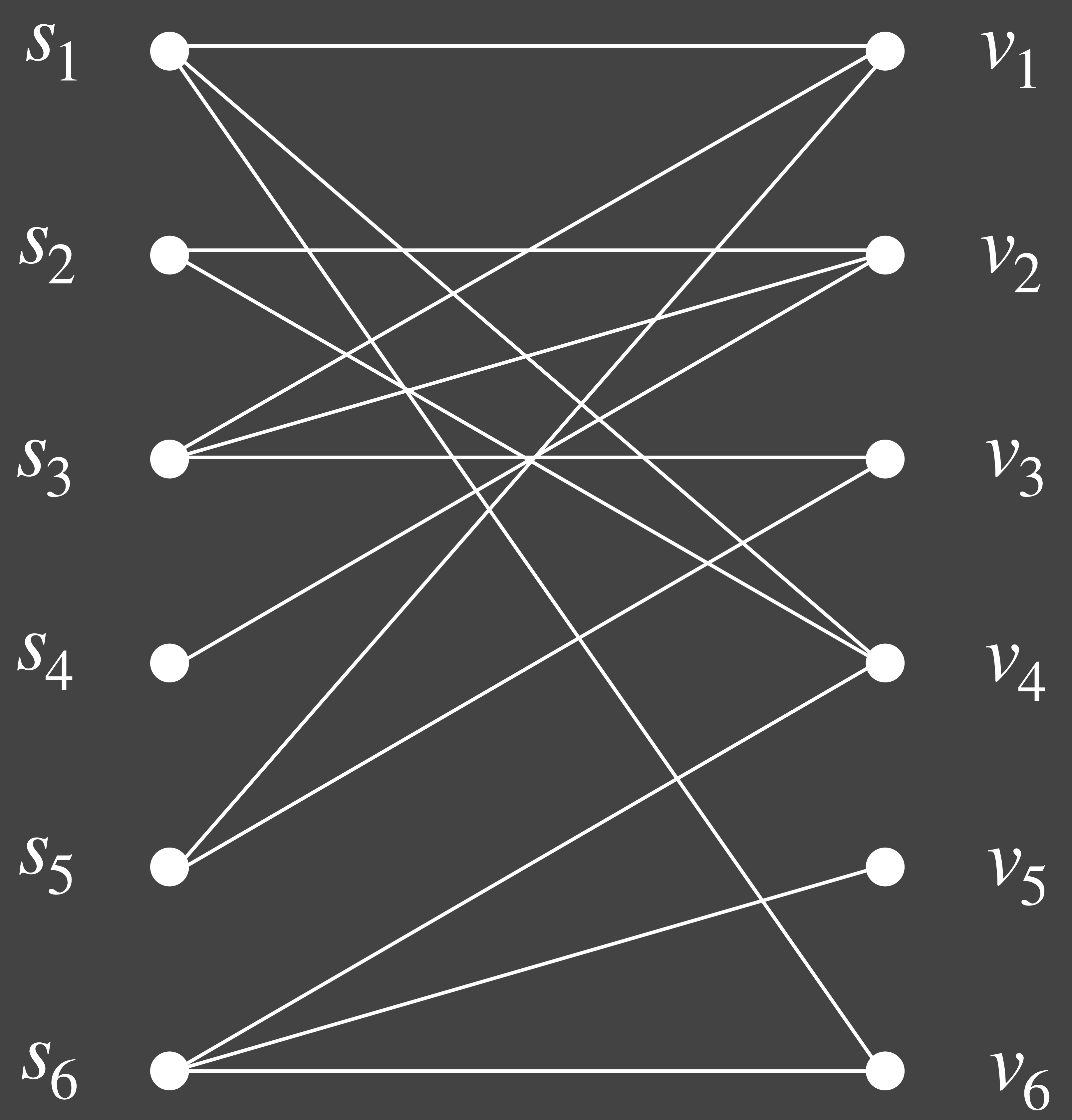
$\mathcal{U}$
n elements

Q: What happens beyond the worst case?

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

s_1 ●

s_2 ●

$\mathcal{S}$

m sets

s_3 ●

$\mathcal{U}$

n elements

s_4 ●

s_5 ●

s_6 ●

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets

s_1 ●

s_2 ●

s_3 ●

s_4 ●

s_5 ●

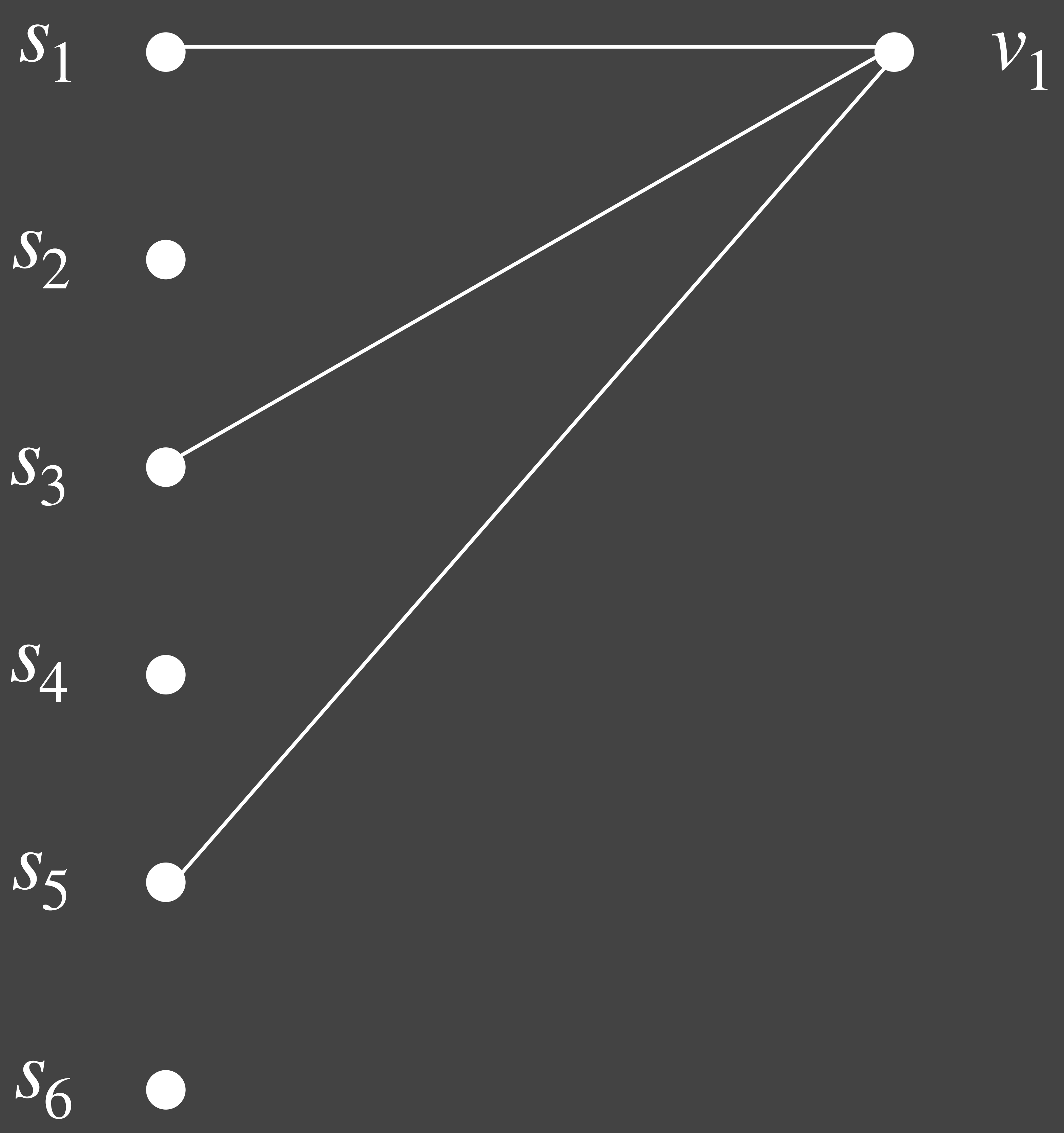
s_6 ●

$\mathcal{U}$

n elements

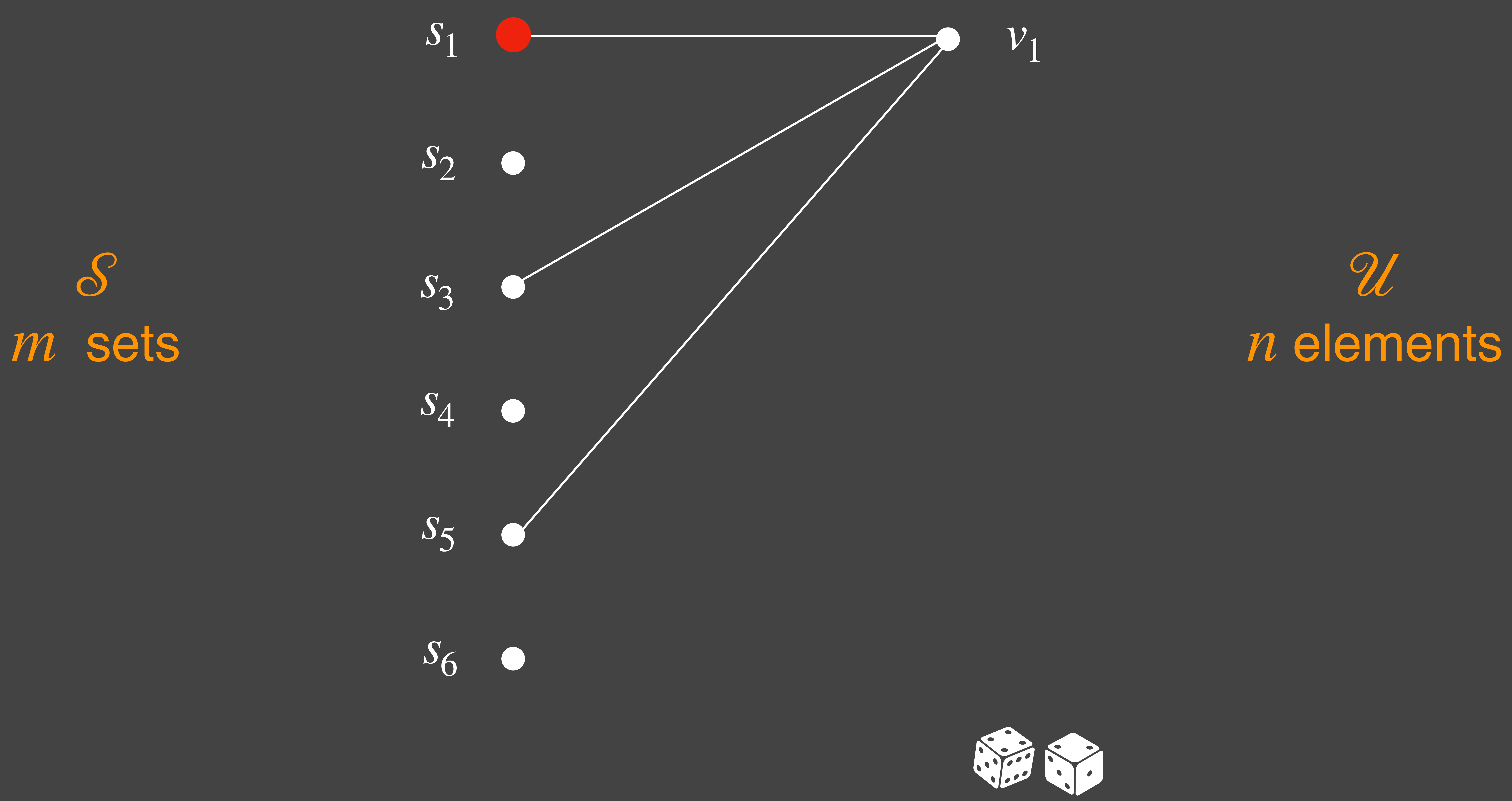
Relaxation 1: Random Order (RO)

$\mathcal{S}$
m sets



$\mathcal{U}$
n elements

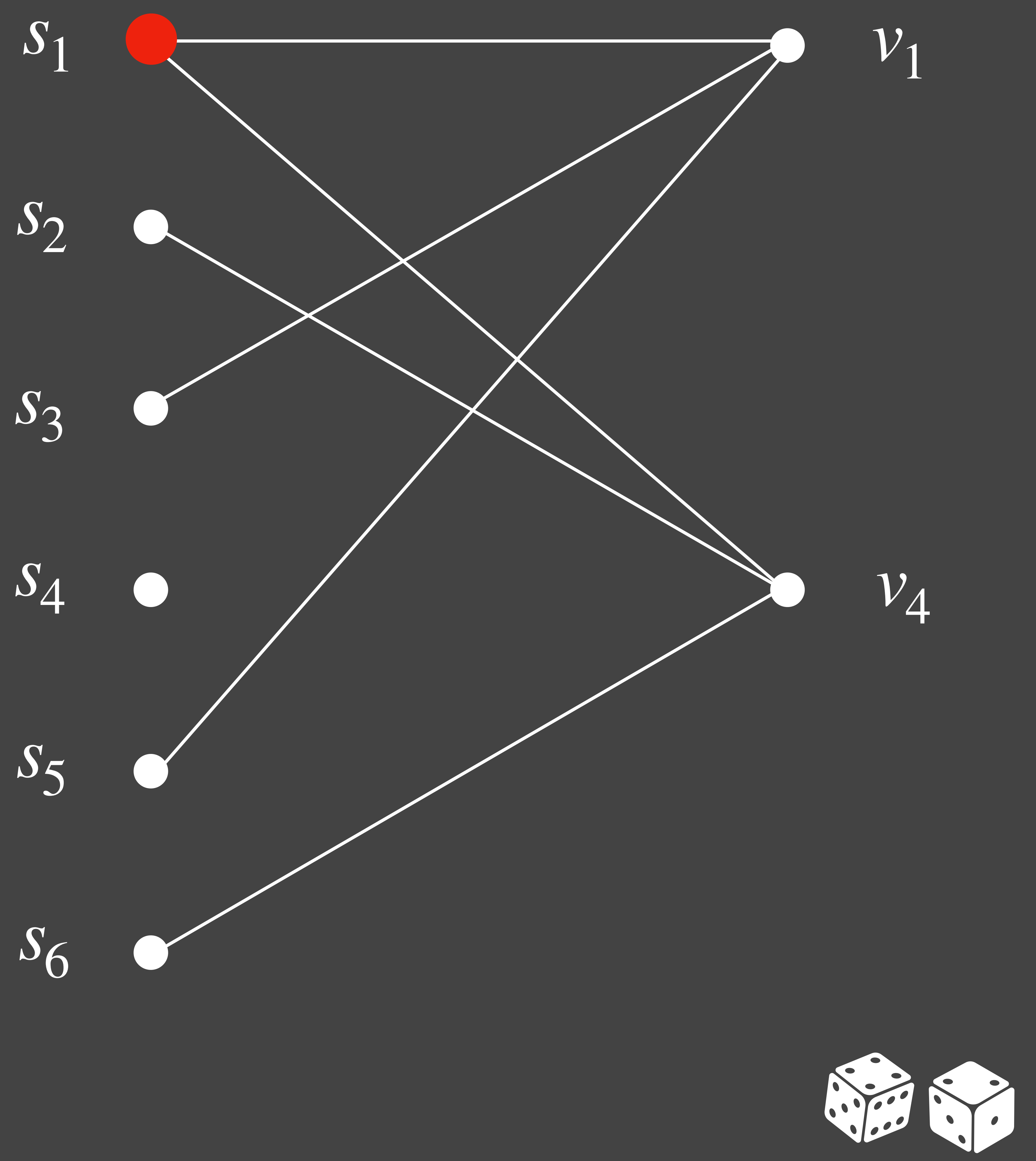
Relaxation 1: Random Order (RO)



Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



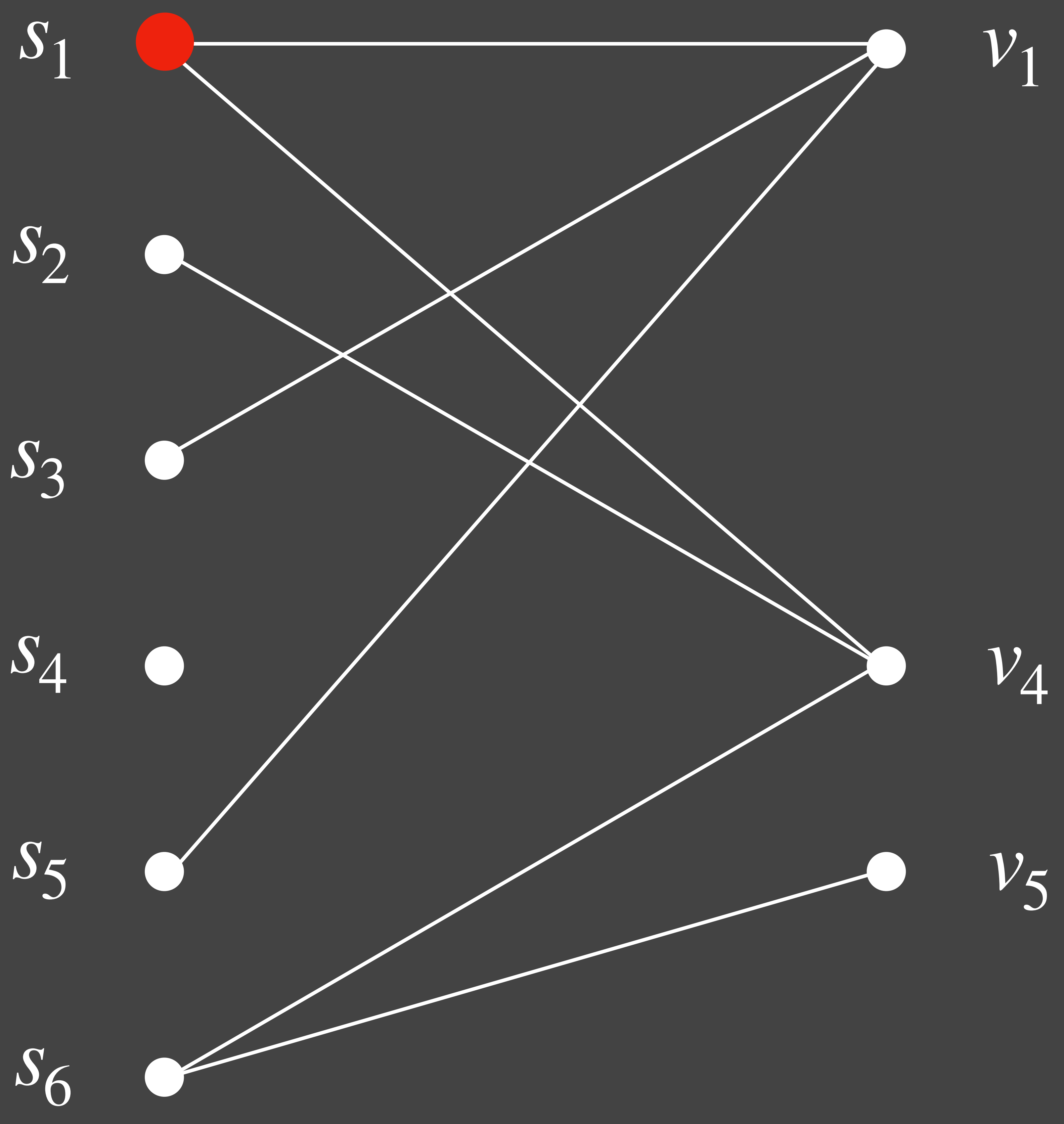
$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets

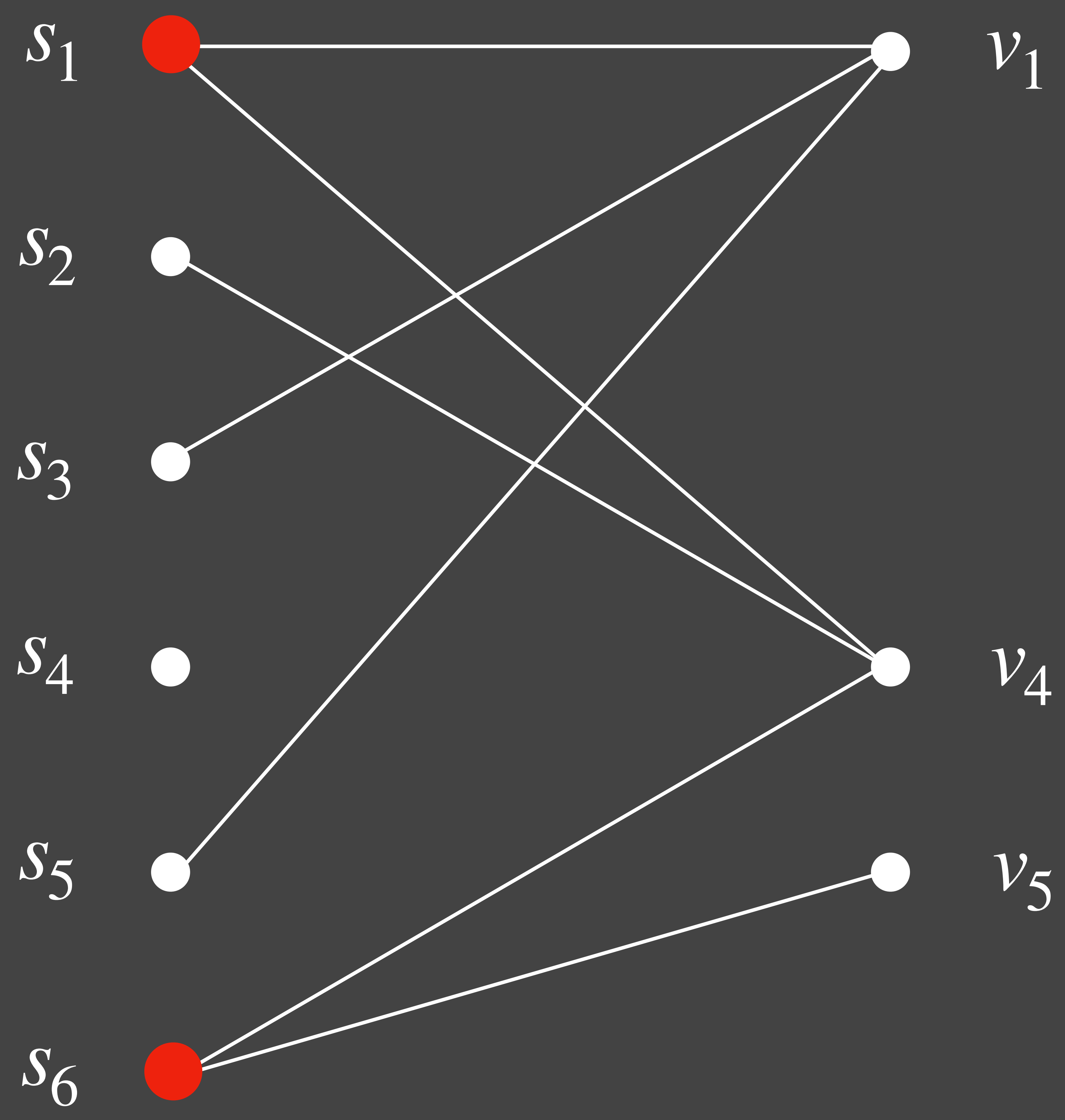


$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$
m sets

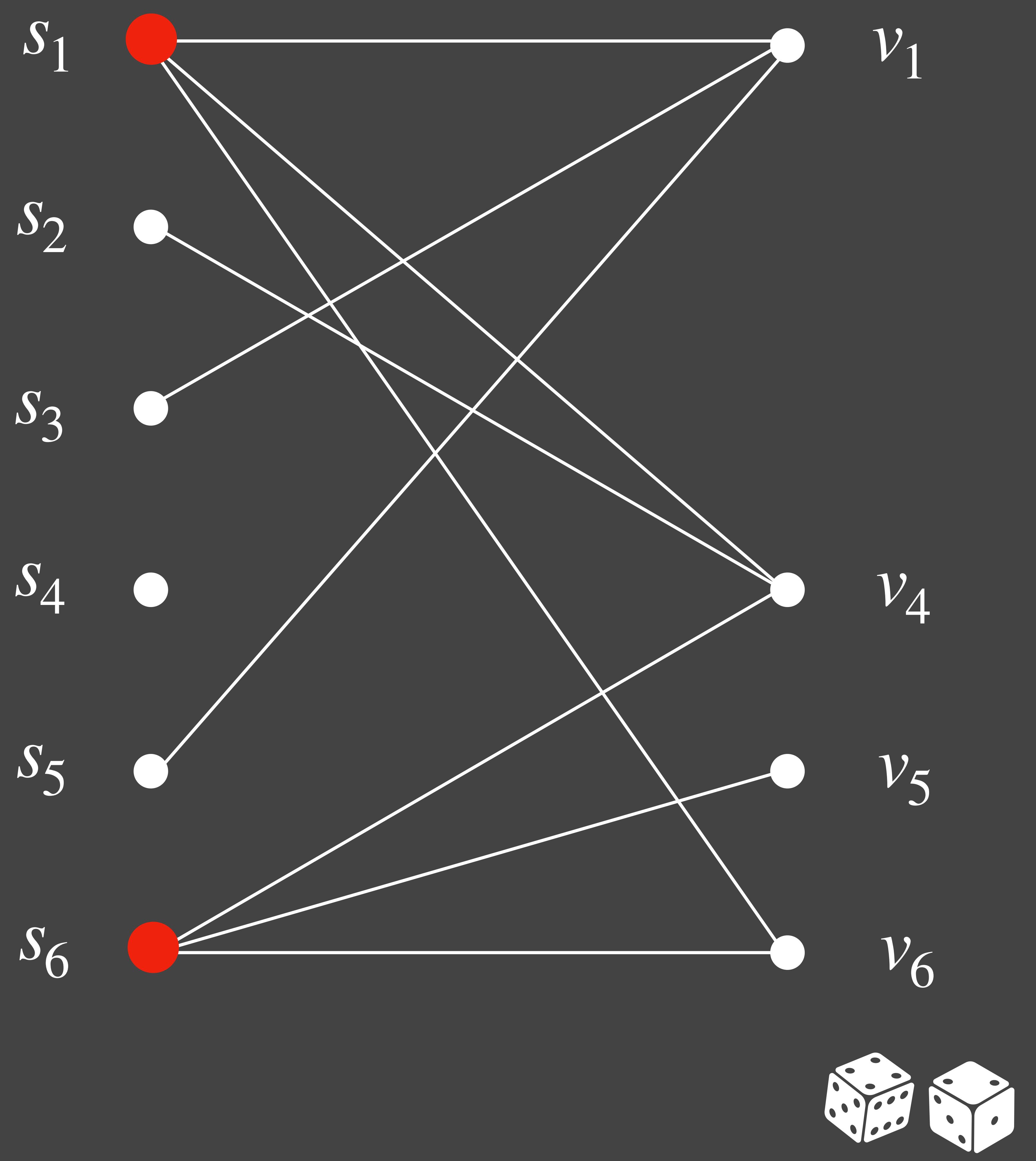


$\mathcal{U}$
n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



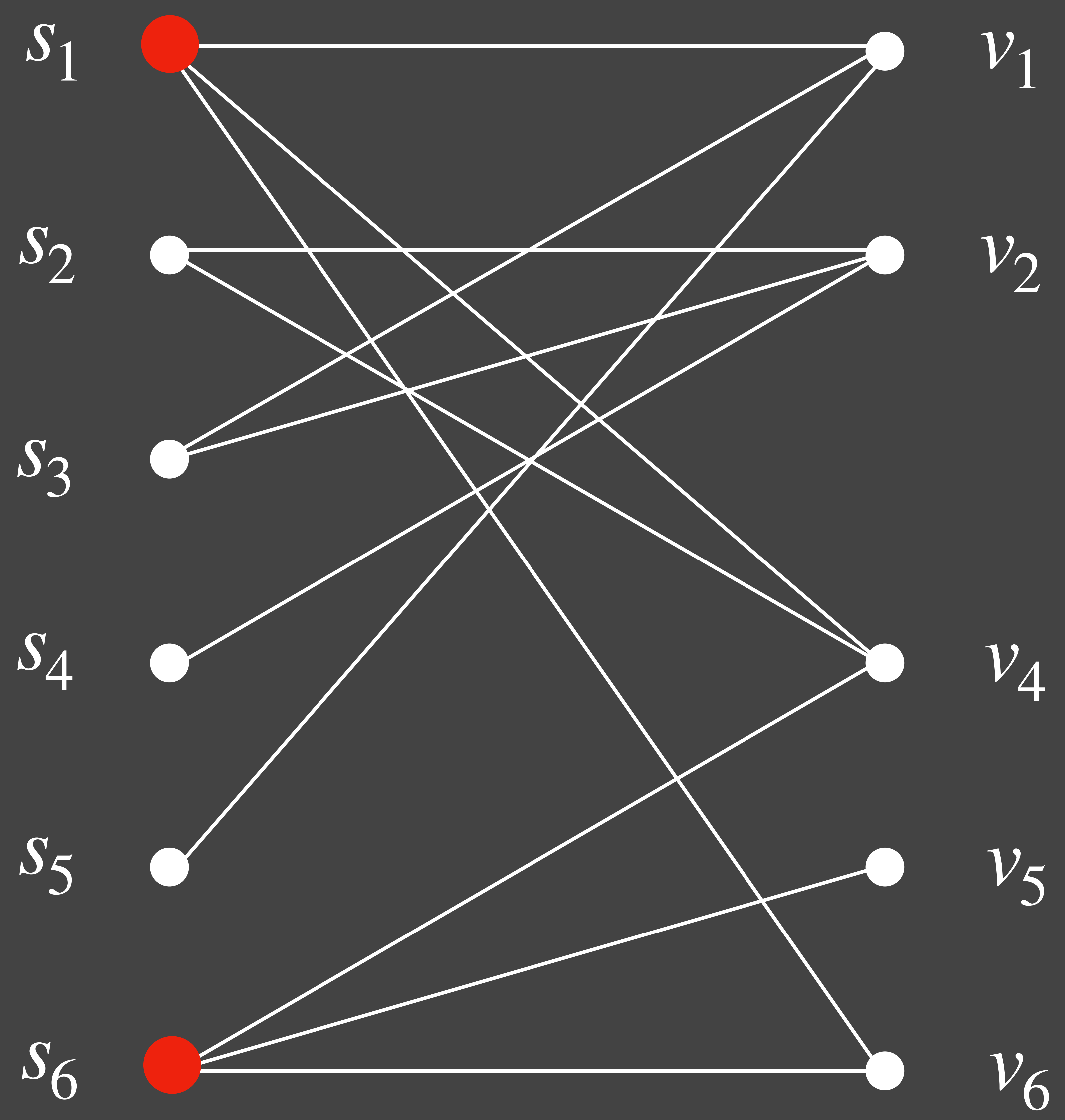
$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



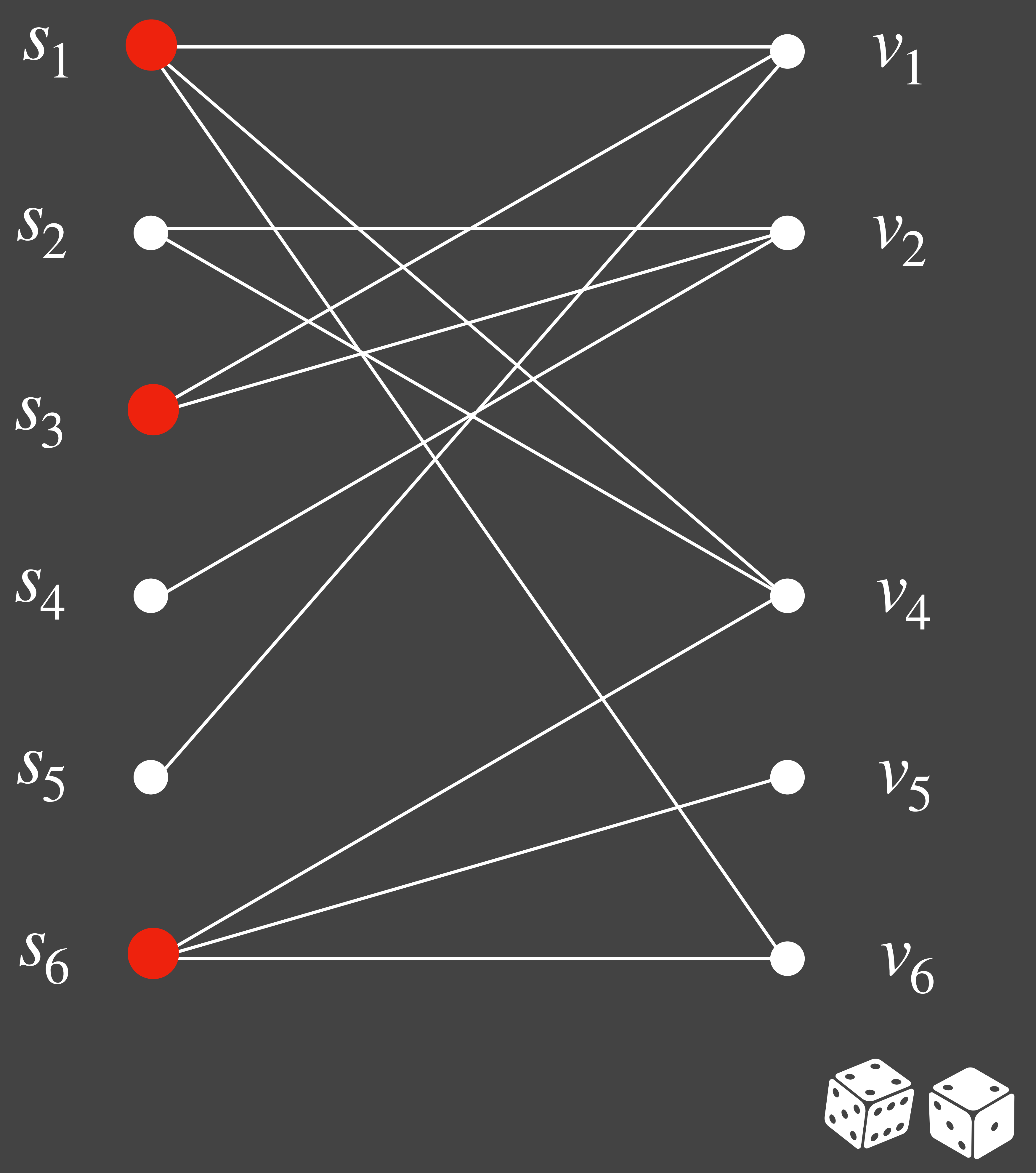
$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



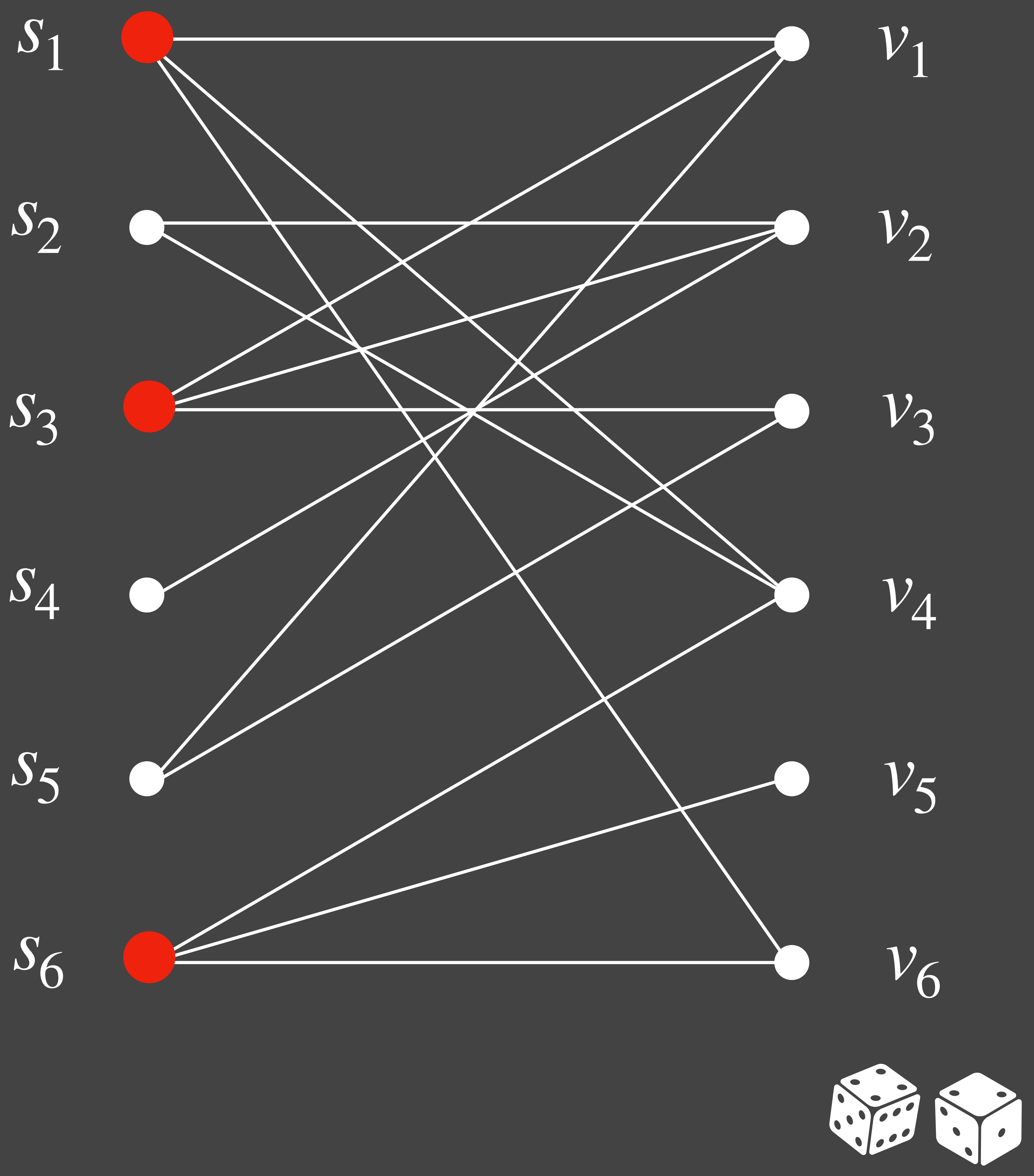
$\mathcal{U}$

n elements

Relaxation 1: Random Order (RO)

$\mathcal{S}$

m sets



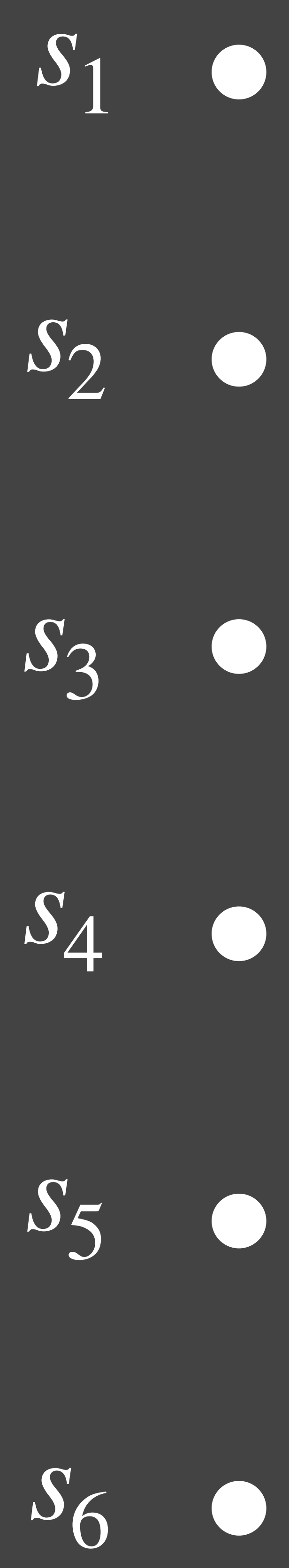
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets



s_1

s_2

s_3

s_4

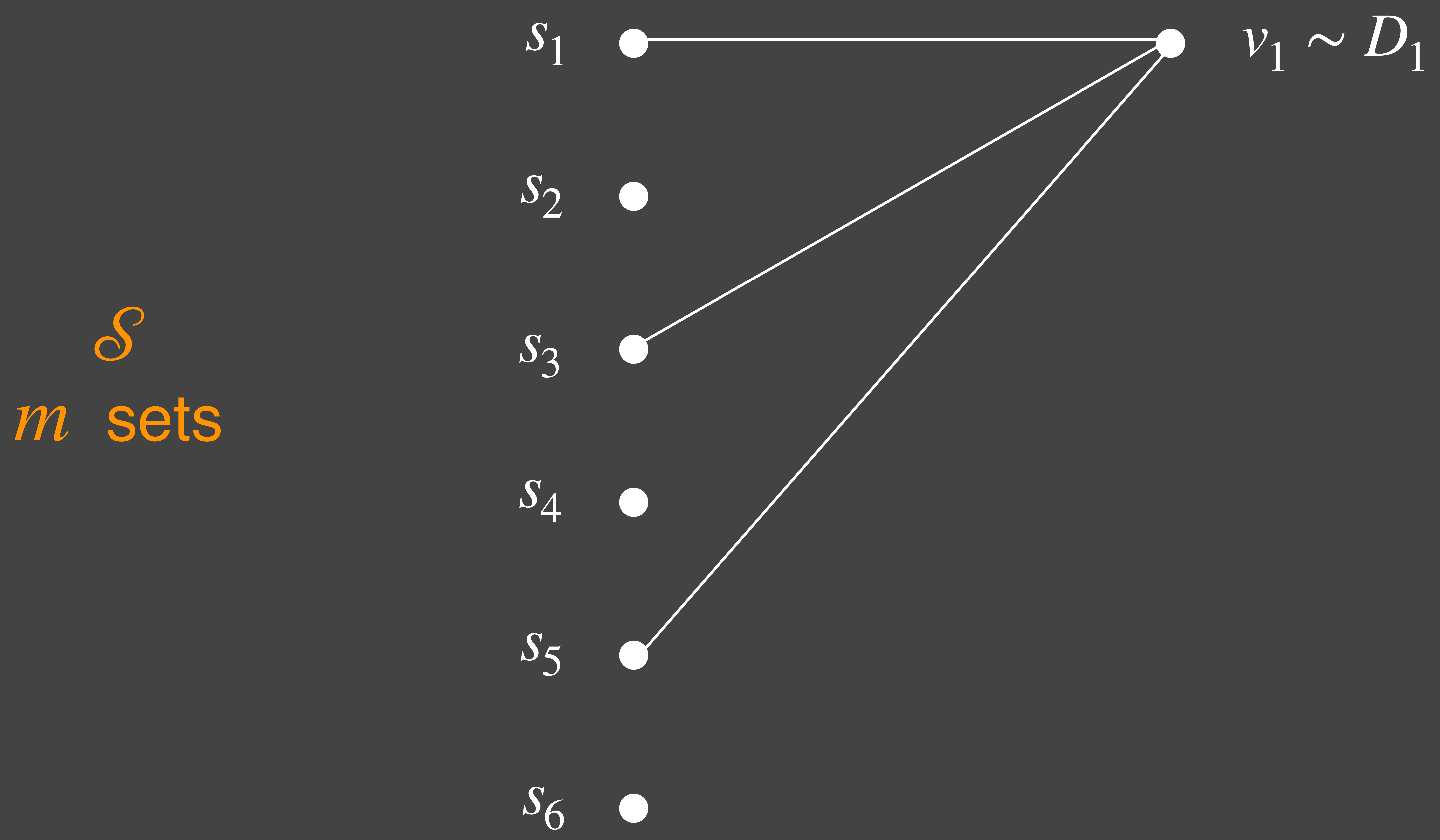
s_5

s_6

$\mathcal{U}$

n elements

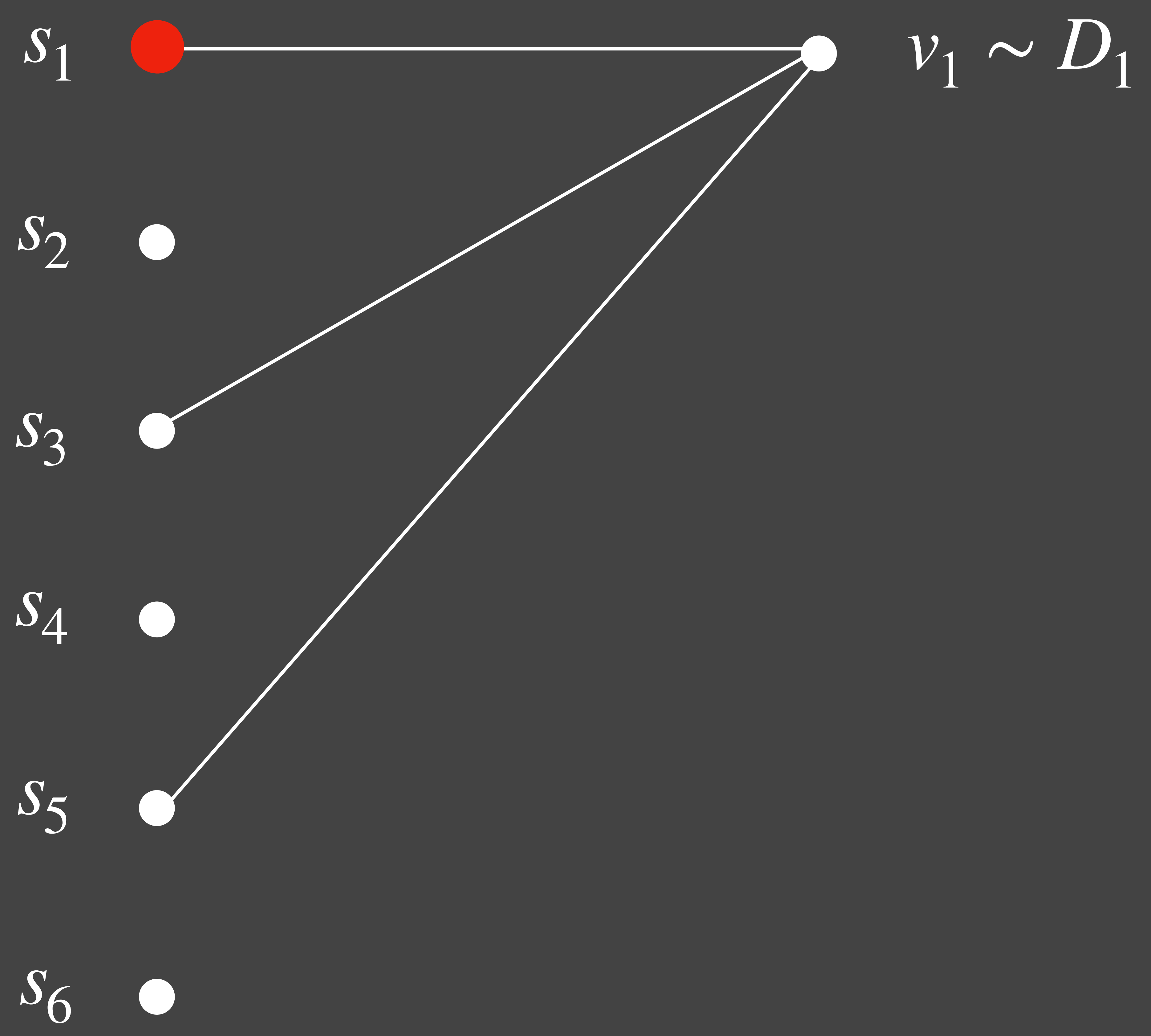
Relaxation 2: Random Instance



Relaxation 2: Random Instance

$\mathcal{S}$

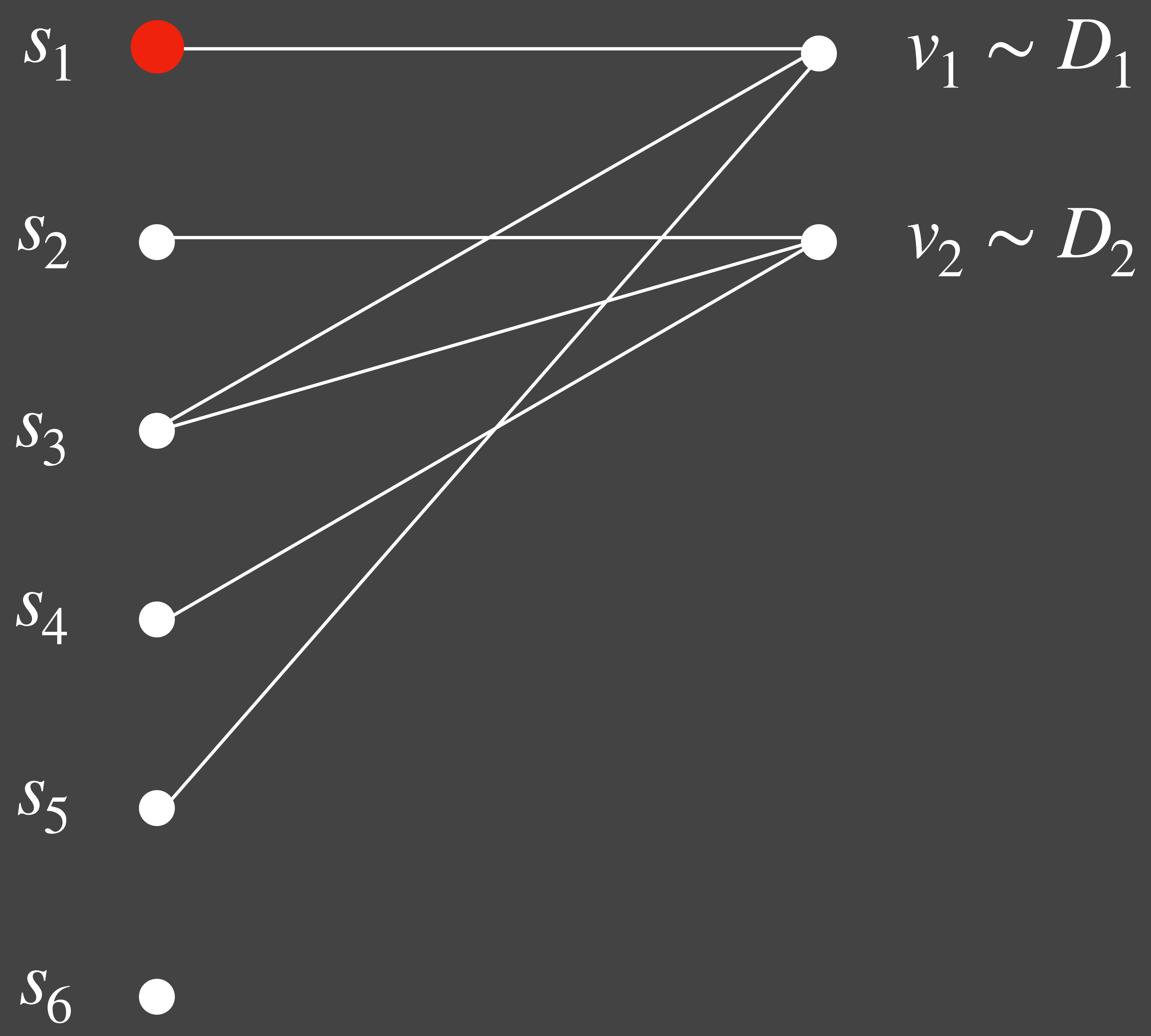
m sets



$\mathcal{U}$

n elements

Relaxation 2: Random Instance



$\mathcal{S}$

m sets

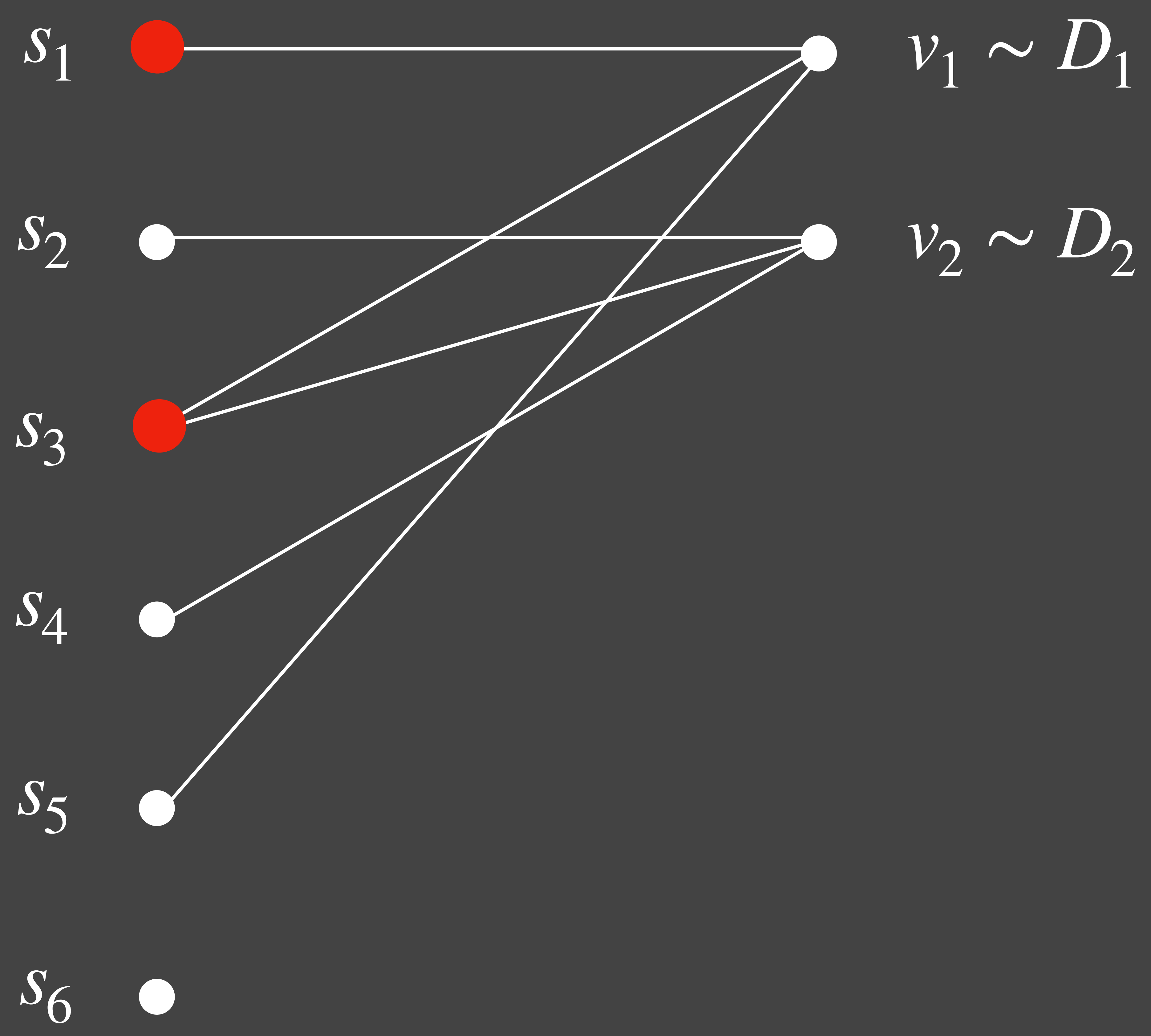
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets



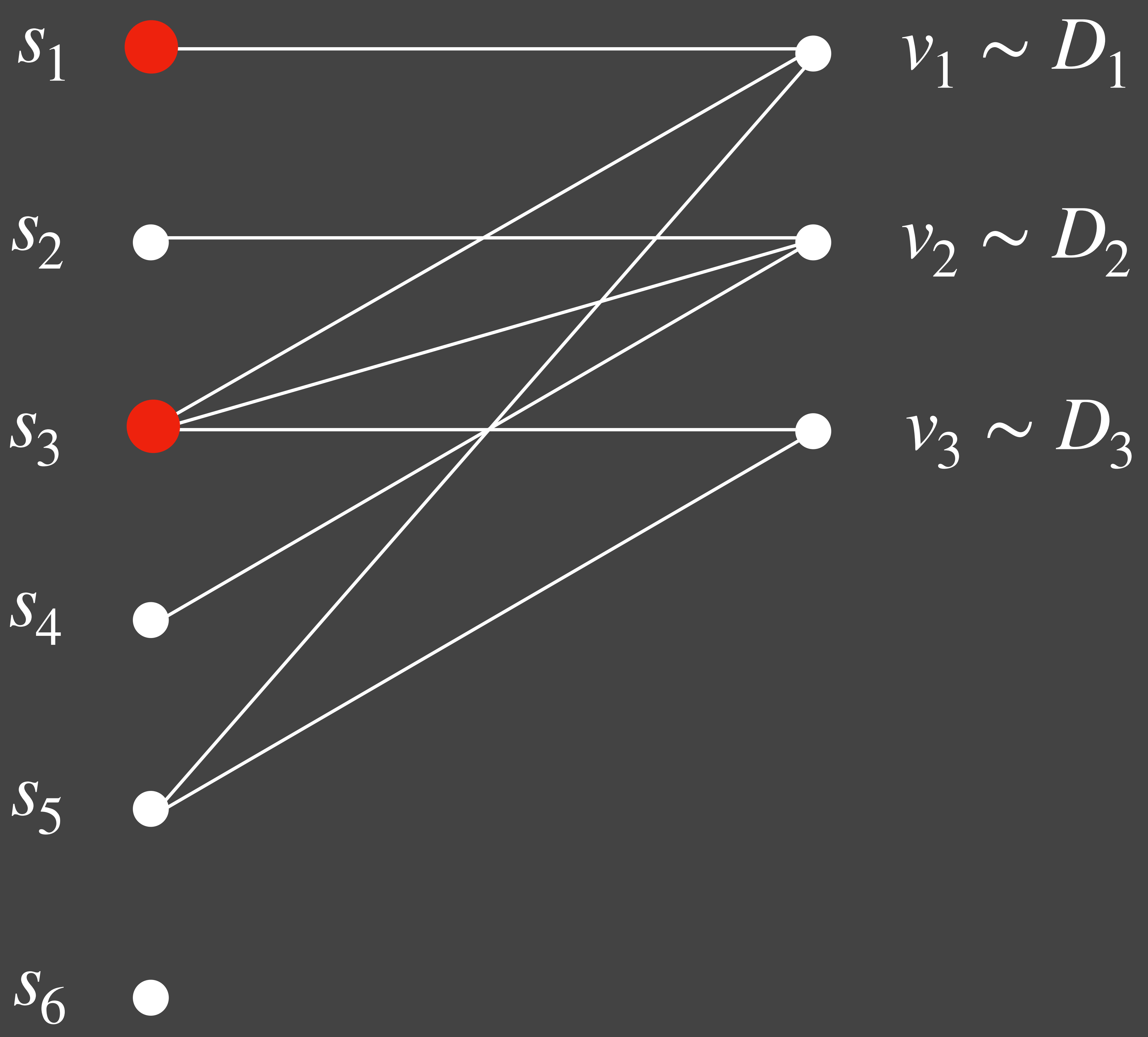
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets



s_1

s_2

s_3

s_4

s_5

s_6

$v_1 \sim D_1$

$v_2 \sim D_2$

$v_3 \sim D_3$

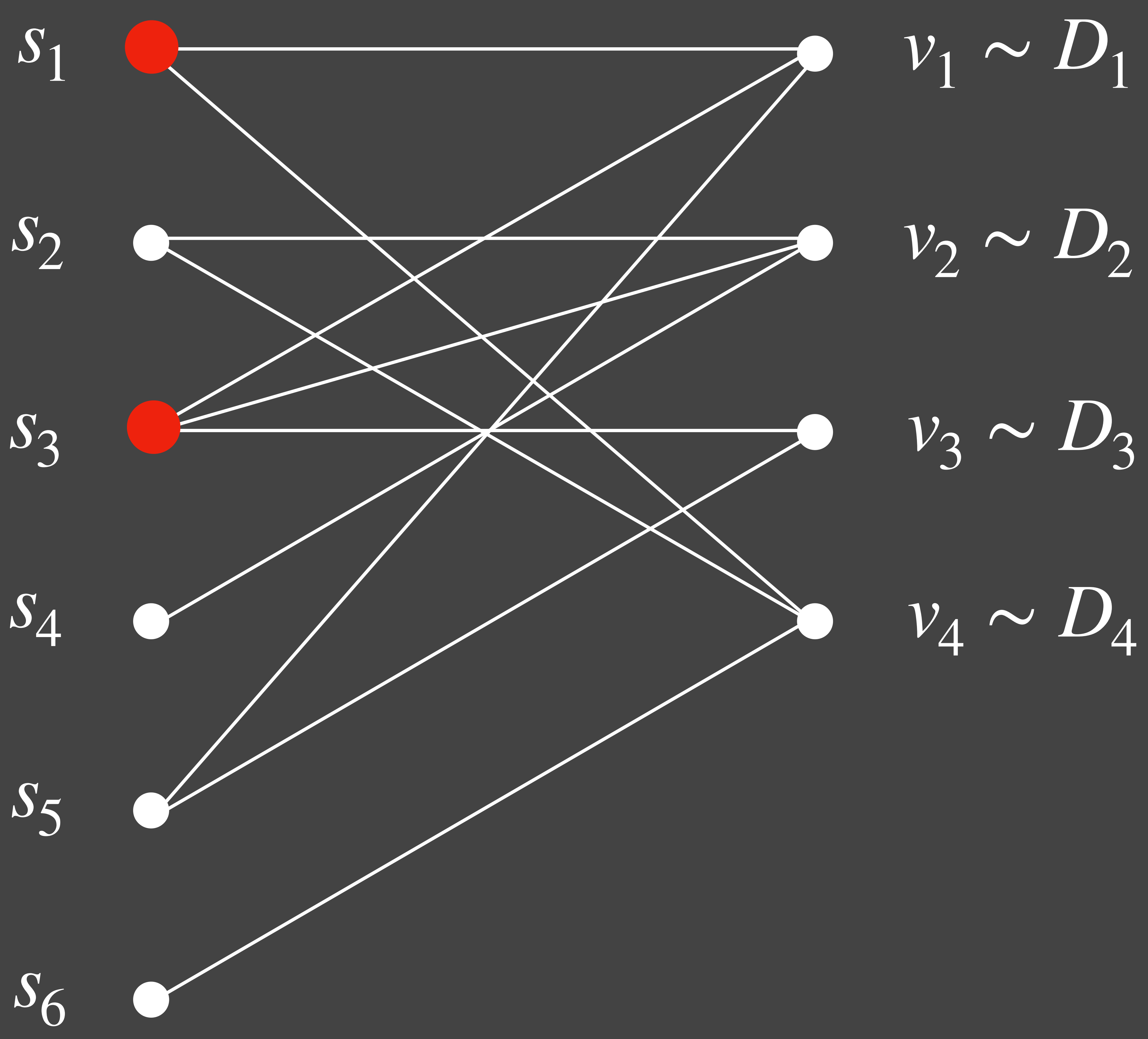
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets



s_1

s_2

s_3

s_4

s_5

s_6

$v_1 \sim D_1$

$v_2 \sim D_2$

$v_3 \sim D_3$

$v_4 \sim D_4$

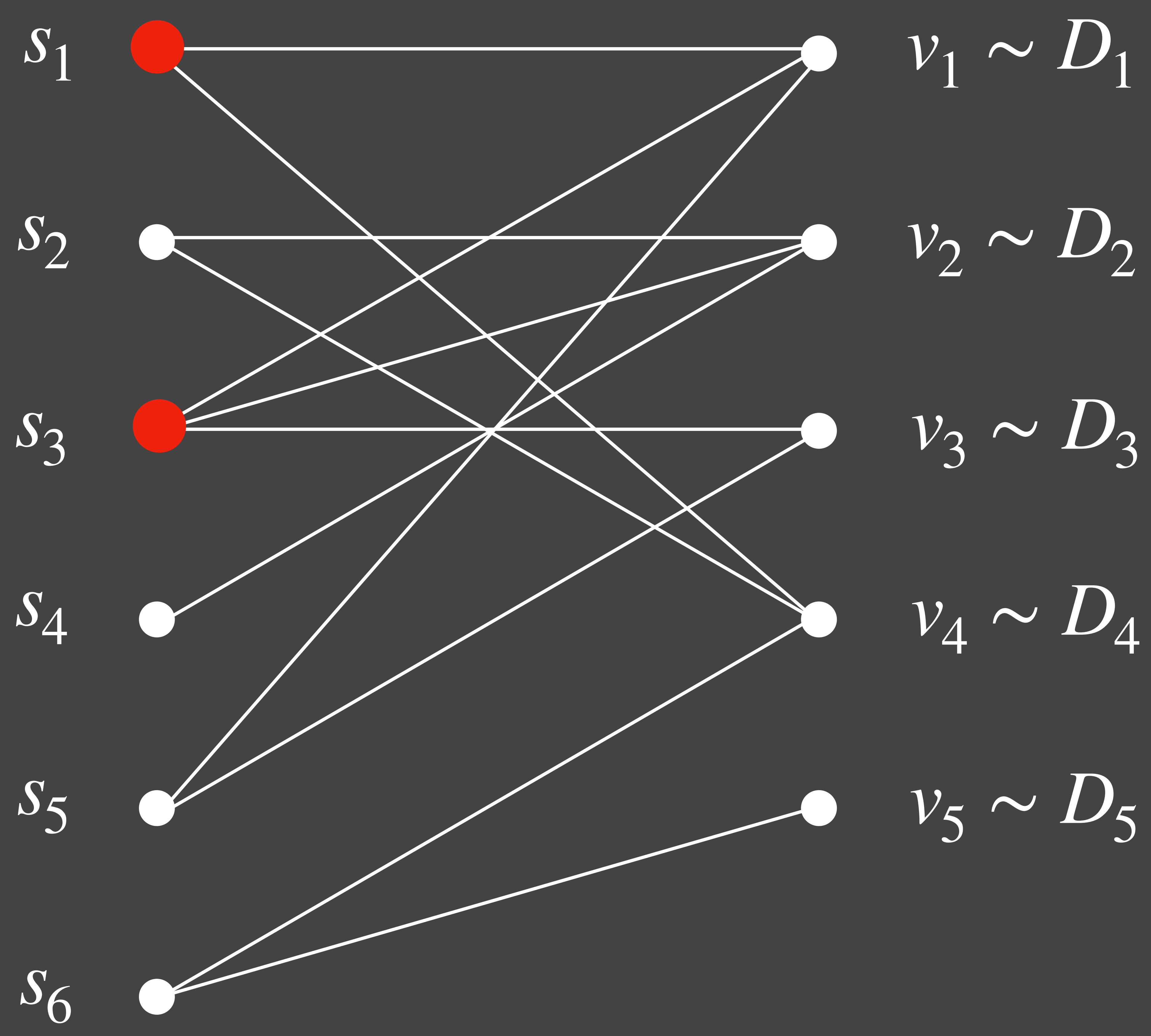
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets



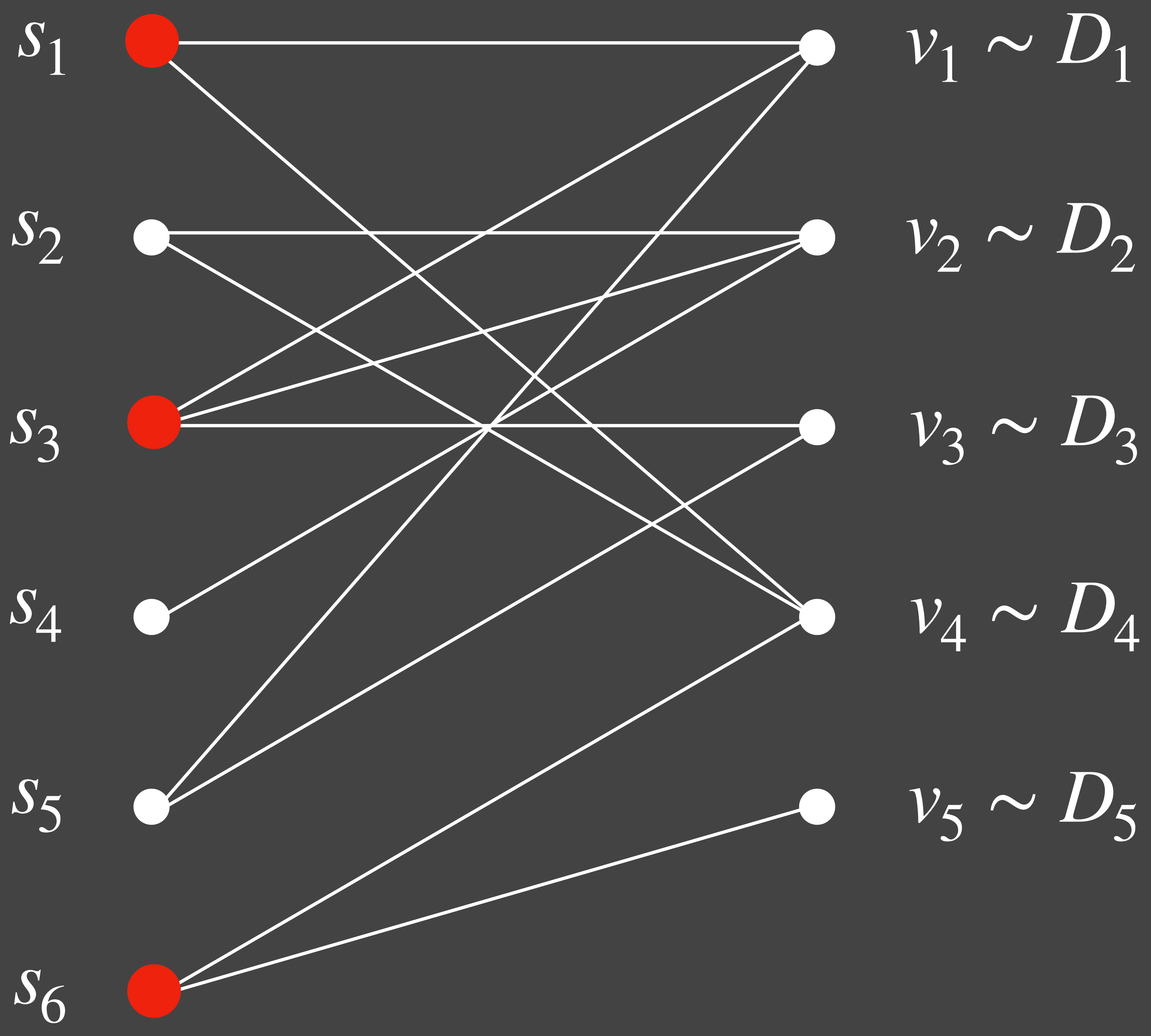
$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$

m sets

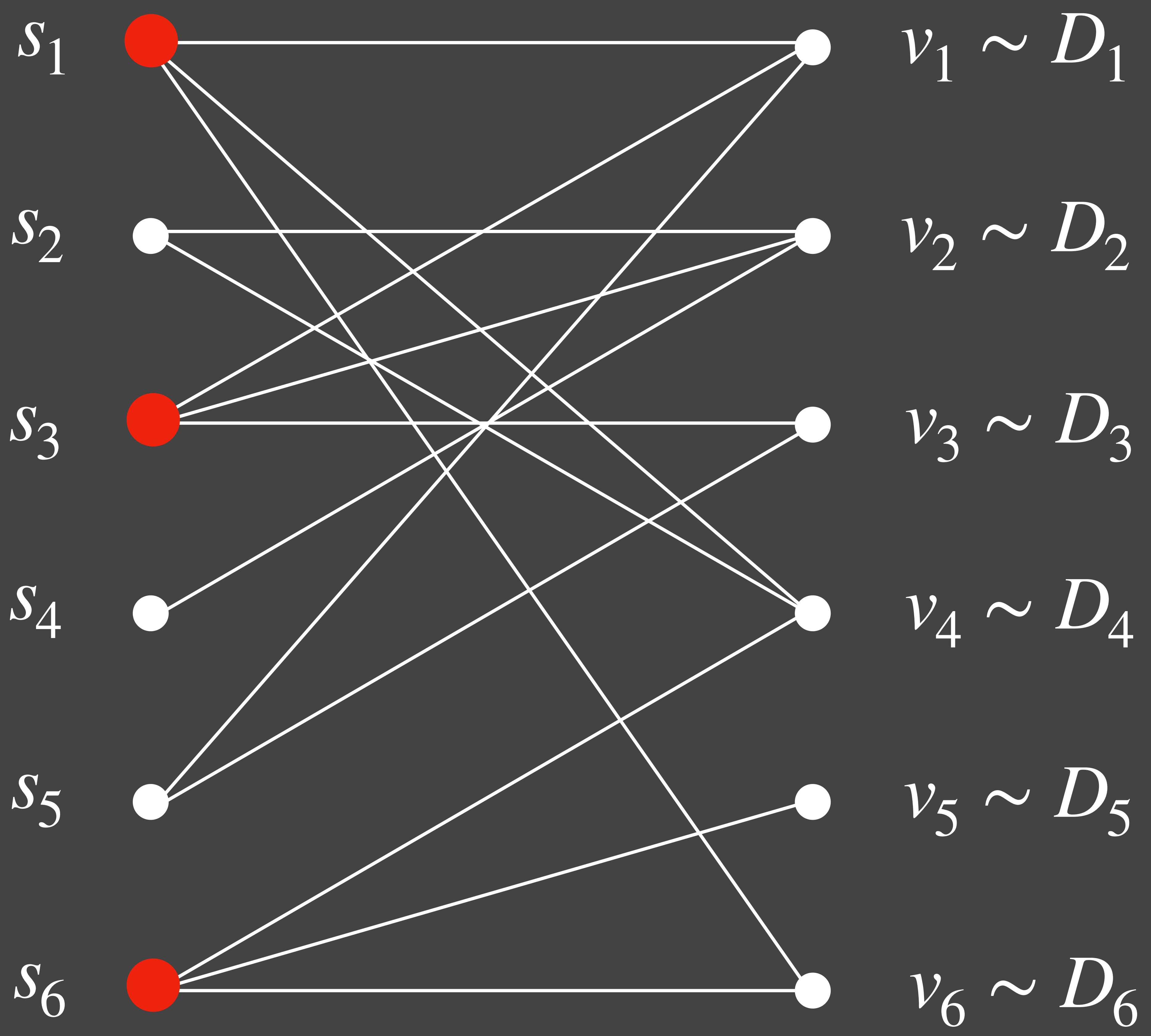


$\mathcal{U}$

n elements

Relaxation 2: Random Instance

$\mathcal{S}$
m sets



$\mathcal{U}$
n elements

The Landscape

$m =$ # sets

$n =$ # elements

The Landscape

$m =$ # sets

$n =$ # elements

Instance

		Random	Adversarial
Arrival Order	Random		
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

The Landscape

$m =$ # sets
$n =$ # elements

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

The Landscape

$m = $ # sets
$n = $ # elements

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

The Landscape

$m =$ # sets
$n =$ # elements

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary
	Adversarial	Prophet	O(log n log m) [Alon+ 03] [Buchbinder Naor

The Landscape

$m =$ # sets
$n =$ # elements

Instance

		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

Secretary

Prophet

Some reasons to believe $o(\log n \log m)$ not possible…

The Landscape

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	O(log mn) Our work
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

Secretary

Prophet

$m =$ # sets

$n =$ # elements

Theorem [Gupta Kehne L. FOCS 21]:

There is a poly time algorithm for secretary Covering IPs with competitive ratio $O(\log mn)$.

The Landscape

$m =$ # sets

$n =$ # elements

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	O(log mn) Our work
	Adversarial		O(log n log m) [Alon+ 03] [Buchbinder Naor

Secretary

Prophet

<u>Theorem</u> [Gupta Kehne L. FOCS 21]:

There is a poly time algorithm for <u>secretary</u> Covering IPs with competitive ratio $O(\log mn)$.

<u>New algorithm</u>! We show how to <u>learn</u> distribution & <u>solve</u> at same time.

The Landscape

$m =$ # sets
$n =$ # elements

		Instance	
		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	O(log mn) Our work
	Adversarial	New! O(log mn) Our work	O(log n log m) [Alon+ 03] [Buchbinder Naor

Secretary

Prophet

Theorem [Gupta Kehne L. FOCS 21]:

There is a poly time algorithm for secretary Covering IPs with competitive ratio $O(\log mn)$.

Theorem [Gupta Kehne L. 22]:

There is a poly time algorithm for prophet Covering IPs with competitive ratio $O(\log mn)$.

The Landscape

Instance

		Random	Adversarial
Arrival Order	Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary O(log mn) Our work
	Adversarial	New! O(log mn) Our work Prophet	O(log n log m) [Alon+ 03] [Buchbinder Naor

Bonus!

1. Only need 1 sample from each D_i!
2. Universal! Gives sample complexity bound $O(n)$.

$m =$ # sets

$n =$ # elements

Theorem [Gupta Kehne L. FOCS 21]:

There is a poly time algorithm for secretary Covering IPs with competitive ratio $O(\log mn)$.

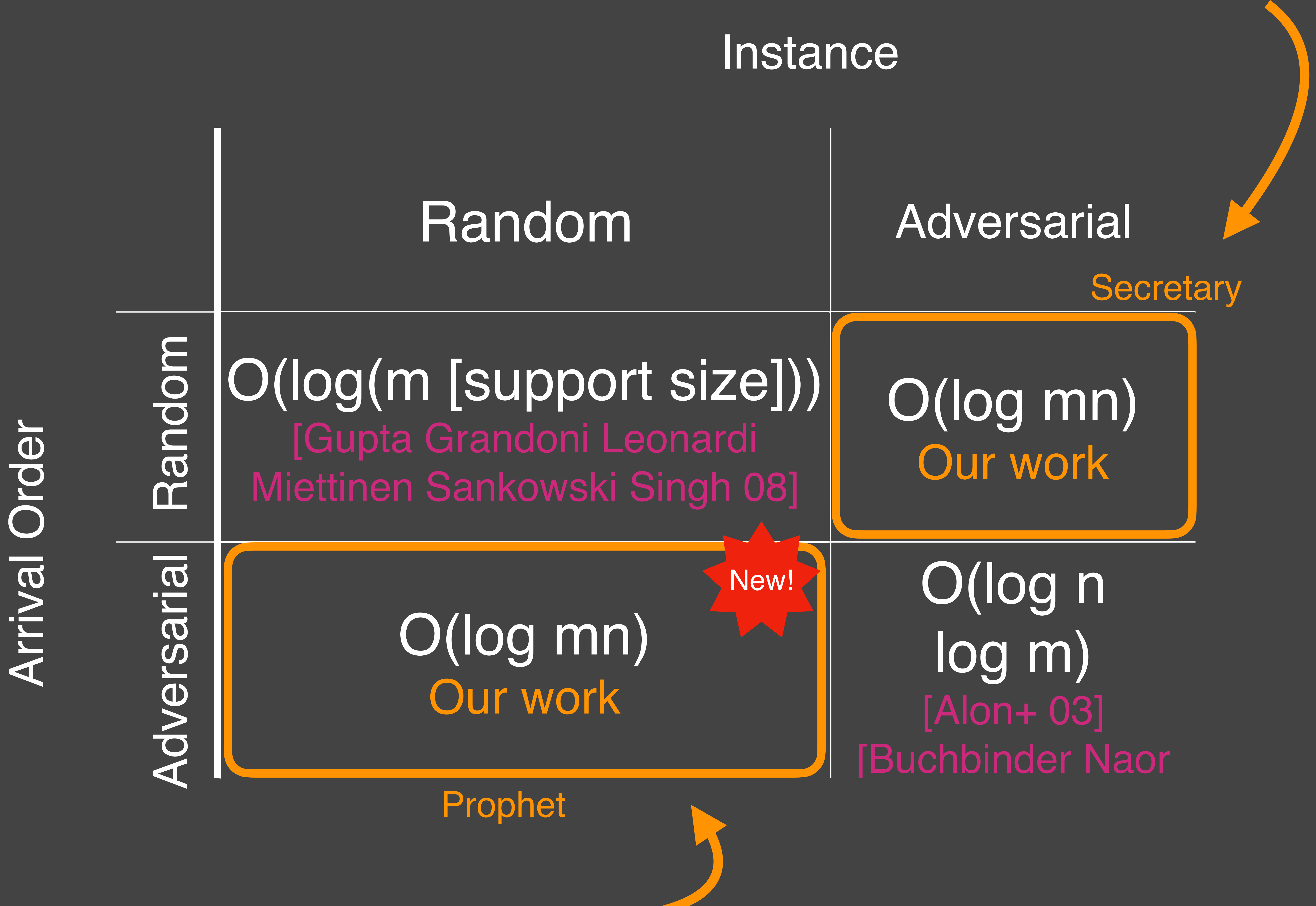
Theorem [Gupta Kehne L. 22]:

There is a poly time algorithm for prophet Covering IPs with competitive ratio $O(\log mn)$.

The Landscape

Bonus!
1-pass Streaming Algorithm with
$O(m)$ space!

m = # sets
n = # elements



Arrival Order \ Instance	Random	Adversarial
Random	O(log(m [support size])) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]	Secretary O(log mn) Our work
Adversarial	New! O(log mn) Our work Prophet	O(log n log m) [Alon+ 03] [Buchbinder Naor

Bonus!
1. Only need 1 sample from each D_i!
2. Universal! Gives sample complexity bound $O(n)$.

Theorem [Gupta Kehne L. FOCS 21]:

There is a poly time algorithm for secretary Covering IPs with competitive ratio $O(\log mn)$.

Theorem [Gupta Kehne L. 22]:

There is a poly time algorithm for prophet Covering IPs with competitive ratio $O(\log mn)$.

Online Covering IPs

$$\min \ c^\intercal x$$

$$a_1^\intercal x \geq 1$$
$$a_2^\intercal x \geq 1$$
$$a_3^\intercal x \geq 1$$
$$a_4^\intercal x \geq 1$$
$$a_5^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \; c^{\intercal} x$$

$$x \in \mathbb{Z}^m_{\geq 0}$$

Online Covering IPs

$$\min \ c^\intercal x$$

$$a_1^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \ c^\intercal x$$

$$a_1^\intercal x \geq 1$$

$$a_2^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \ c^\intercal x$$

$$a_1^\intercal x \geq 1$$

$$a_2^\intercal x \geq 1$$

$$a_3^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \quad c^\intercal x$$

$$a_1^\intercal x \geq 1$$

$$a_2^\intercal x \geq 1$$

$$a_3^\intercal x \geq 1$$

$$a_4^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\min \ c^\intercal x$$

$$a_1^\intercal x \geq 1$$
$$a_2^\intercal x \geq 1$$
$$a_3^\intercal x \geq 1$$
$$a_4^\intercal x \geq 1$$
$$a_5^\intercal x \geq 1$$

$$x \in \mathbb{Z}_{\geq 0}^m$$

Online Covering IPs

$$\begin{aligned} &\min \; c^\intercal x \\ &a_1^\intercal x \geq 1 \\ &a_2^\intercal x \geq 1 \\ &a_3^\intercal x \geq 1 \\ &a_4^\intercal x \geq 1 \\ &a_5^\intercal x \geq 1 \\ &x \in \mathbb{Z}_{\geq 0}^m \end{aligned}$$

Goal: Maintain feasible solution x that is *monotonically* increasing.

Online Covering IPs

$$\begin{aligned} &\min\ c^\intercal x \\ &a_1^\intercal x \geq 1 \\ &a_2^\intercal x \geq 1 \\ &a_3^\intercal x \geq 1 \\ &a_4^\intercal x \geq 1 \\ &a_5^\intercal x \geq 1 \\ &x \in \mathbb{Z}_{\geq 0}^m \end{aligned}$$

Goal: Maintain feasible solution x that is *monotonically* increasing.

Set Cover is the special case where constraint matrix A is 0/1.

Talk Outline

➡ Intro

Secretary

- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

➔ Secretary

- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Set Cover via Random Rounding

Set Cover via Random Rounding

2 Stage algorithm!

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

(II) Round.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathscr{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathscr{S} : \quad x_S \geq 0$$

(II) Round.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathscr{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathscr{S} : \quad x_S \geq 0$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\begin{aligned} &\min \sum_S x_S \\ \forall v \in \mathscr{U} : \quad &\sum_{S \ni v} x_S \geq 1 \\ \forall S \in \mathscr{S} : \quad &x_S \geq 0 \end{aligned}$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Can show $\forall v \in \mathscr{U}$, covered with constant prob.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathscr{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathscr{S} : \quad x_S \geq 0$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Can show $\forall v \in \mathscr{U}$, covered with constant prob.

Repeat $O(\log n)$ times, union bound.

Set Cover via Random Rounding

2 Stage algorithm!

(I) Solve LP.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

This is relaxation, so $c(x) \leq c(\text{OPT})$.

(II) Round.

Buy S with probability x_S.

Expected cost is $c(x)$!

Can show $\forall v \in \mathcal{U}$, covered with constant prob.

Repeat $O(\log n)$ times, union bound.

Expected Cost: $O(\log n) \cdot \text{OPT}$

How [Alon+ 03] works

How [Alon+ 03] works

Same 2 Stages!

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

(II) Round Online.

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(II) Round Online.

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

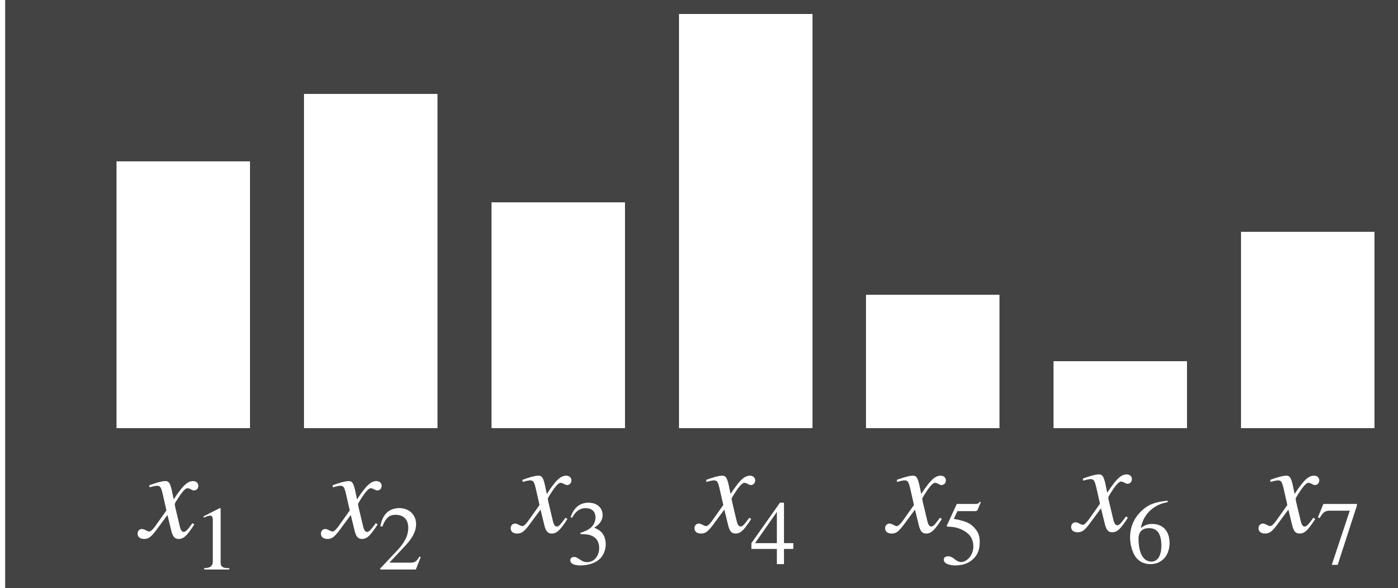
$$\min \sum_S x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.



How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

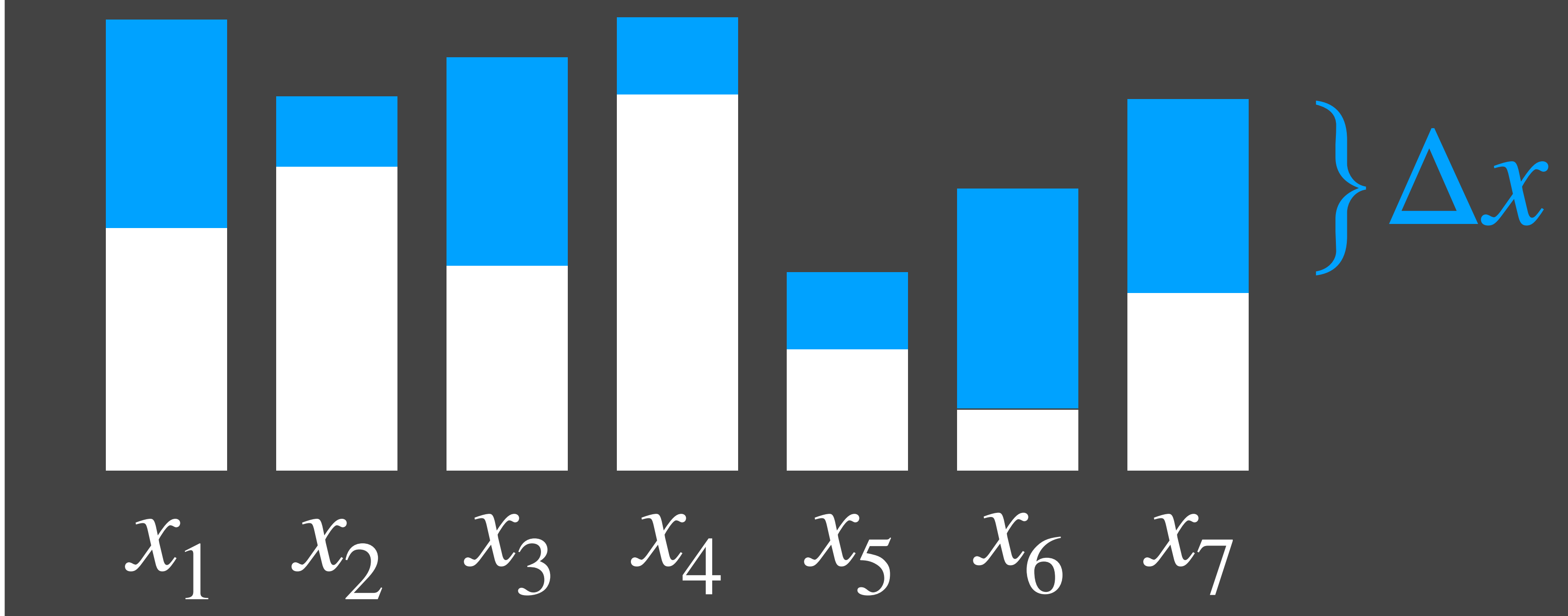
$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.



How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

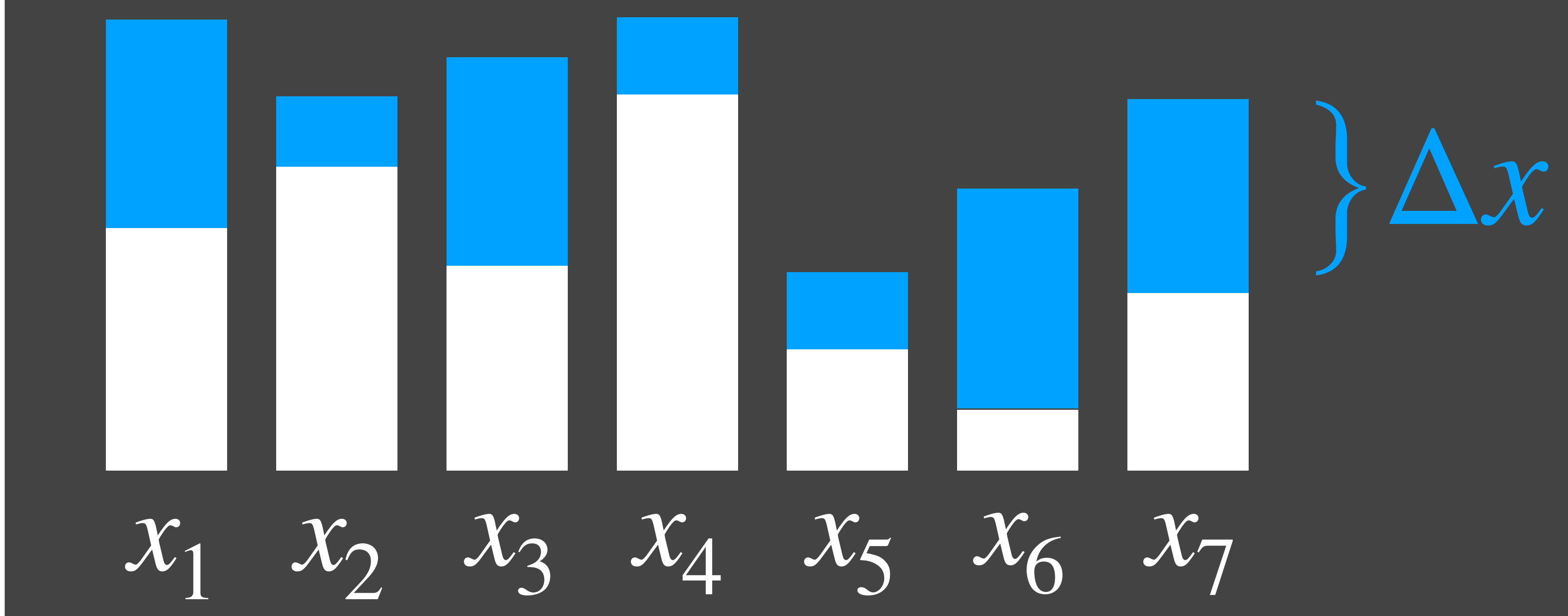
$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.



Take S with prob. $\propto \Delta x_S$.

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

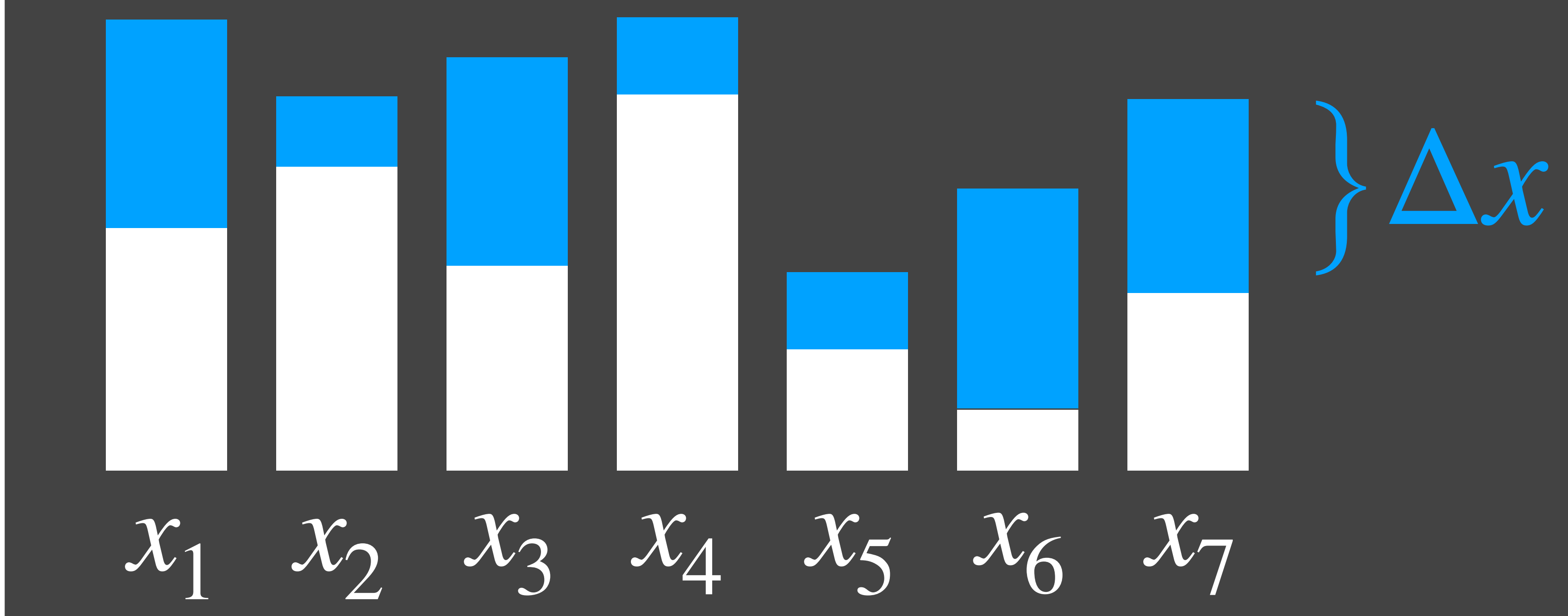
$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.



Take S with prob. $\propto \Delta x_S$.

Suffices to analyze ***offline*** rounding. Repeat $\log n$ times, union bound.

How [Alon+ 03] works

Same 2 Stages!

(I) Solve LP Online.

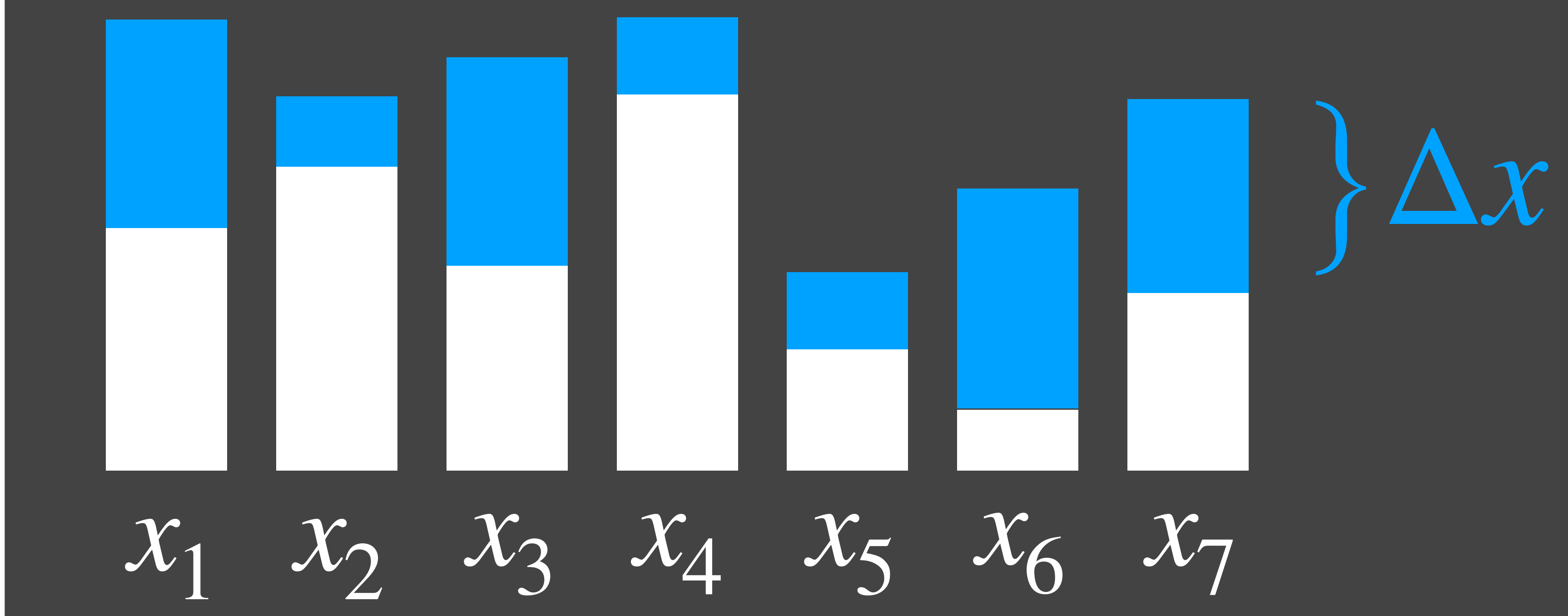
$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

Can guarantee x is $O(\log m)$-apx, and only increases ***monotonically.***

(II) Round Online.



Take S with prob. $\propto \Delta x_S$.

Suffices to analyze ***offline*** rounding. Repeat $\log n$ times, union bound.

Expected Cost: $O(\log n \log m) \cdot \text{OPT}$

Neither stage of [Alon+ 03] can be improved!

Neither stage of [Alon+ 03] can be improved!

Independent rounding loses $\Omega(\log n)$.

Neither stage of [Alon+ 03] can be improved!

Independent rounding loses $\Omega(\log n)$.

Theorem [Gupta Kehne L.]: $\Omega(\log m)$ **for fractional algos in RO.**

Neither stage of [Alon+ 03] can be improved!

Independent rounding loses $\Omega(\log n)$.

Theorem [Gupta Kehne L.]: $\Omega(\log m)$ for fractional algos in RO.

Theorem [Gupta Kehne L.]: algo of [Alon+ 03] gets $\Omega(\log m \log n)$ in RO.

Neither stage of [Alon+ 03] can be improved!

Independent rounding loses $\Omega(\log n)$.

Theorem [Gupta Kehne L.]: $\Omega(\log m)$ for fractional algos in RO.

Theorem [Gupta Kehne L.]: algo of [Alon+ 03] gets $\Omega(\log m \log n)$ in RO.

New algorithm needed!

Neither stage of [Alon+ 03] can be improved!

Independent rounding loses $\Omega(\log n)$.

Theorem [Gupta Kehne L.]: $\Omega(\log m)$ for fractional algos in RO.

Theorem [Gupta Kehne L.]: algo of [Alon+ 03] gets $\Omega(\log m \log n)$ in RO.

New algorithm needed!

We maintain coarse solution x, neither feasible nor monotone,
but round x anyway…

Talk Outline

Intro

➡ Secretary

LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

Secretary

➡ LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

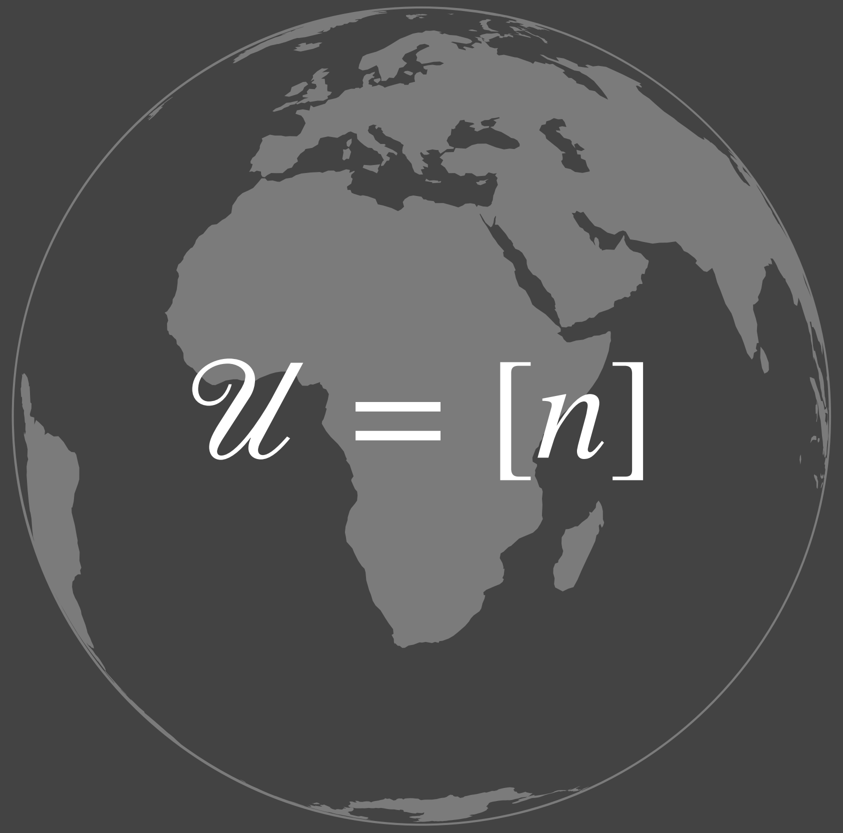
Conclusion & Extensions

LearnOrCover

(Unit cost, exp time warmup)

LearnOrCover

(Unit cost, exp time warmup)



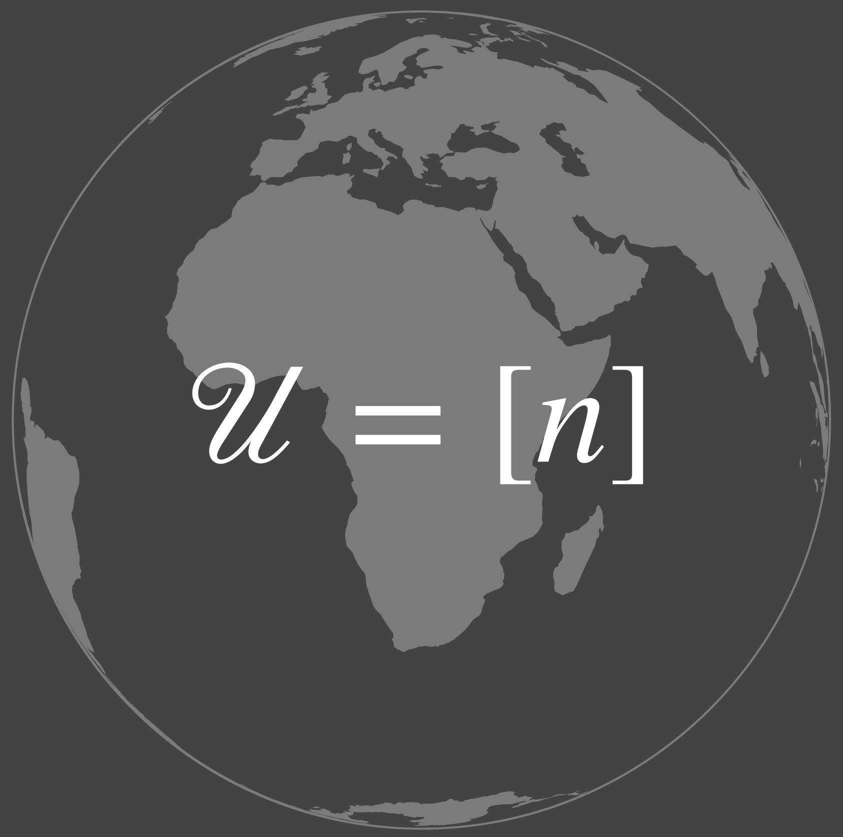
$\mathscr{U} = [n]$

$\mathscr{P} = \binom{\mathcal{S}}{k}$

$k := |OPT|$

LearnOrCover

(Unit cost, exp time warmup)



$$\mathscr{U} = [n]$$

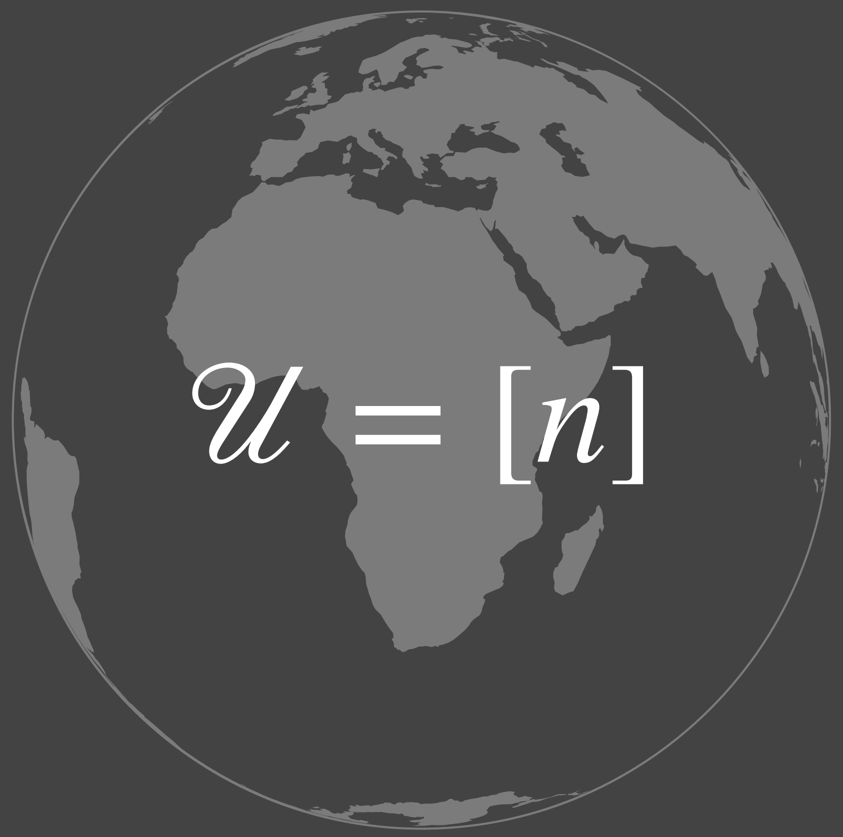
$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$k := |OPT|$

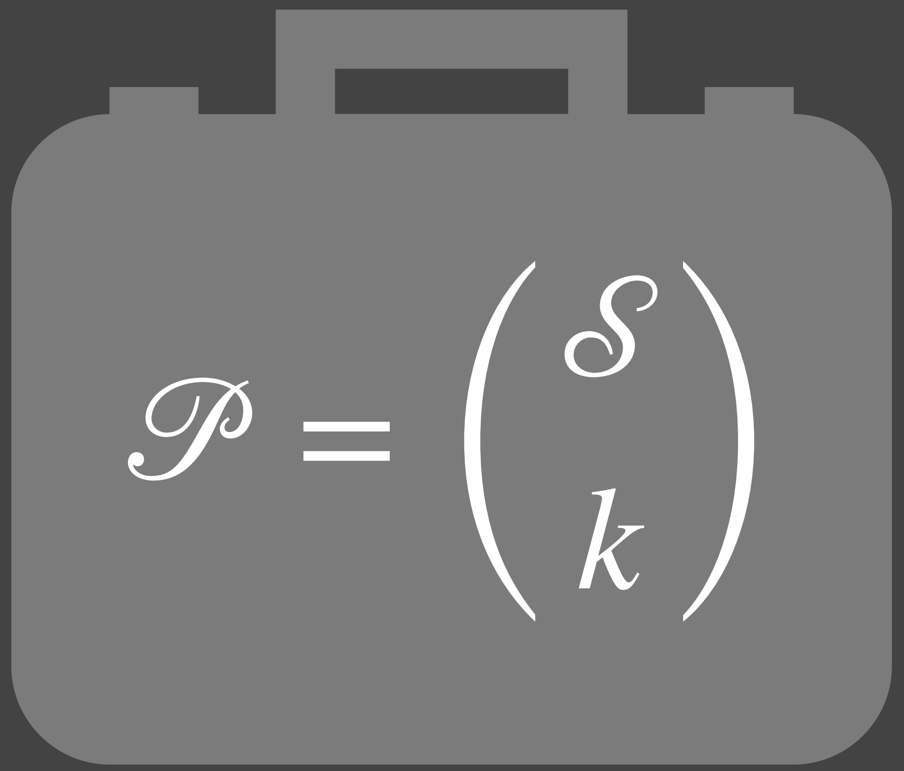
@ time t, element v arrives:

LearnOrCover

(Unit cost, exp time warmup)



$$\mathscr{U} = [n]$$



$$\mathscr{P} = \binom{\mathcal{S}}{k}$$

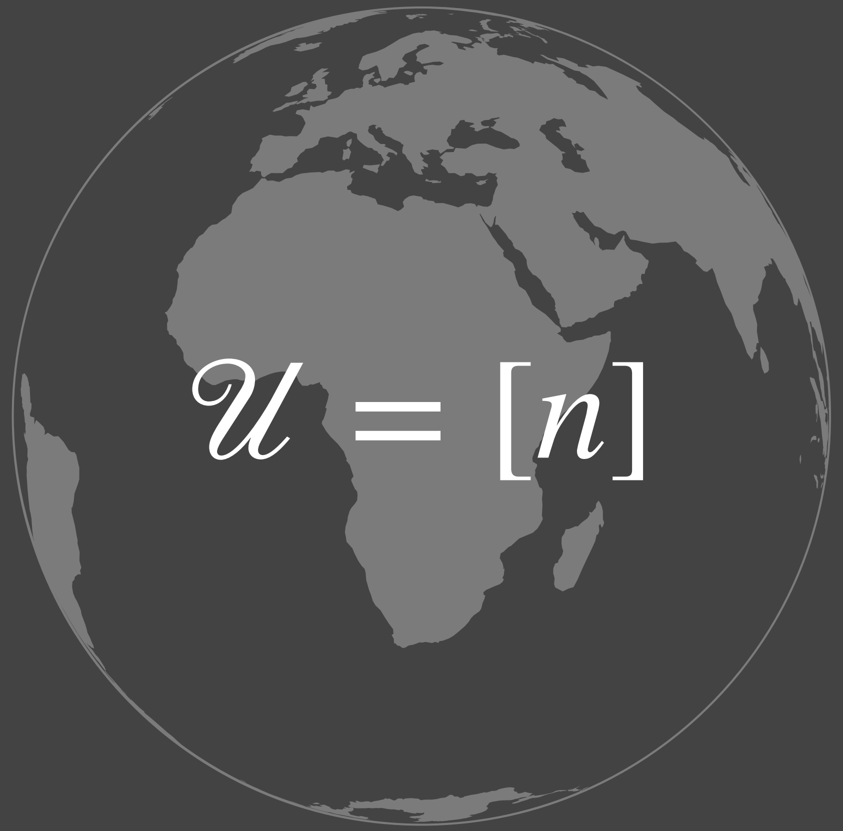
$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$

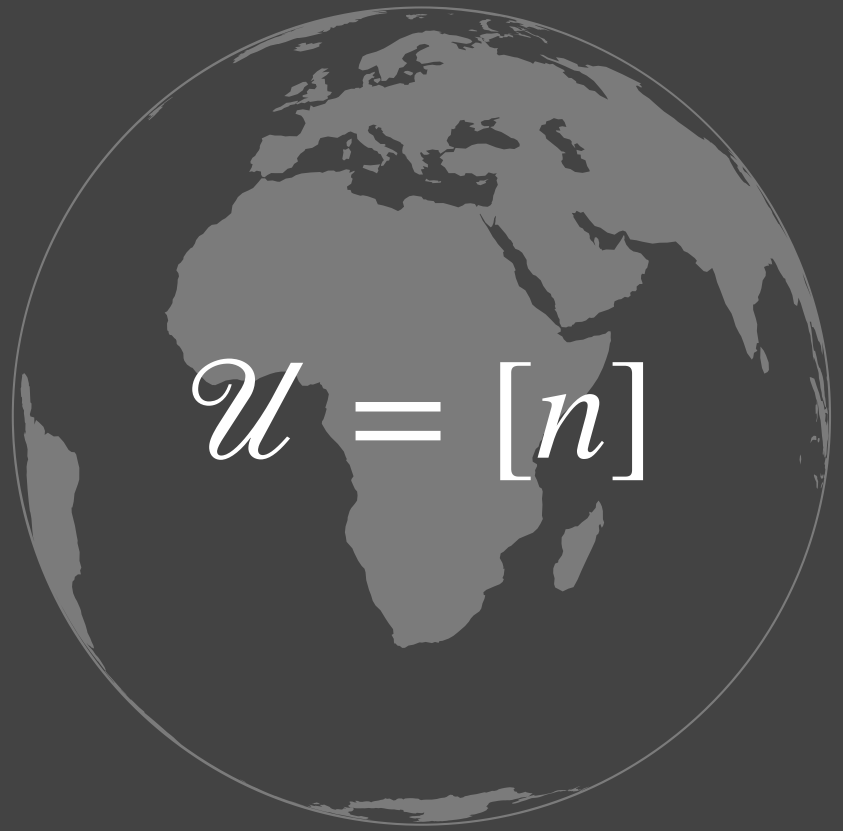
$\mathscr{P} = \binom{\mathcal{S}}{k}$

$k := |OPT|$

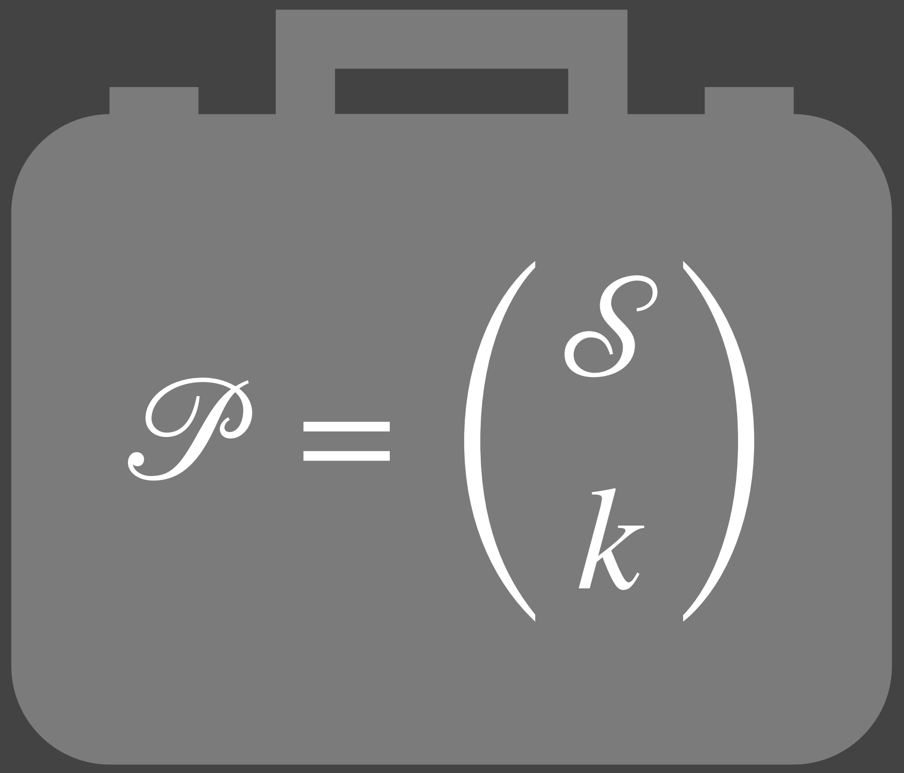
```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.
```

LearnOrCover

(Unit cost, exp time warmup)



$$\mathscr{U} = [n]$$



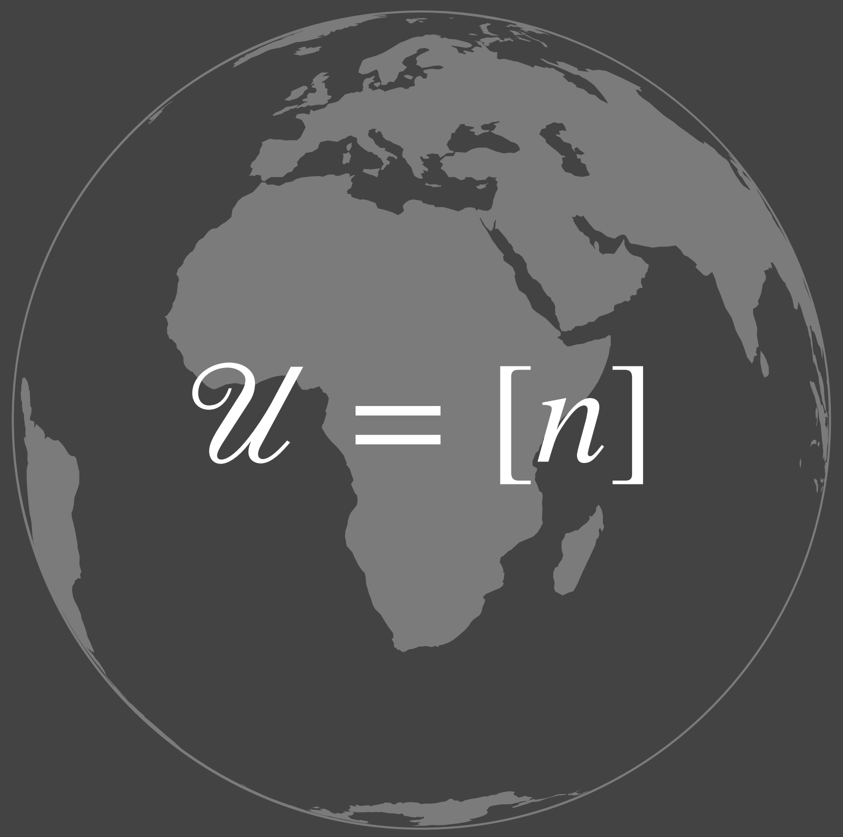
$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$k := |OPT|$

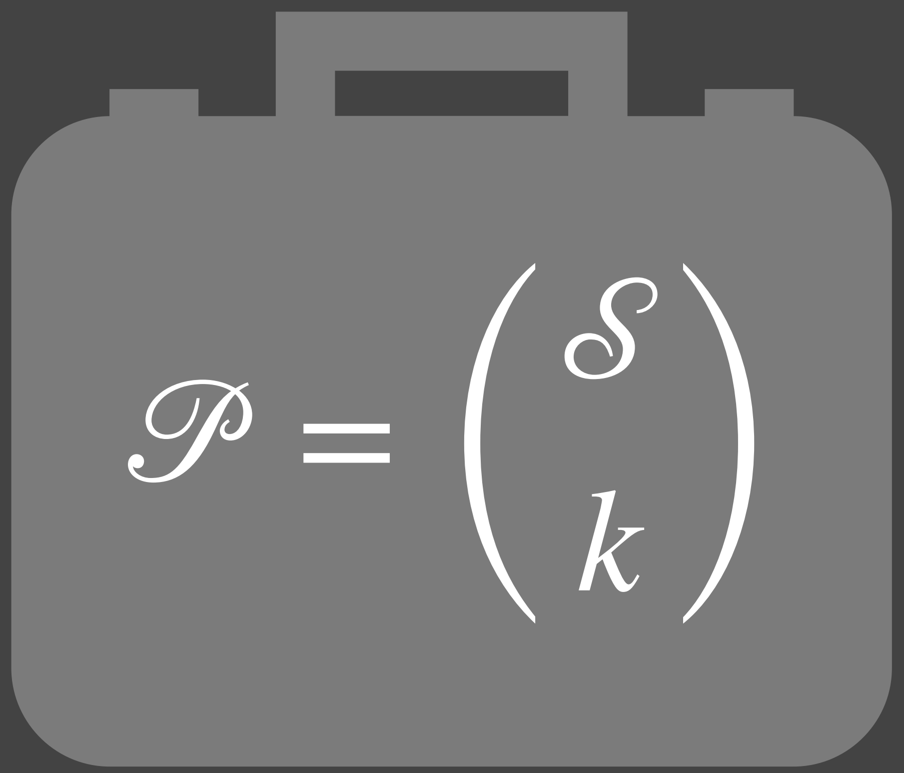
```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

LearnOrCover

(Unit cost, exp time warmup)



$$\mathscr{U} = [n]$$



$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$k := |OPT|$

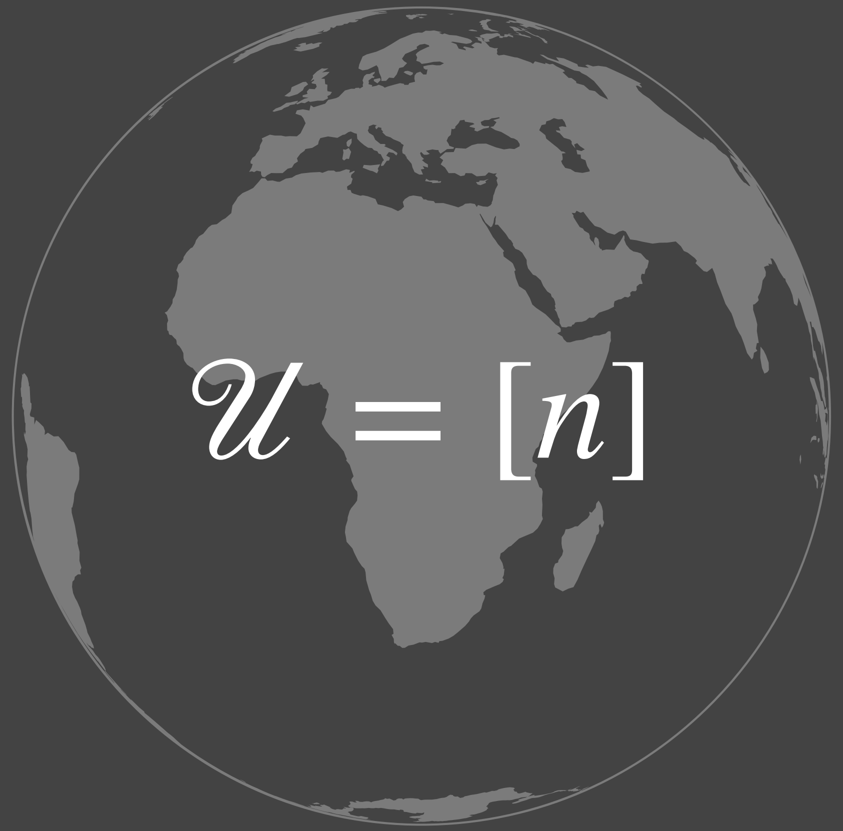
```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$

$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
(II) “Prune” $P \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

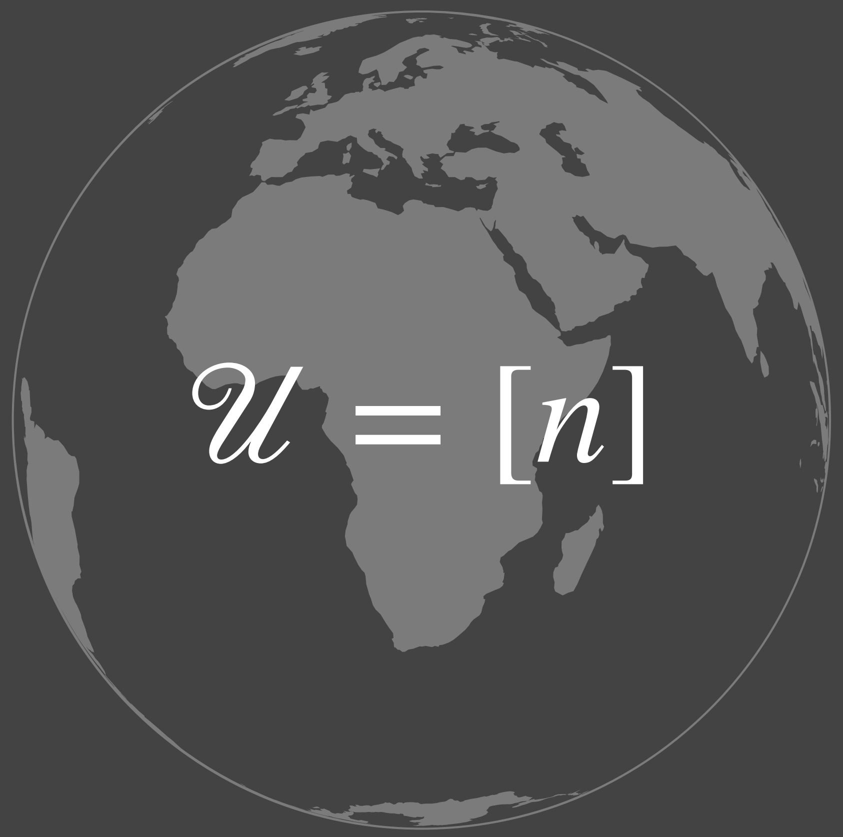
Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

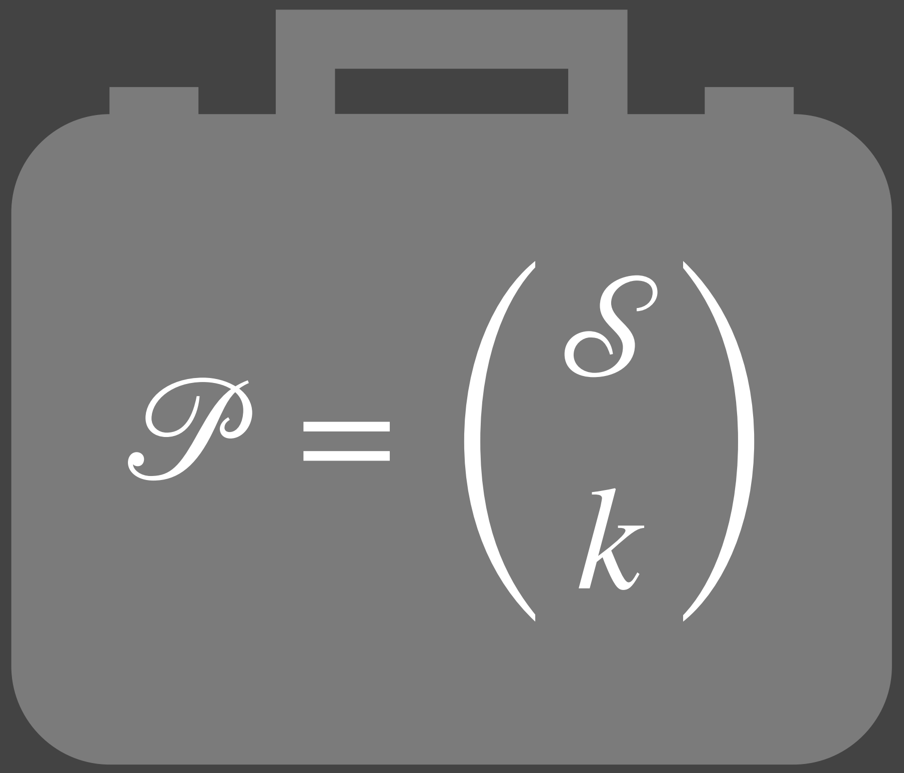
Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.

(II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

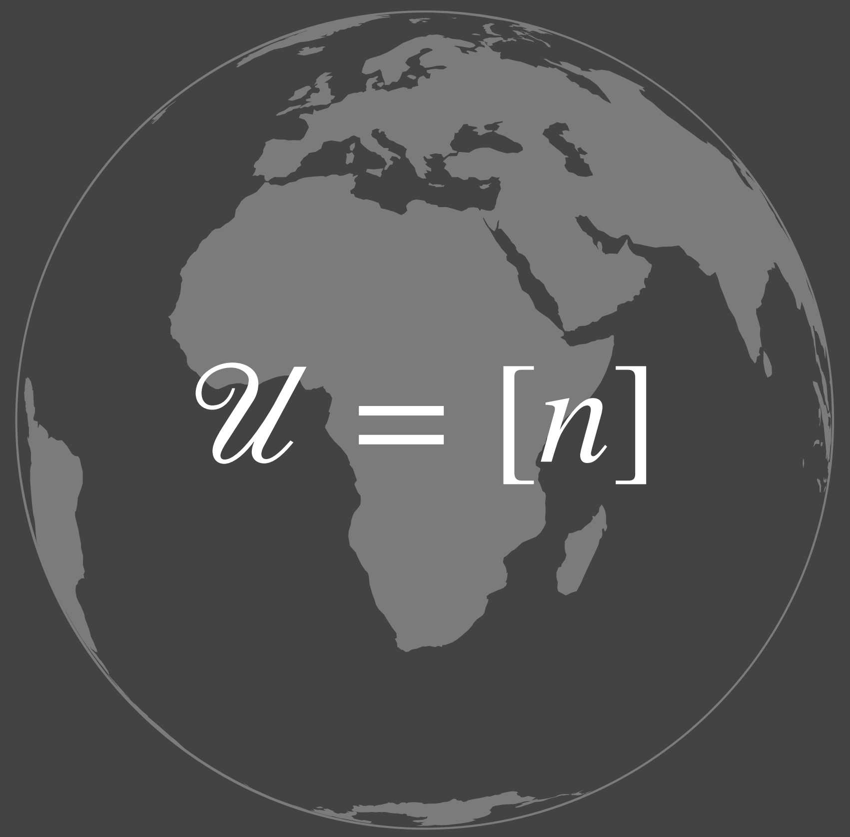
R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$

$\mathscr{P} = \binom{\mathcal{S}}{k}$

$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.

(II) “Prune” $P \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ of $P \in \mathscr{P}$ cover $\geq 1/2$ of $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

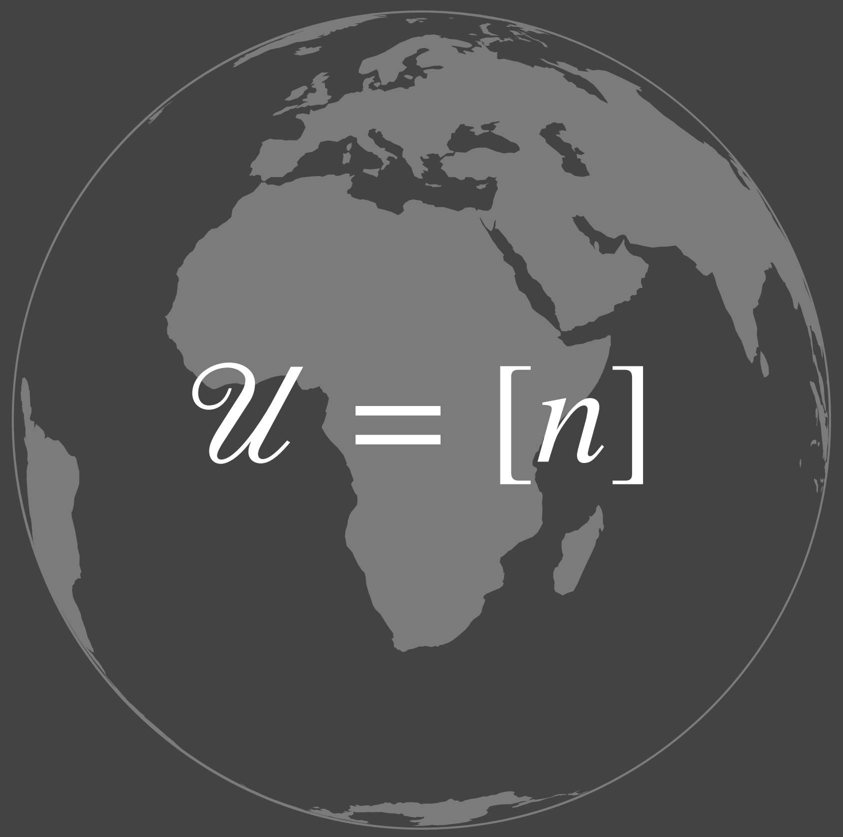
$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ of $P \in \mathscr{P}$ cover $< 1/2$ of $\mathscr{U}$.

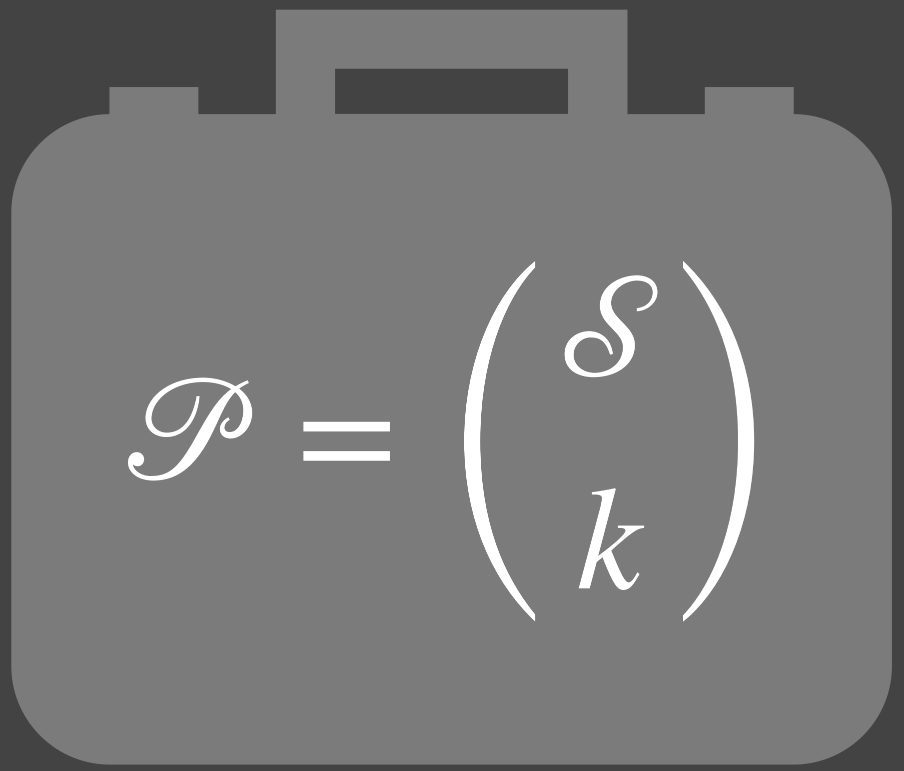
$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

```
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$**.**

R covers $\frac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$**.**

$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

$\mathscr{P}$ shrinks by $3/4$ in expectation.

LearnOrCover

(Unit cost, exp time warmup)

$\mathscr{U} = [n]$

$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

R covers $\frac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

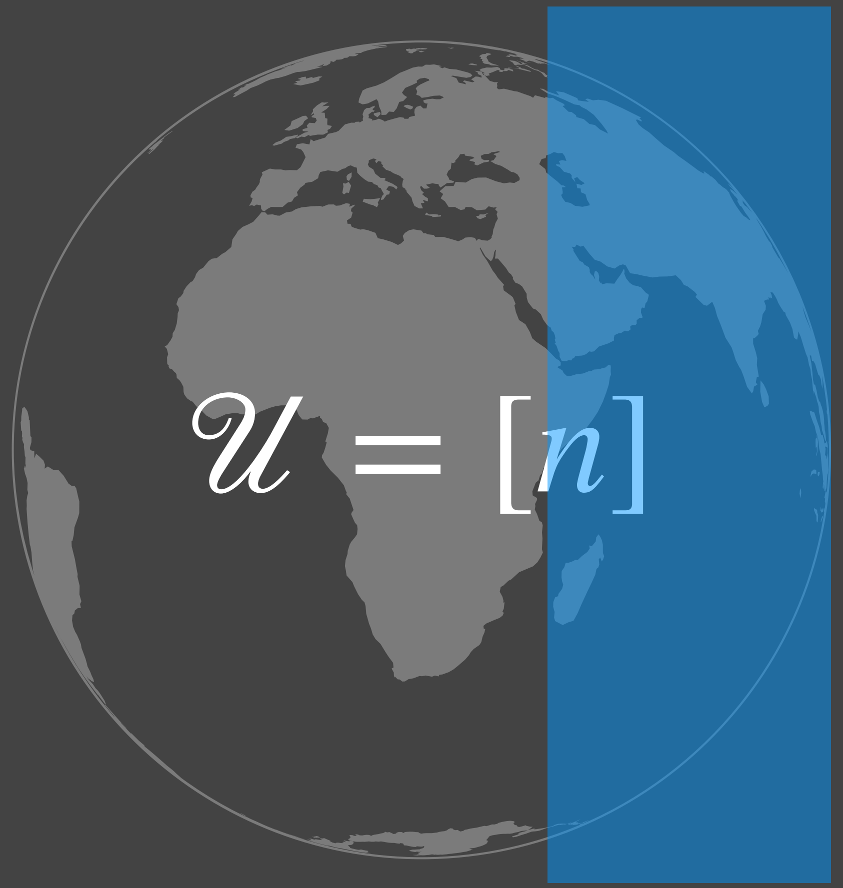
Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

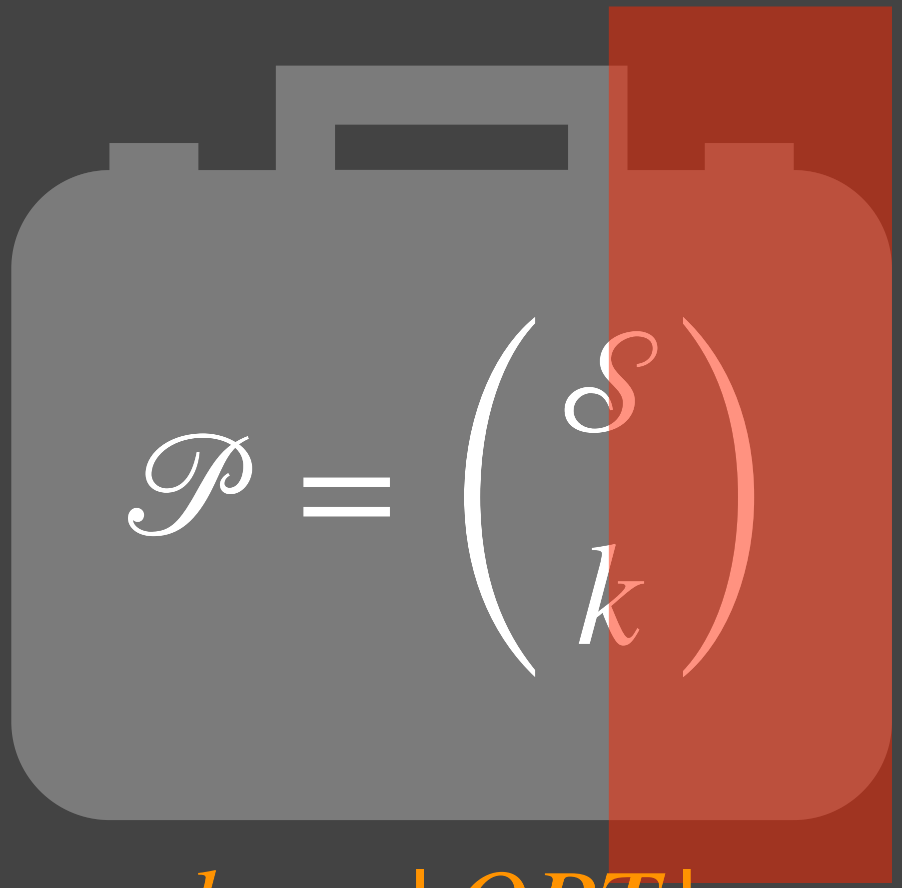
$\mathscr{P}$ shrinks by $3/4$ in expectation.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
(II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ of $P \in \mathscr{P}$ cover $\geq 1/2$ of $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

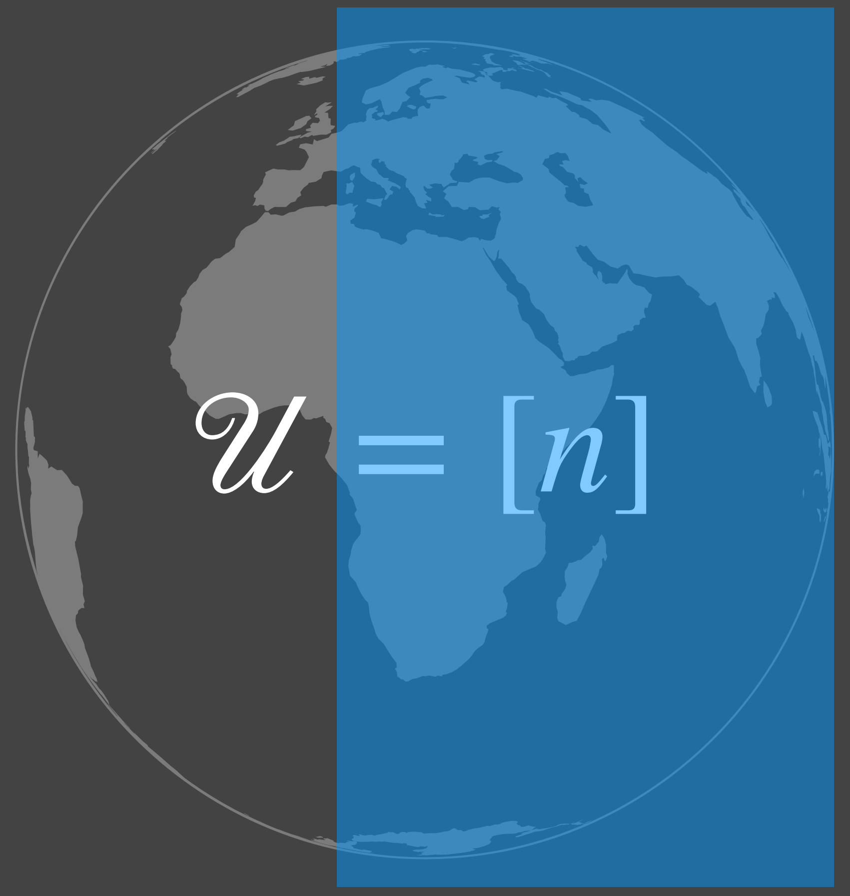
Case 2: $> 1/2$ of $P \in \mathscr{P}$ cover $< 1/2$ of $\mathscr{U}$.

$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

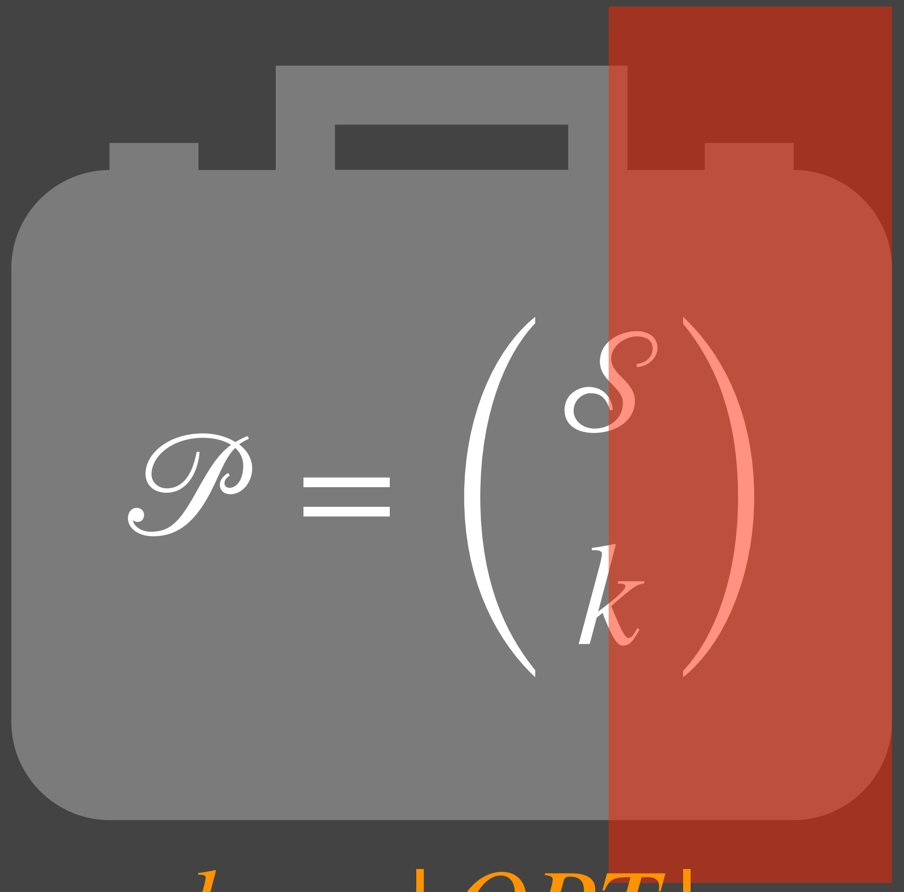
$\mathscr{P}$ shrinks by $3/4$ in expectation.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

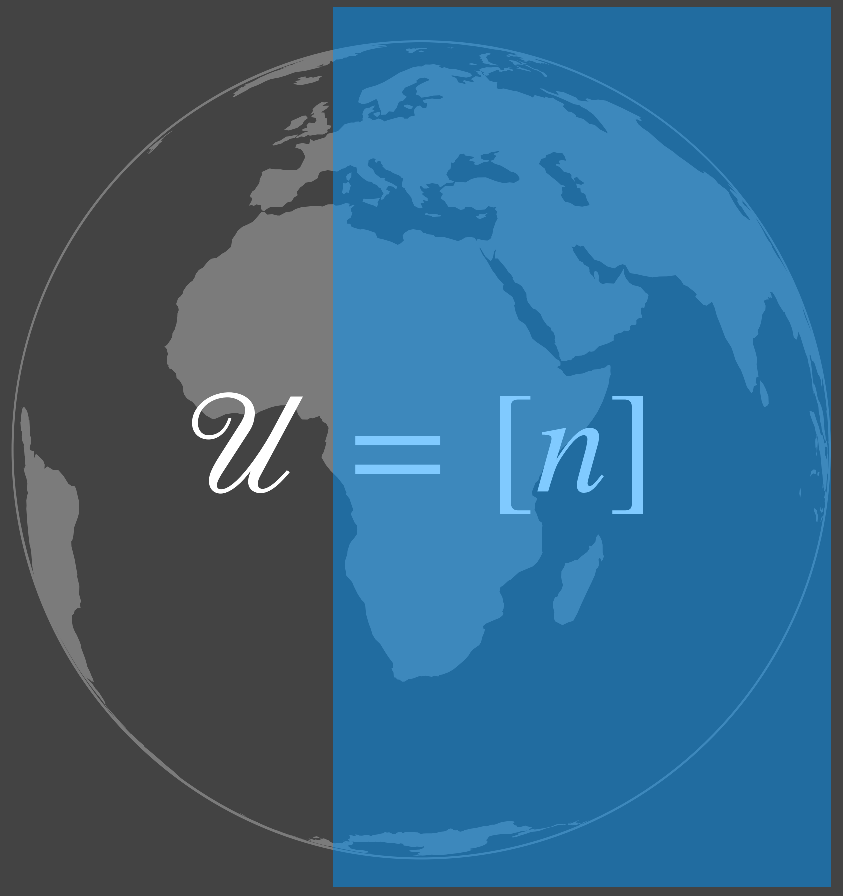
Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

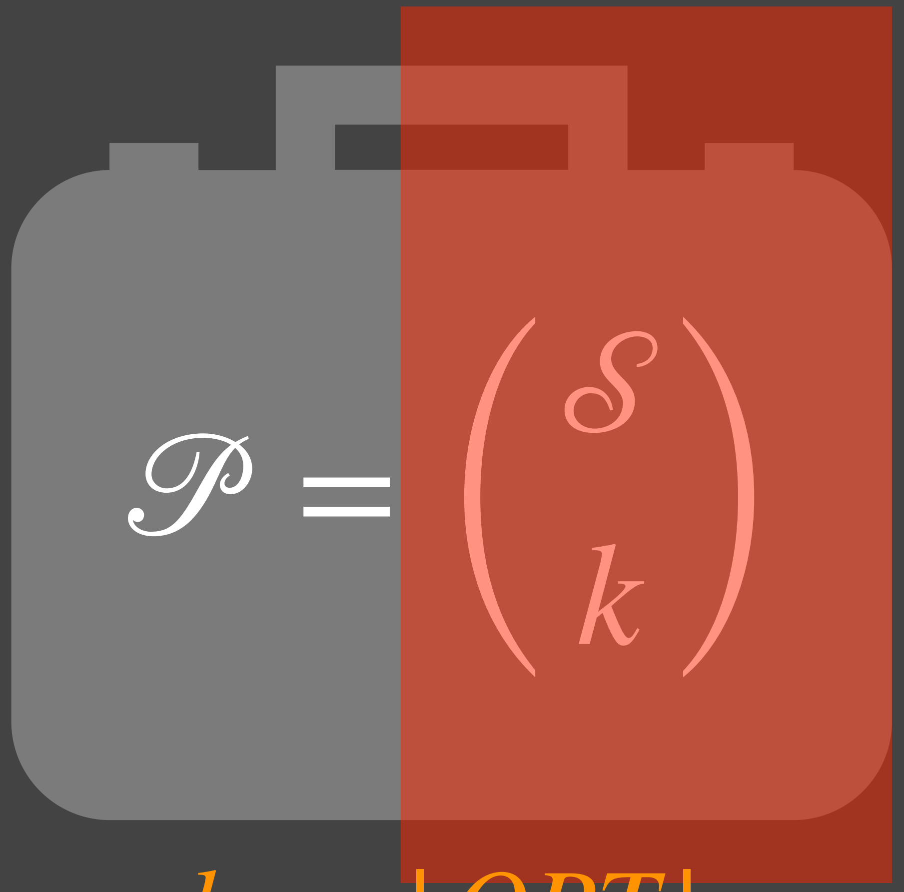
$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

$\mathscr{P}$ shrinks by $3/4$ in expectation.

LearnOrCover

(Unit cost, exp time warmup)





$k := |OPT|$

@ time t, element v arrives:

If v covered, do nothing.

Else:

- (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
- (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

Buy arbitrary set to cover v.

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

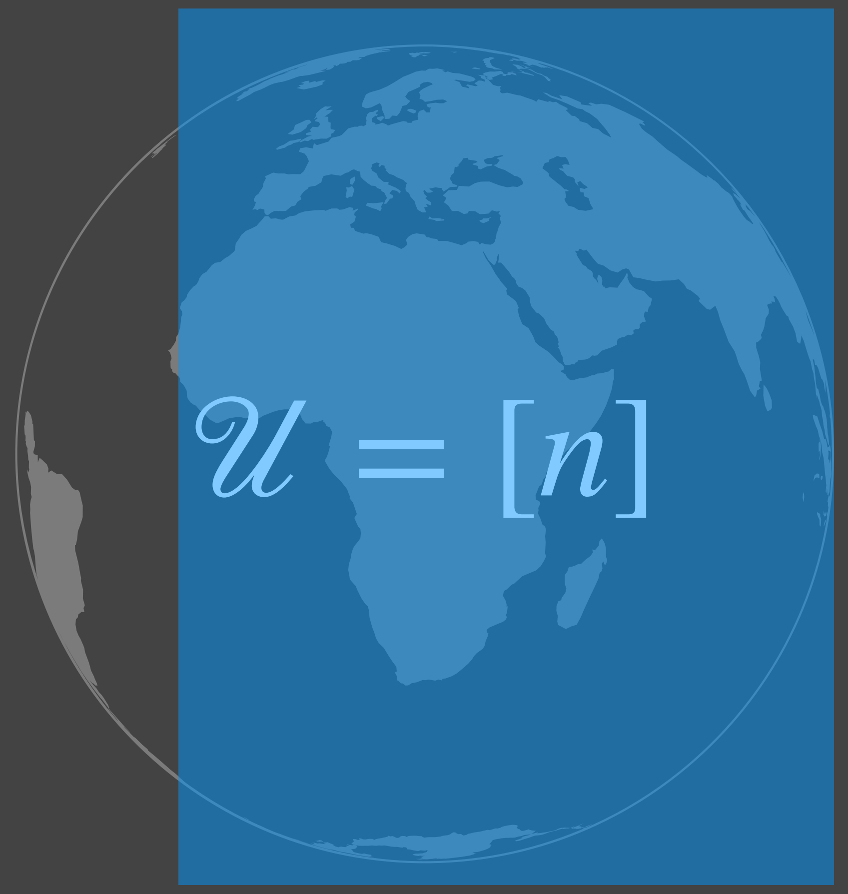
Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

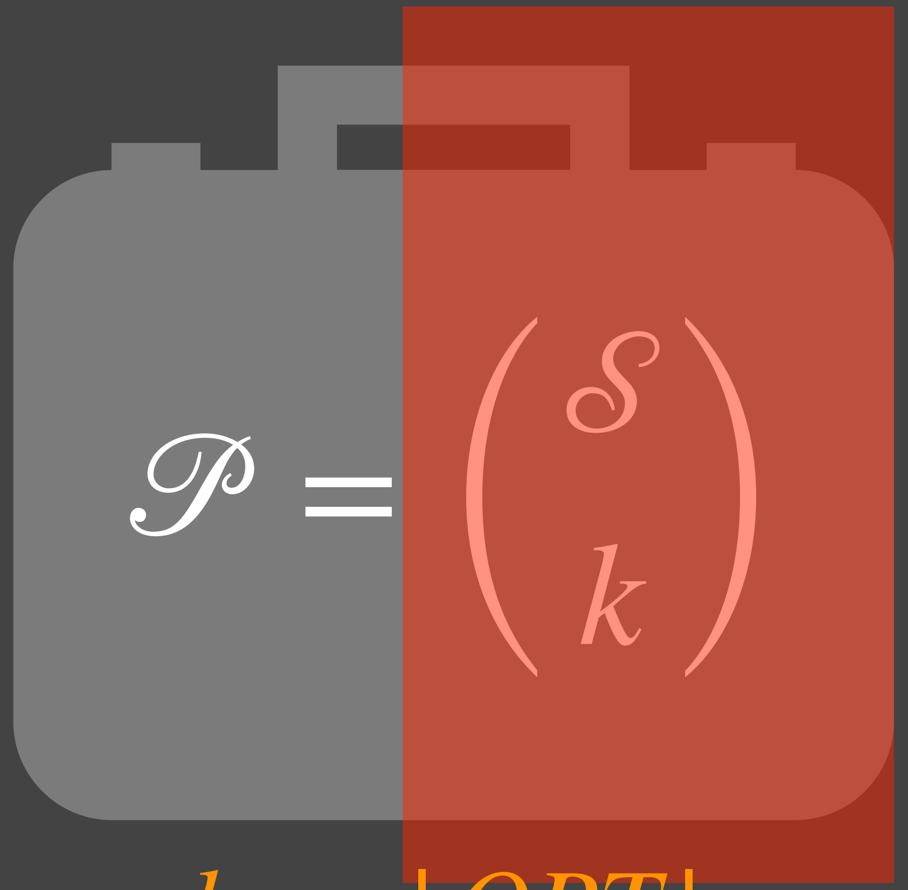
$\mathscr{P}$ shrinks by $3/4$ in expectation.

LearnOrCover

(Unit cost, exp time warmup)



$\mathscr{U} = [n]$



$\mathscr{P} = \binom{\mathscr{S}}{k}$

$k := |OPT|$

```
@ time t, element $v$ arrives:
If $v$ covered, do nothing.
Else:
    (I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.
    (II) "Prune" $P \not\ni v$ from $\mathscr{P}$.

    Buy arbitrary set to cover $v$.
```

Case 1: $\geq 1/2$ **of** $P \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

R covers $\frac{|\mathscr{U}|}{4k}$ in expectation.

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: $> 1/2$ **of** $P \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

$\geq 1/2$ of $P \in \mathscr{P}$ pruned w.p. $1/2$.

$\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

|$\mathscr{U}$| initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

$|\mathscr{U}|$ initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

$|\mathscr{P}|$ initially $\binom{m}{k} \approx m^k$, $\Rightarrow$ $O(k \log m)$ LEARN steps suffice.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

|$\mathscr{U}$| initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

|$\mathscr{P}$| initially $\binom{m}{k} \approx m^k$, $\Rightarrow$ $O(k \log m)$ LEARN steps suffice.

$\Rightarrow O(k \log mn)$ steps suffice.

RO Set Cover

(Exponential Time Warmup)

Case 1: (COVER)

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: (LEARN)

$\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

$$\boxed{\Phi = \frac{1}{k} \log |\mathscr{P}| + \log |\mathscr{U}|}$$

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

$$\boxed{\Phi = \frac{1}{k}\log|\mathscr{P}| + \log|\mathscr{U}|}$$

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

$$\boxed{\Phi = \frac{1}{k} \log |\mathscr{P}| + \log |\mathscr{U}|}$$

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\Omega\left(\frac{1}{k}\right)$.

RO Set Cover

(Exponential Time Warmup)

Case 1: **(COVER)**

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: **(LEARN)**

$\mathscr{P}$ shrinks by $3/4$ in expectation.

$$\boxed{\Phi = \frac{1}{k}\log|\mathscr{P}| + \log|\mathscr{U}|}$$

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\Omega\left(\frac{1}{k}\right)$.

But how to make polytime?

Can we reuse LEARN/ COVER intuition?

Talk Outline

Intro

Secretary

➡ LearnOrCover in Exponential Time

LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

Secretary

- LearnOrCover in Exponential Time
- ➡ LearnOrCover in Poly Time

(Single Sample) Prophet

Conclusion & Extensions

LearnOrCover

(Unit cost)

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
```

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
```

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
```

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
```

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
      Buy arbitrary set to cover $v$.
```

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
        Buy arbitrary set to cover $v$.
```

Idea! Measure convergence with potential function:

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
        Buy arbitrary set to cover $v$.
```

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) + c_2 \, \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
        Buy arbitrary set to cover $v$.
```

$$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$$

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) \; + c_2 \, \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
        Buy arbitrary set to cover $v$.
```

$$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$$

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) \; + c_2 \, \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
        Buy arbitrary set to cover $v$.
```

$$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$$

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) + c_2 \, \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\frac{1}{k}$.

(Recall $k = |OPT|$)

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.

    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.

        Buy arbitrary set to cover $v$.
```

$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* || x^t) + c_2 \, \log |\mathcal{U}^t|}$$

$\mathcal{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\frac{1}{k}$.

(Recall $k = |OPT|$)

Bound $E_R[\Delta \log |\mathcal{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v.

LearnOrCover

(Unit cost)

```
Init. $x \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy random $R \sim x$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
             Renormalize $x \leftarrow x/||x||_1$.
      Buy arbitrary set to cover $v$.
```

$$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$$

Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* || x^t) + c_2 \, \log |\mathscr{U}^t|}$$

$\mathscr{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\frac{1}{k}$.

(Recall $k = |OPT|$)

Bound $E_R[\Delta \log |\mathscr{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v. ⟵ This is where we use RO!

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \le -E_v\left[\sum_{S \ni v} x_S\right].$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

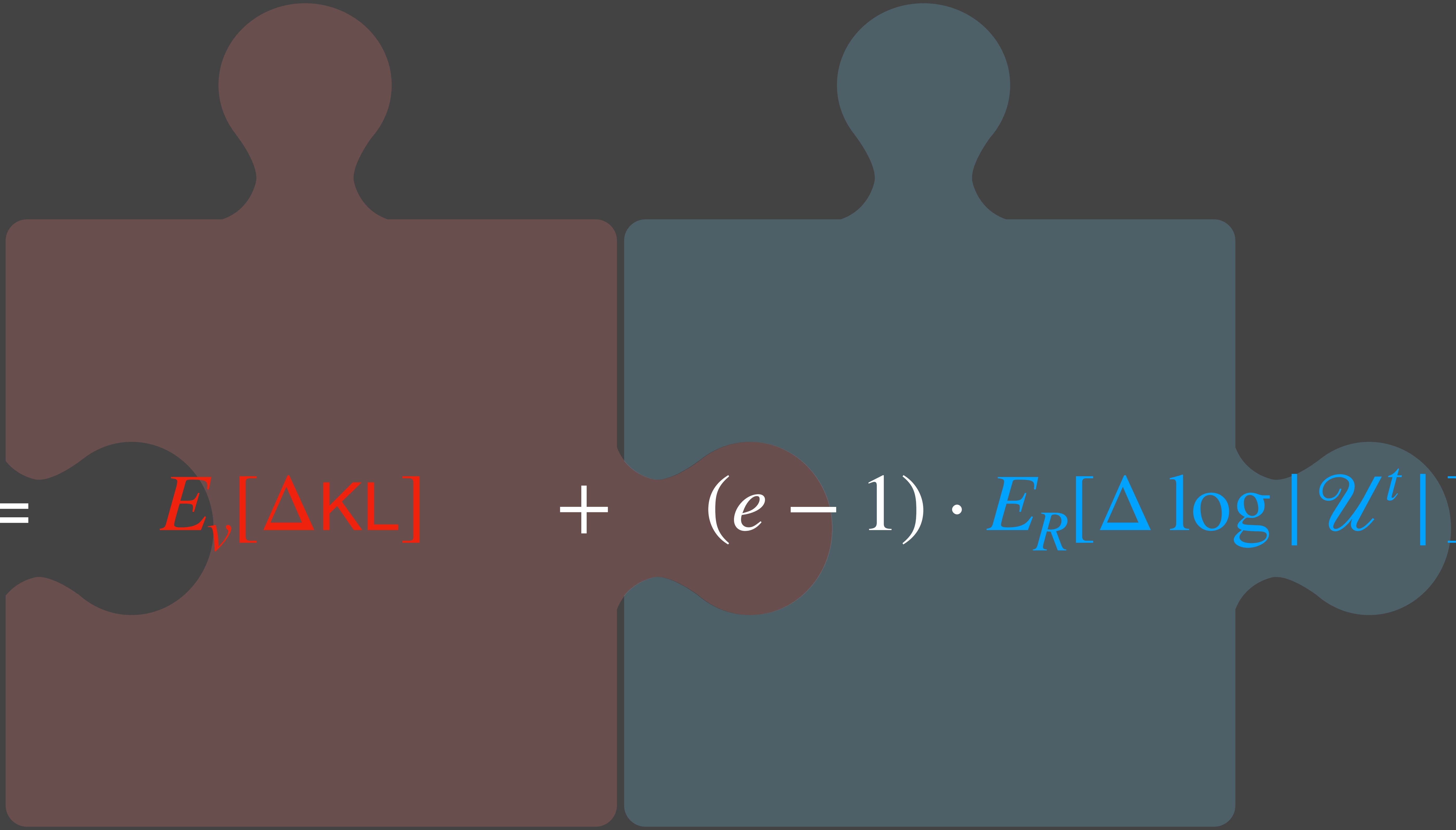
$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathcal{U}^t|] \leq -\frac{1}{k}$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$



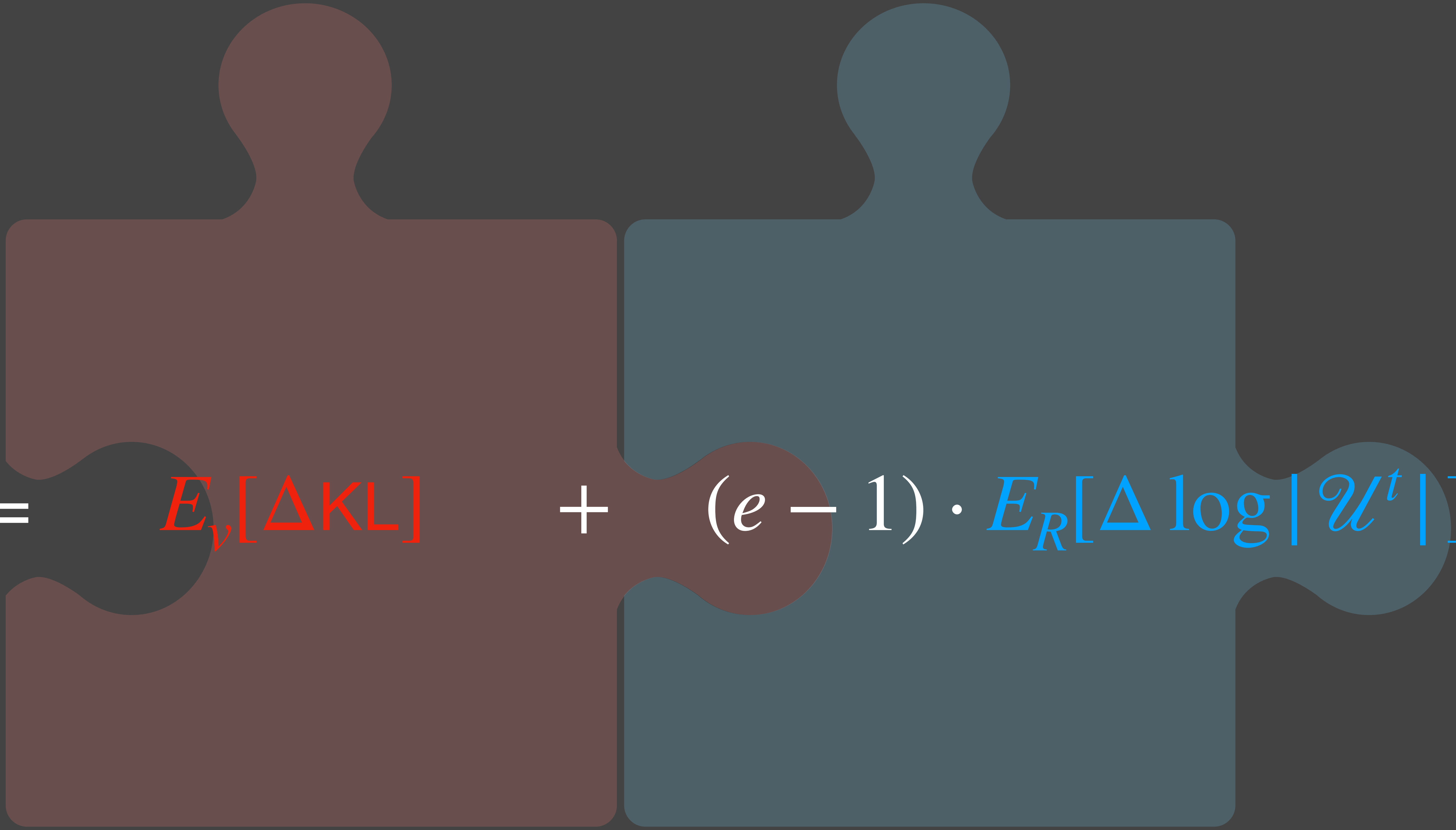
$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1)\cdot E_R[\Delta \log|\mathcal{U}^t|] \le -\frac{1}{k}$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$



$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1)\cdot E_R[\Delta \log|\mathcal{U}^t|] \le -\frac{1}{k}$$

Since $\Phi(0) = O(\log(mn))$, total cost is $k\log(mn)$.

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\mathbf{KL}(x^* \,||\, x^t) - \mathbf{KL}(x^* \,||\, x^{t-1})$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq - E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \sum_S x_S^* \log \|x\|_1 - \sum_{S \ni v} x_S^* \log e$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{= 1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S \ni v} x_S^* \log e$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S \ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x^*_S \log\left(\frac{x^*_S}{x^t_S}\right) - \sum_S x^*_S \log\left(\frac{x^*_S}{x^{t-1}_S}\right)$$

$$= \sum_S x^*_S \log\left(\frac{x^{t-1}_S}{x^t_S}\right)$$

$$= \underbrace{\cancel{\sum_S x^*_S}}_{=1} \log \|x\|_1 - \sum_{S \ni v} x^*_S \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\sum_S x^{t-1}_S + \sum_{S \ni v} (e-1) \cdot x^{t-1}_S\right) - \sum_{S \ni v} x^*_S$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta\mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underbrace{\cancel{\sum_S x_S^*}}_{=1} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v} (e-1)\cdot x_S^{t-1}\right) - \sum_{S\ni v} x_S^*$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathscr{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v} (e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\ge 1/k}$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S \ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\right) - \underbrace{\sum_{S \ni v} x_S^*}_{\geq 1/k}$$

$$\leq \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S \ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\Bigg(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\Bigg) - \underbrace{\sum_{S \ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \le z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \le -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \cancel{\sum_S x_S^*}_{=1} \log \|x\|_1 - \sum_{S \ni v} x_S^* \cancel{\log e}_{=1}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\right) - \underbrace{\sum_{S \ni v} x_S^*}_{\geq 1/k}$$

$$\leq \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \leq z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

$$\log|\mathscr{U}^t| - \log|\mathscr{U}^{t-1}|$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underbrace{\cancel{\sum_S x_S^*}}_{=1} \log \|x\|_1 - \sum_{S \ni v} x_S^* \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\right) - \underbrace{\sum_{S \ni v} x_S^*}_{\geq 1/k}$$

$$\leq \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \leq z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

$$\log|\mathscr{U}^t| - \log|\mathscr{U}^{t-1}|$$

$$= \log\left(1 - \frac{|\mathscr{U}^{t-1}| - |\mathscr{U}^t|}{|\mathscr{U}^{t-1}|}\right)$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S \ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S \ni v} (e-1) \cdot x_S^{t-1}\right) - \underbrace{\sum_{S \ni v} x_S^*}_{\geq 1/k}$$

$$\leq \log\left(1 + \sum_{S \ni v} (e-1) \cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \leq z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

$$\log |\mathscr{U}^t| - \log |\mathscr{U}^{t-1}|$$

$$= \log\left(1 - \frac{|\mathscr{U}^{t-1}| - |\mathscr{U}^t|}{|\mathscr{U}^{t-1}|}\right)$$

Use $\log(1-z) \leq -z$.

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underbrace{\cancel{\sum_S x_S^*}}_{=1} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\geq 1/k}$$

$$\leq \log\left(1 + \sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \leq z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathscr{U}^t|] \leq -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathscr{U}^t| - \log|\mathscr{U}^{t-1}|$$

$$= \log\left(1 - \frac{|\mathscr{U}^{t-1}| - |\mathscr{U}^t|}{|\mathscr{U}^{t-1}|}\right)$$

Use $\log(1-z) \leq -z$.

$$\leq -\frac{1}{|\mathscr{U}^{t-1}|}\sum_{v\in\mathscr{U}^{t-1}} \mathbb{1}\{R\ni v\}.$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log\|x\|_1 - \sum_{S\ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1+\sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z)\le z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathcal{U}^t| - \log|\mathcal{U}^{t-1}|$$

$$= \log\left(1 - \frac{|\mathcal{U}^{t-1}| - |\mathcal{U}^t|}{|\mathcal{U}^{t-1}|}\right)$$

Use $\log(1-z)\le -z$.

$$\le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}} \mathbb{1}\{R\ni v\}.$$

Take expectation over R.

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1+\sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z) \le z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathcal{U}^t| - \log|\mathcal{U}^{t-1}|$$

$$= \log\left(1 - \frac{|\mathcal{U}^{t-1}| - |\mathcal{U}^t|}{|\mathcal{U}^{t-1}|}\right)$$

Use $\log(1-z) \le -z$.

$$\le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}} \mathbb{1}\{R\ni v\}.$$

Take expectation over R.

$$E_R[\Delta \log|\mathcal{U}^t|] \le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_R x_R \sum_{v\in\mathcal{U}^{t-1}} \mathbb{1}\{R\ni v\}$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underset{=1}{\cancel{\sum_S x_S^*}} \log\|x\|_1 - \sum_{S\ni v} x_S^* \underset{=1}{\cancel{\log e}}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1+\sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z)\le z$, take expectation over v. ∎

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathcal{U}^t| - \log|\mathcal{U}^{t-1}|$$

$$= \log\left(1-\frac{|\mathcal{U}^{t-1}|-|\mathcal{U}^t|}{|\mathcal{U}^{t-1}|}\right)$$

Use $\log(1-z)\le -z$.

$$\le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}} \mathbb{1}\{R\ni v\}.$$

Take expectation over R.

$$E_R[\Delta \log|\mathcal{U}^t|] \le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_R x_R \sum_{v\in\mathcal{U}^{t-1}}\mathbb{1}\{R\ni v\}$$

$$= -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}}\sum_{R\ni v} x_R.$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1)\cdot E_v\left[\sum_{S\ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

$$= \sum_S x_S^* \log\left(\frac{x_S^{t-1}}{x_S^t}\right)$$

$$= \underbrace{\cancel{\sum_S x_S^*}}_{=1} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\underbrace{\sum_S x_S^{t-1}}_{=1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \underbrace{\sum_{S\ni v} x_S^*}_{\ge 1/k}$$

$$\le \log\left(1+\sum_{S\ni v}(e-1)\cdot x_S\right) - \frac{1}{k}.$$

Use $\log(1+z)\le z$, take expectation over v. ■

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log|\mathcal{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

$$\log|\mathcal{U}^t| - \log|\mathcal{U}^{t-1}|$$

$$= \log\left(1-\frac{|\mathcal{U}^{t-1}|-|\mathcal{U}^t|}{|\mathcal{U}^{t-1}|}\right)$$

Use $\log(1-z)\le -z$.

$$\le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}} \mathbb{1}\{R\ni v\}.$$

Take expectation over R.

$$E_R[\Delta\log|\mathcal{U}^t|] \le -\frac{1}{|\mathcal{U}^{t-1}|}\sum_R x_R \sum_{v\in\mathcal{U}^{t-1}}\mathbb{1}\{R\ni v\}$$

$$= -\frac{1}{|\mathcal{U}^{t-1}|}\sum_{v\in\mathcal{U}^{t-1}}\sum_{R\ni v} x_R.$$

■

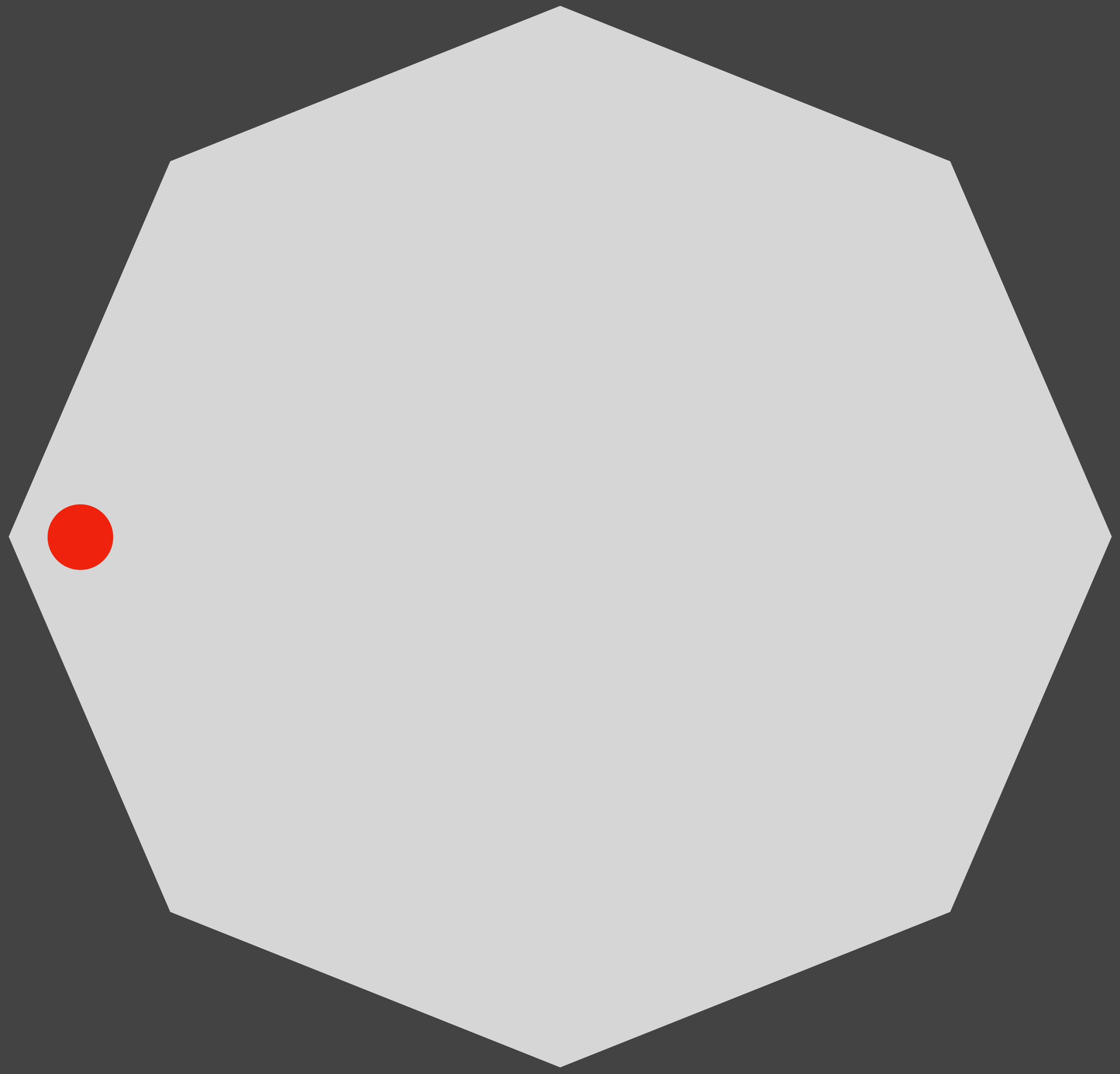
LearnOrCover

(Some philosophy)

LearnOrCover

(Some philosophy)

Perspective 1:



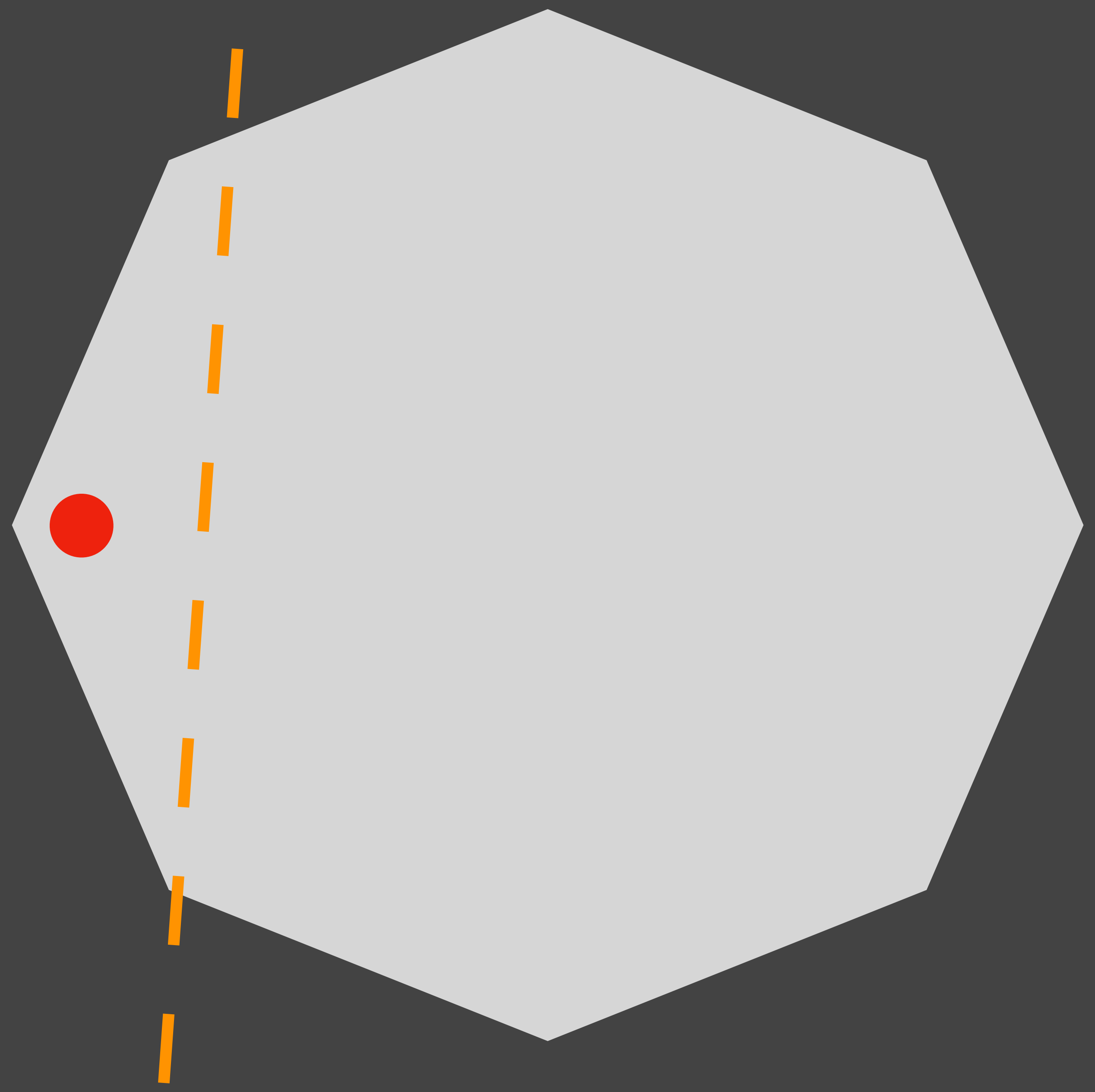
[Alon+ 03]

[Buchbinder Gupta Molinaro Naor 19]

LearnOrCover

(Some philosophy)

Perspective 1:



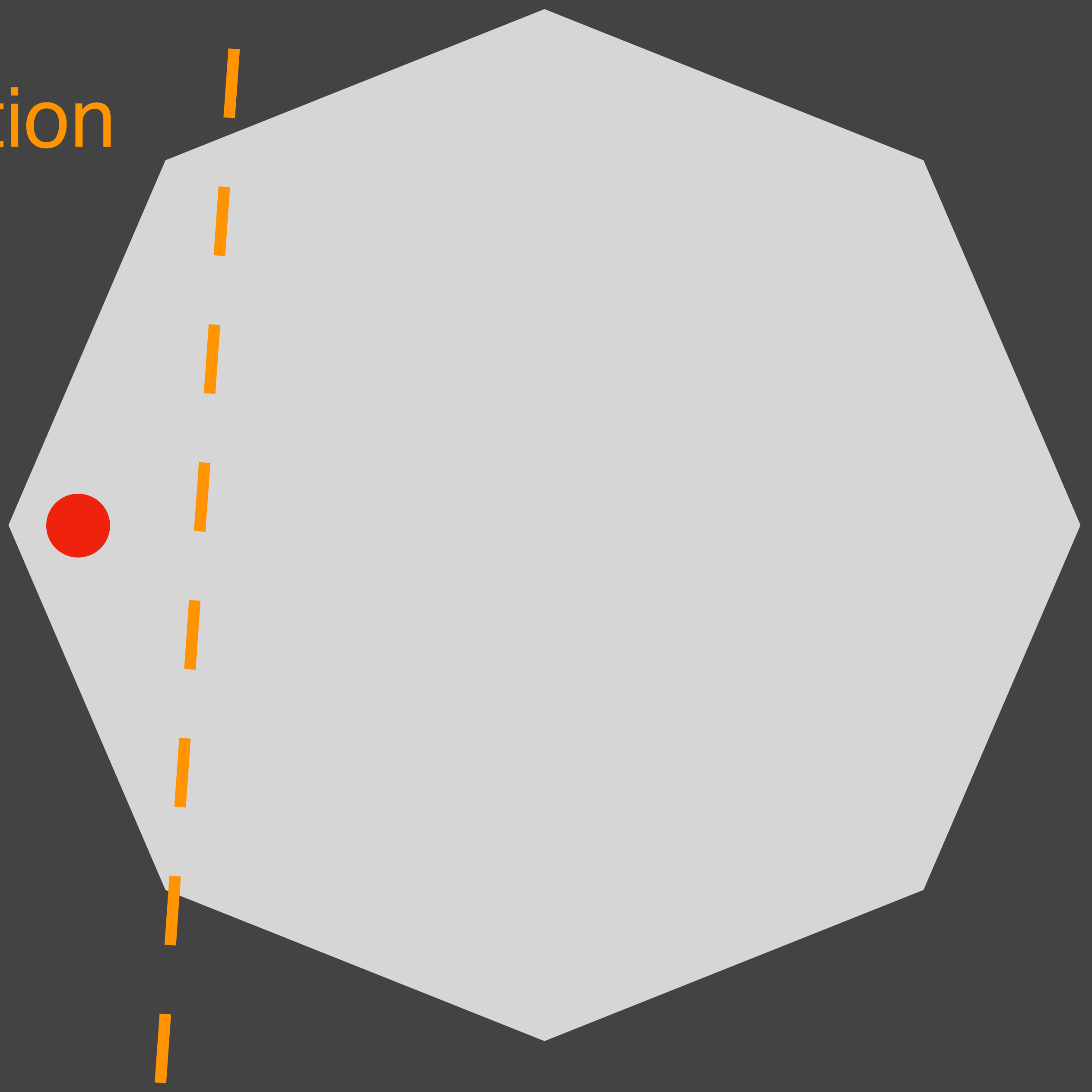
[Alon+ 03]

[Buchbinder Gupta Molinaro Naor 19]

LearnOrCover

(Some philosophy)

Perspective 1:

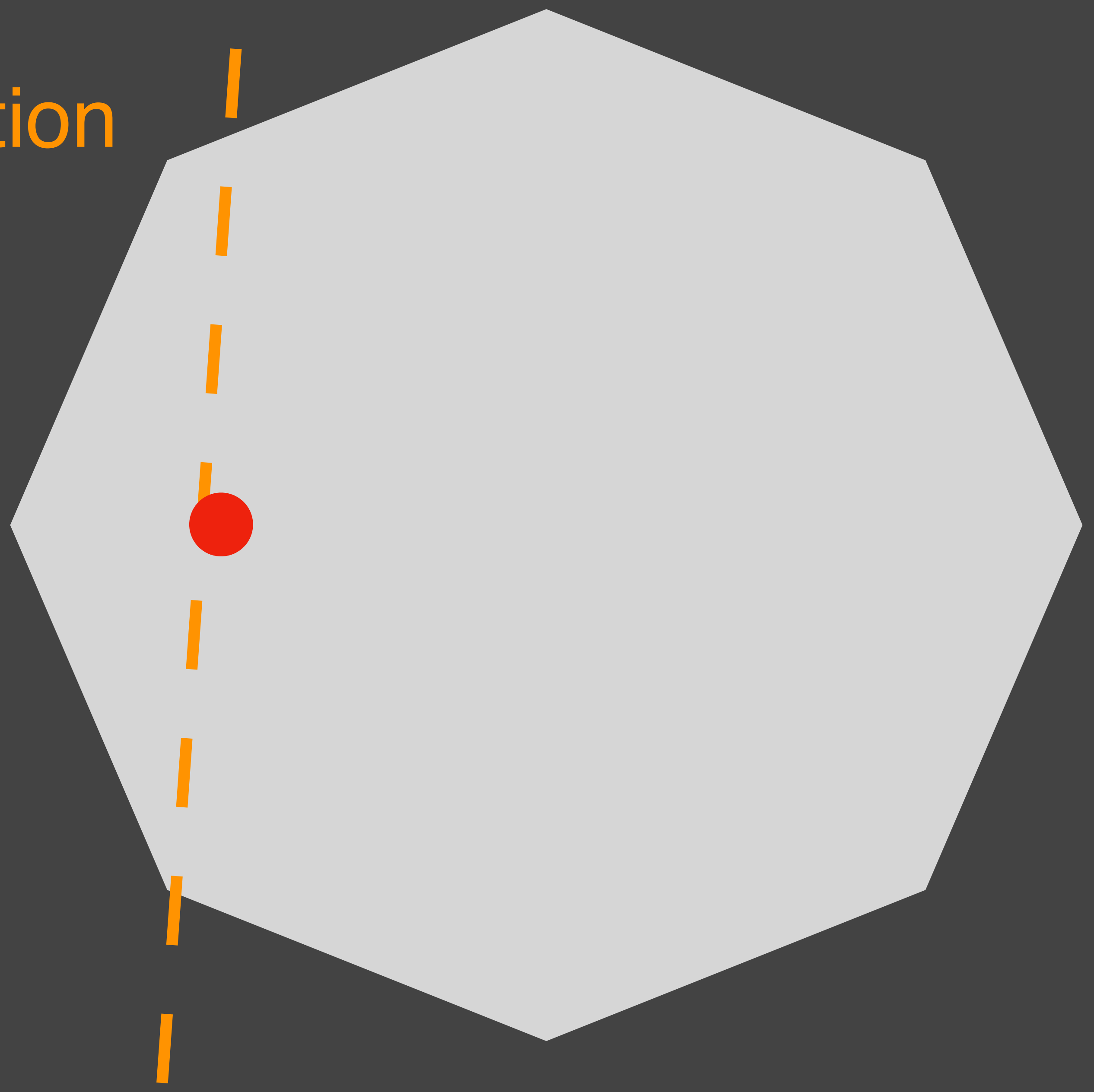


[Alon+ 03]
[Buchbinder Gupta Molinaro Naor 19]

LearnOrCover

(Some philosophy)

Perspective 1:



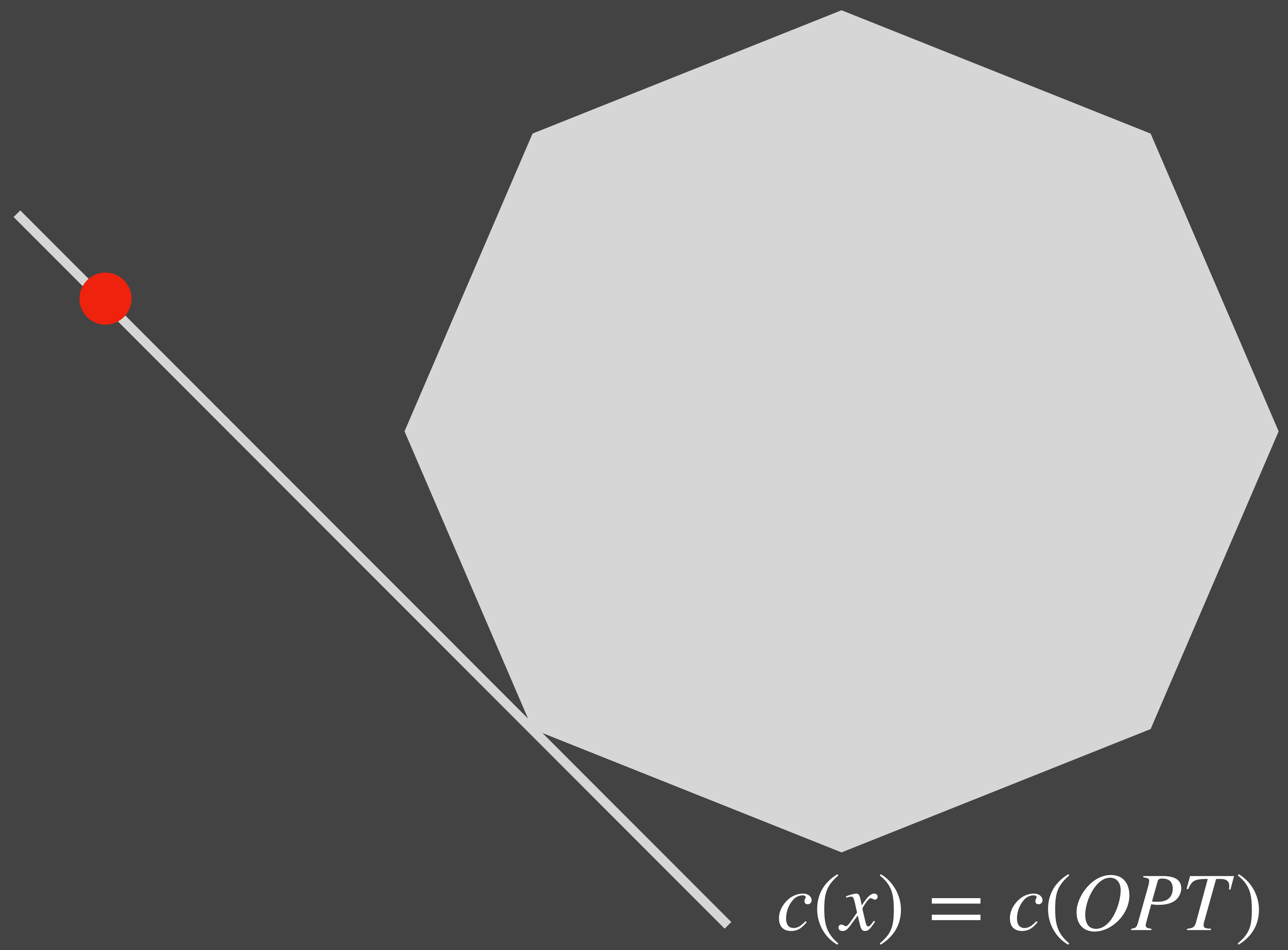
[Alon+ 03]

[Buchbinder Gupta Molinaro Naor 19]

LearnOrCover

(Some philosophy)

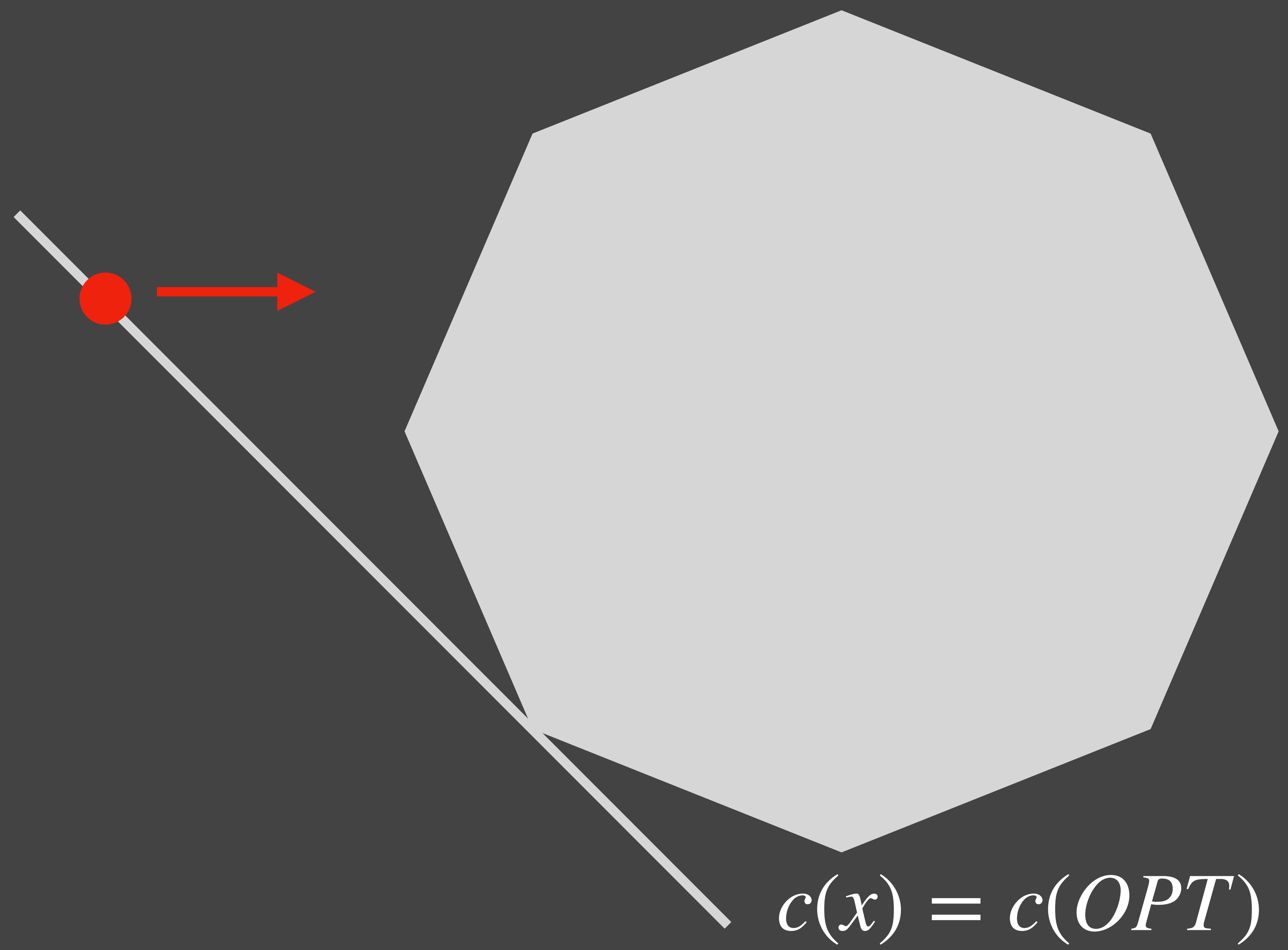
Perspective 1:



LearnOrCover

(Some philosophy)

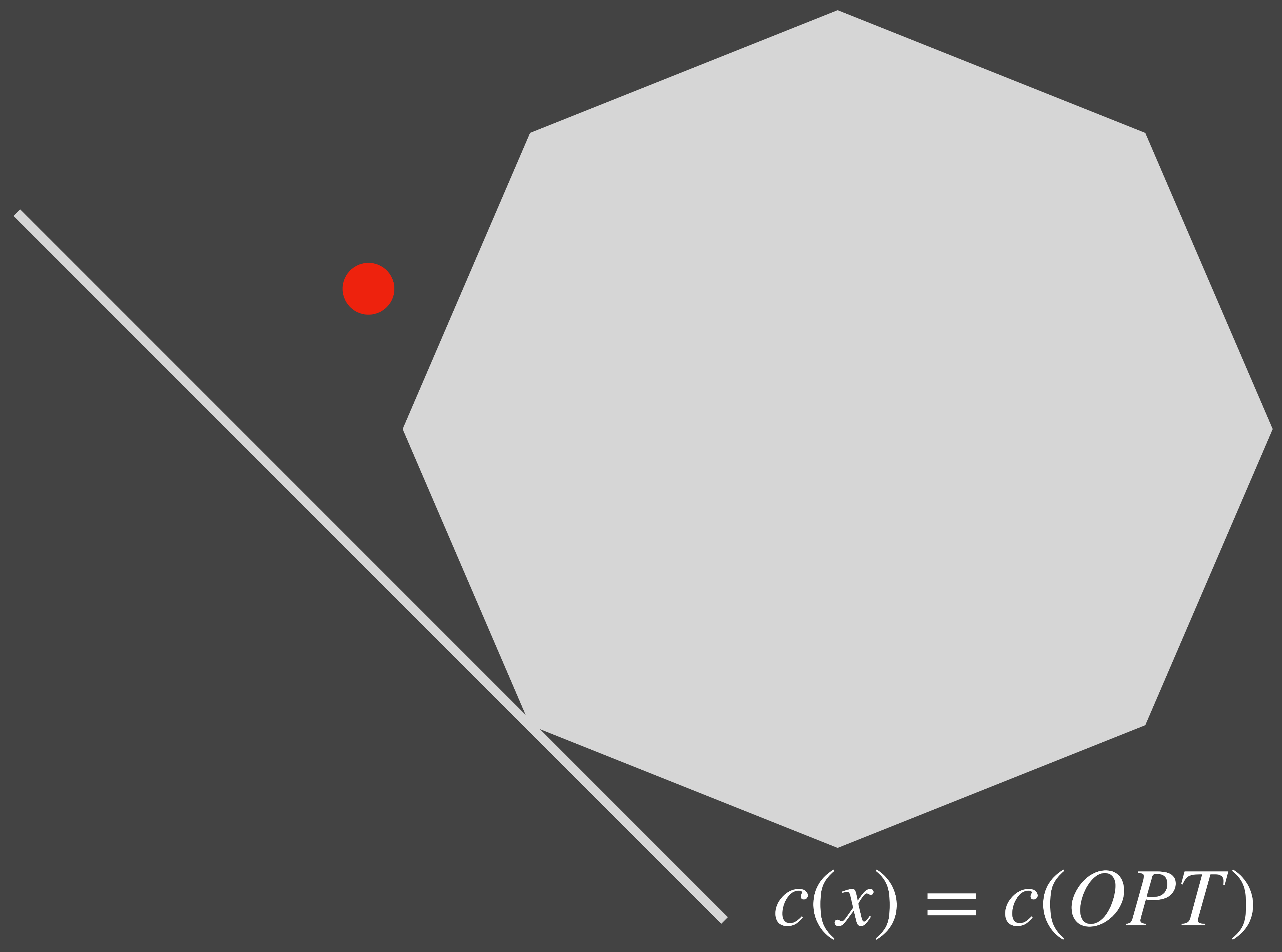
Perspective 1:



LearnOrCover

(Some philosophy)

Perspective 1:

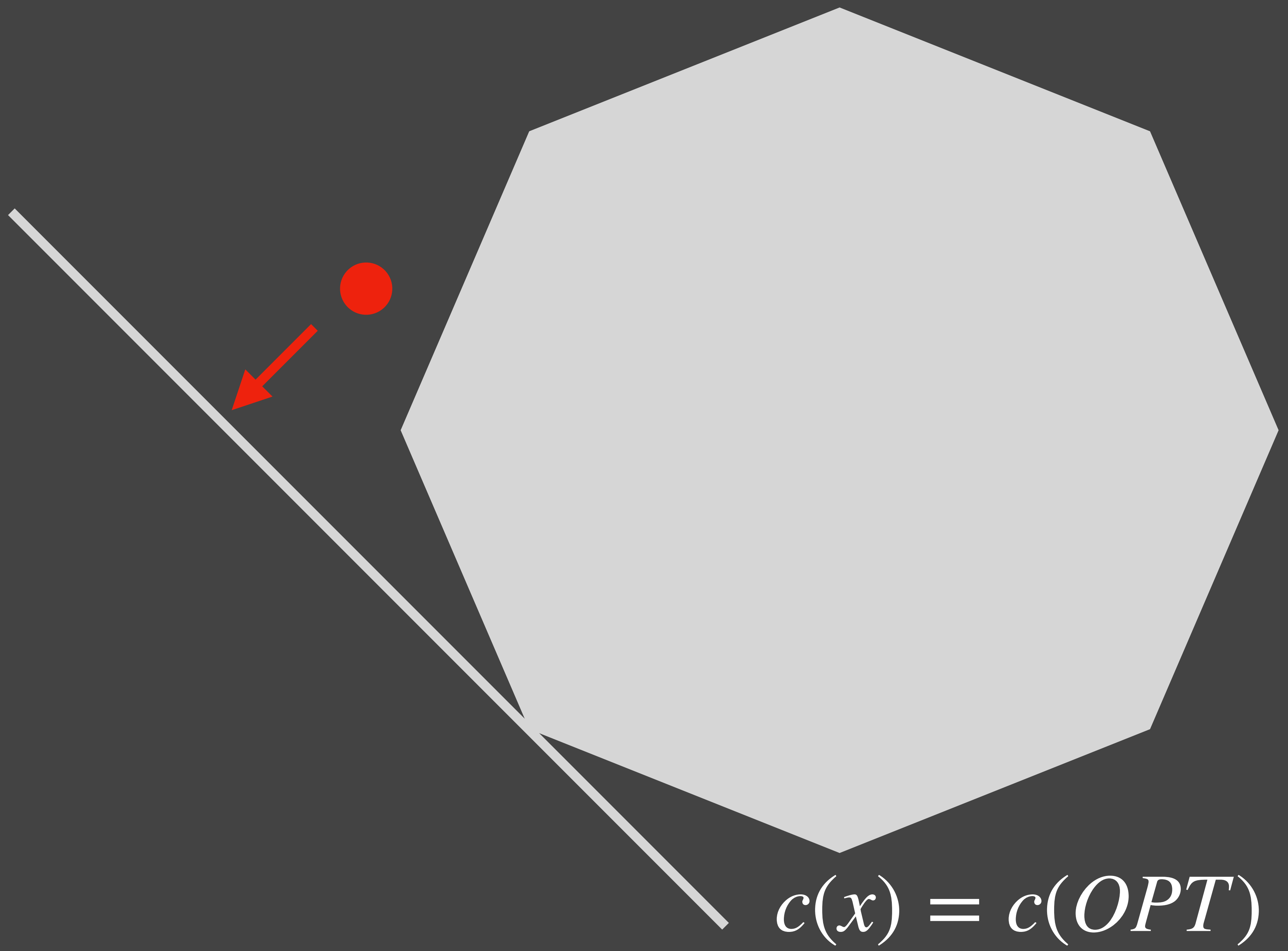


$c(x) = c(OPT)$

LearnOrCover

(Some philosophy)

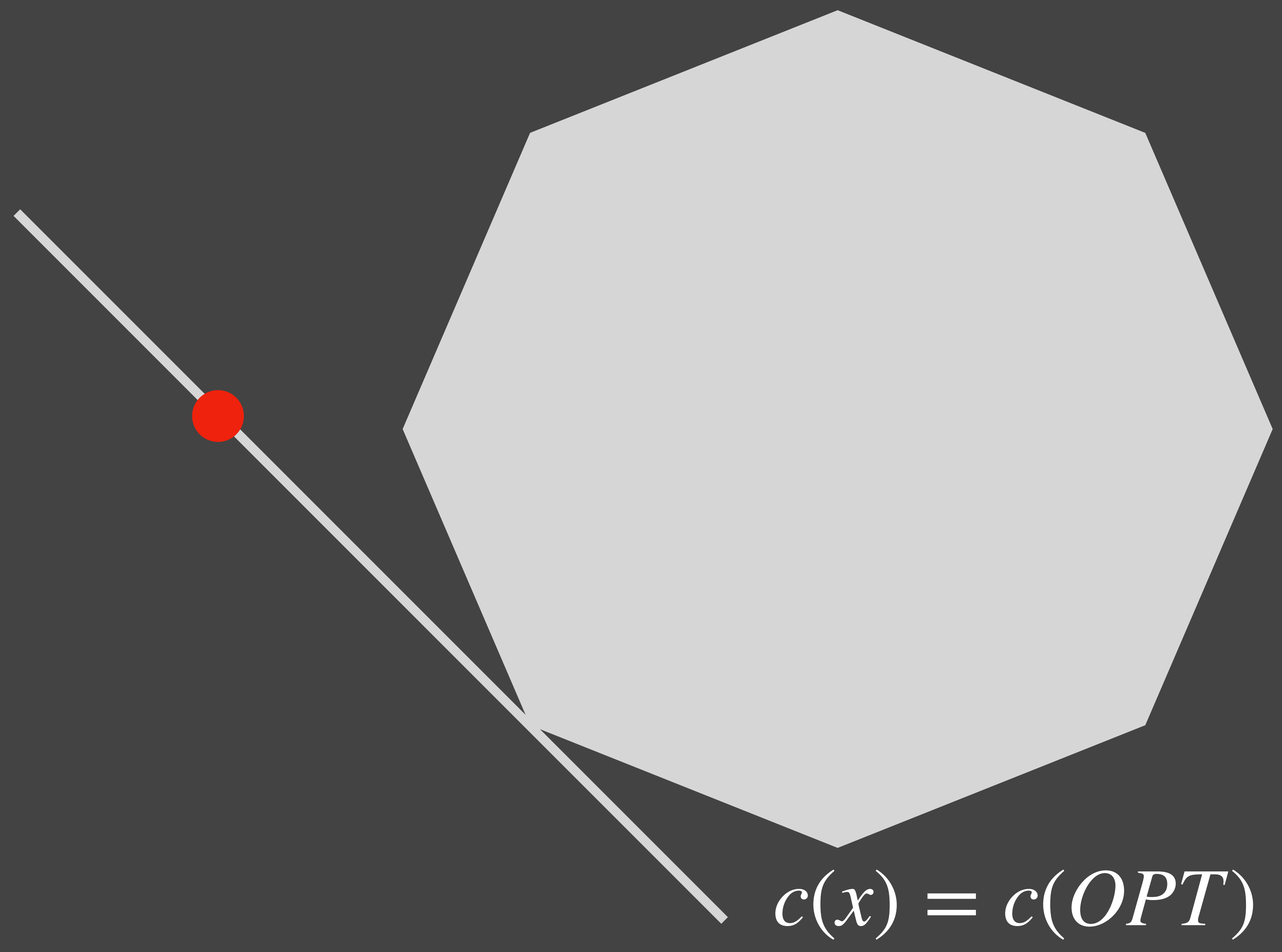
Perspective 1:



LearnOrCover

(Some philosophy)

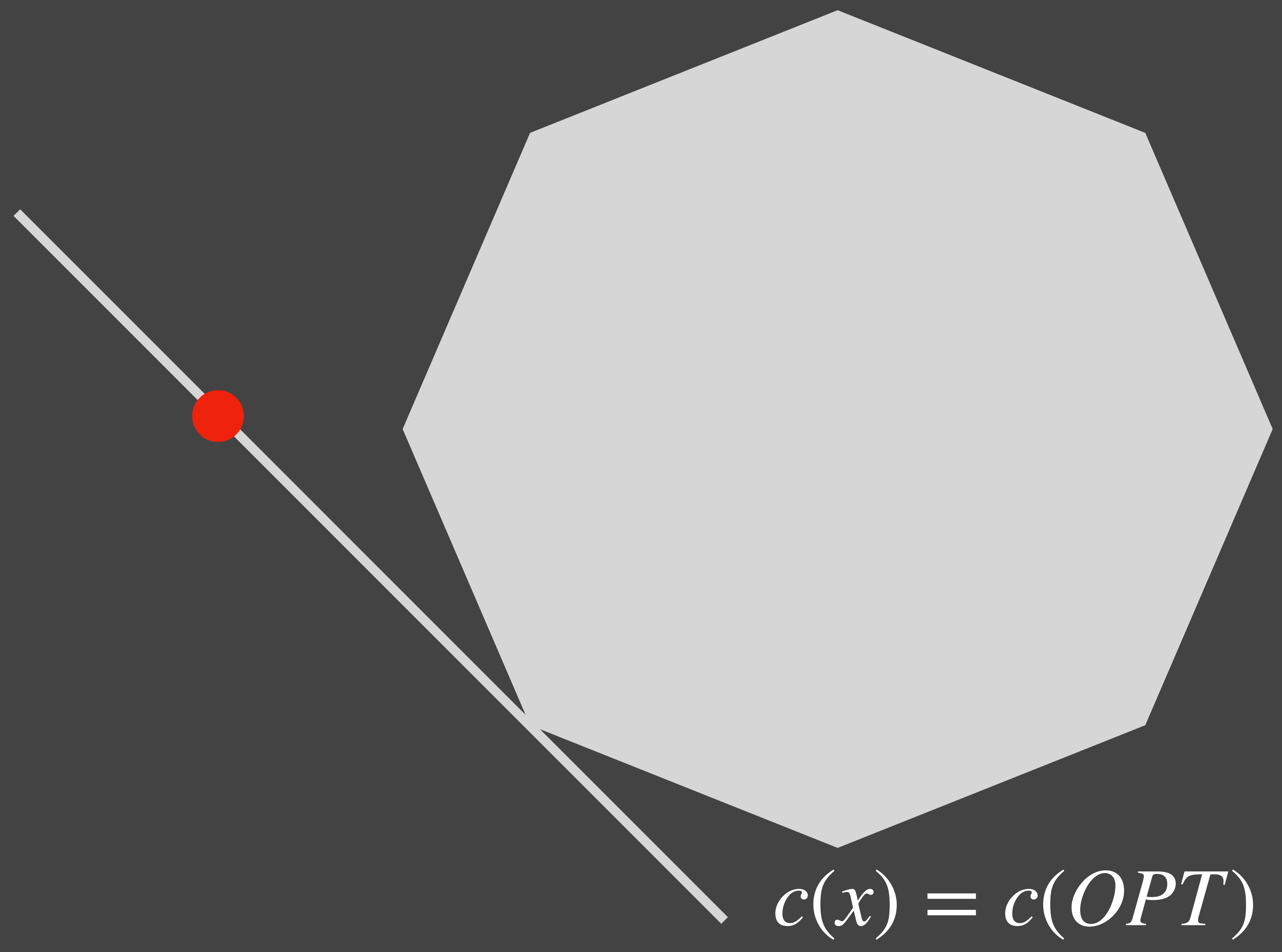
Perspective 1:



LearnOrCover

(Some philosophy)

Perspective 1:



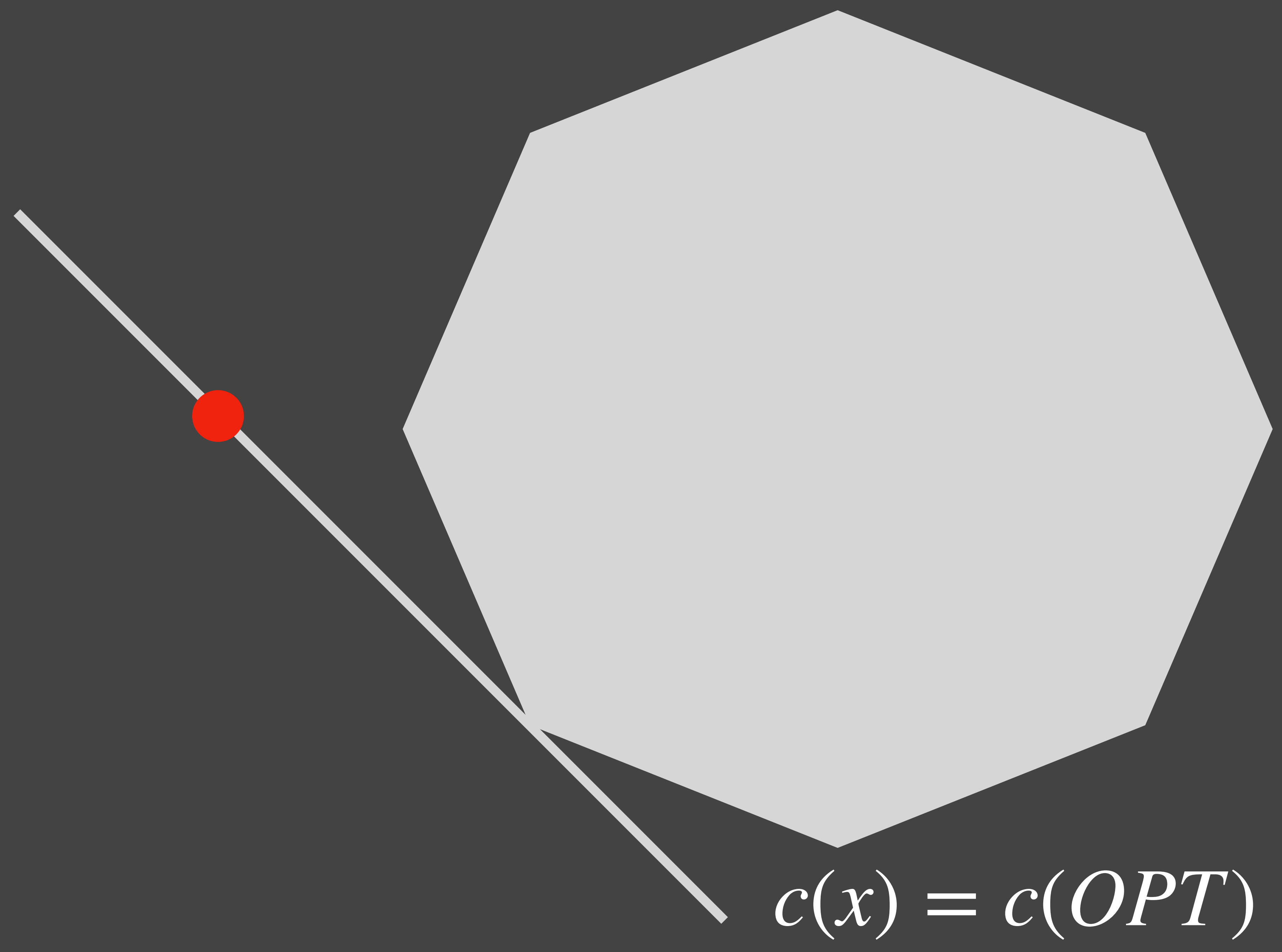
LearnOrCover

Perspective 2:

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

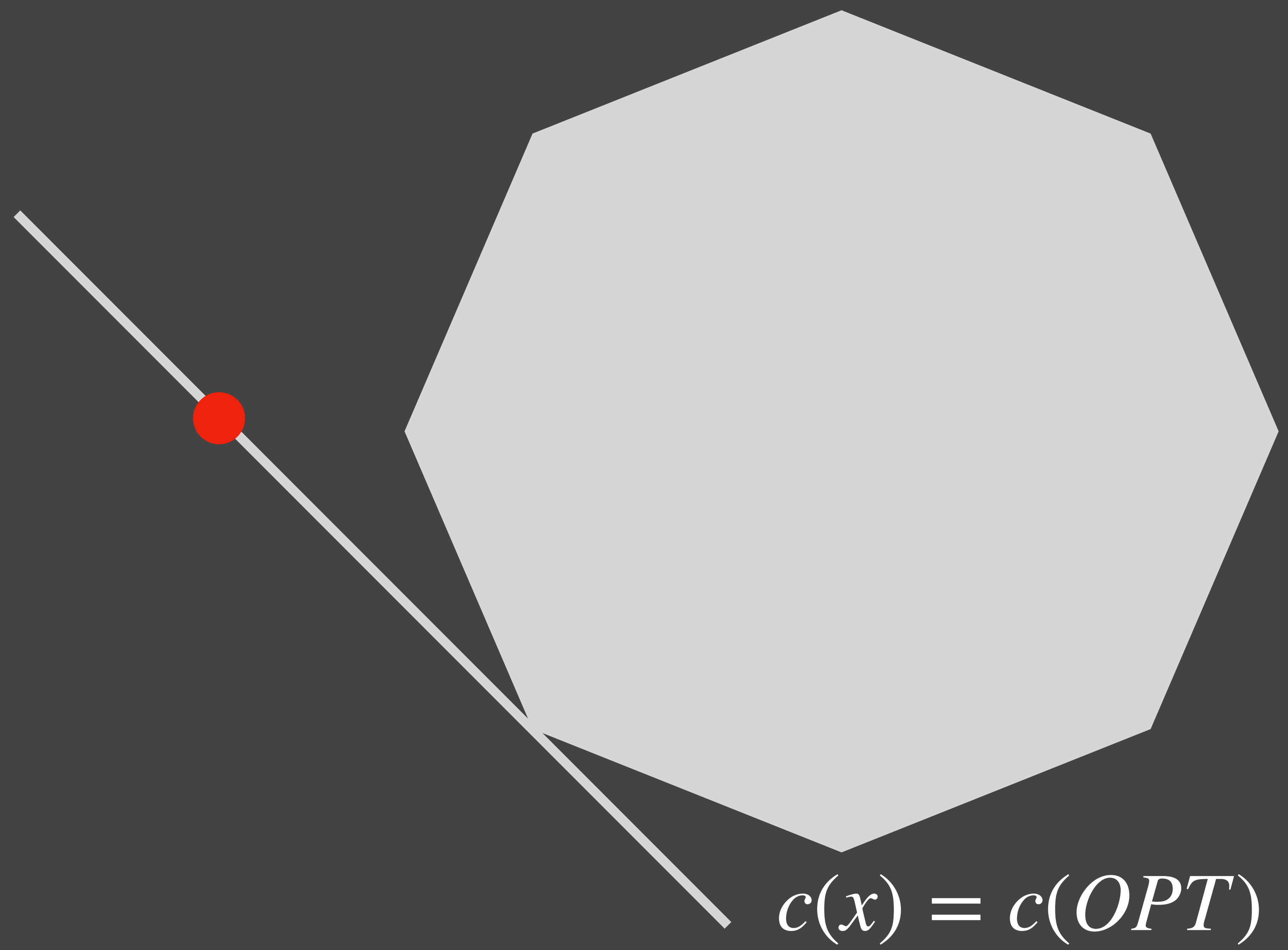
Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

Define

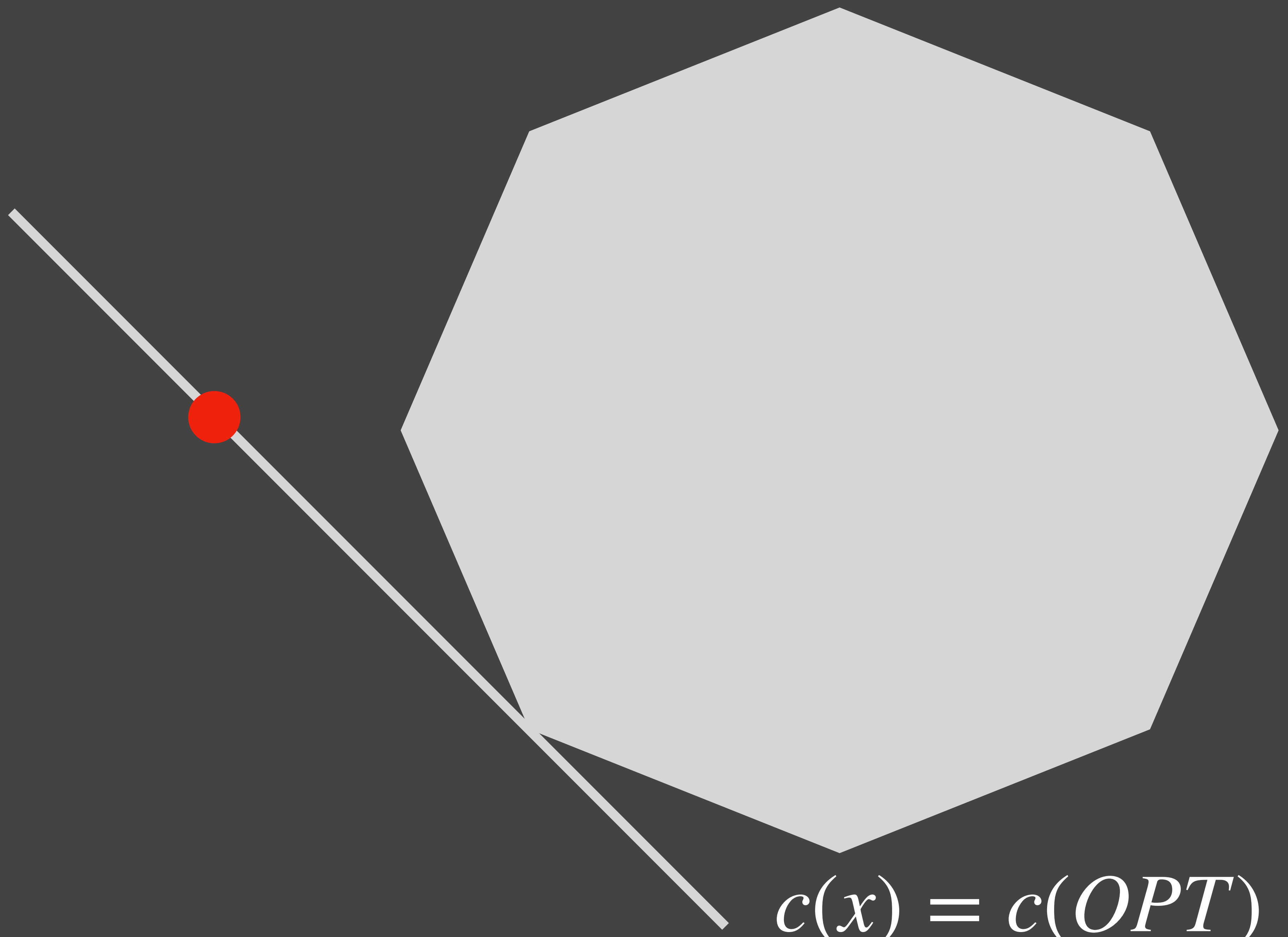
$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

(Goal is to minimize f in smallest # of steps)

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

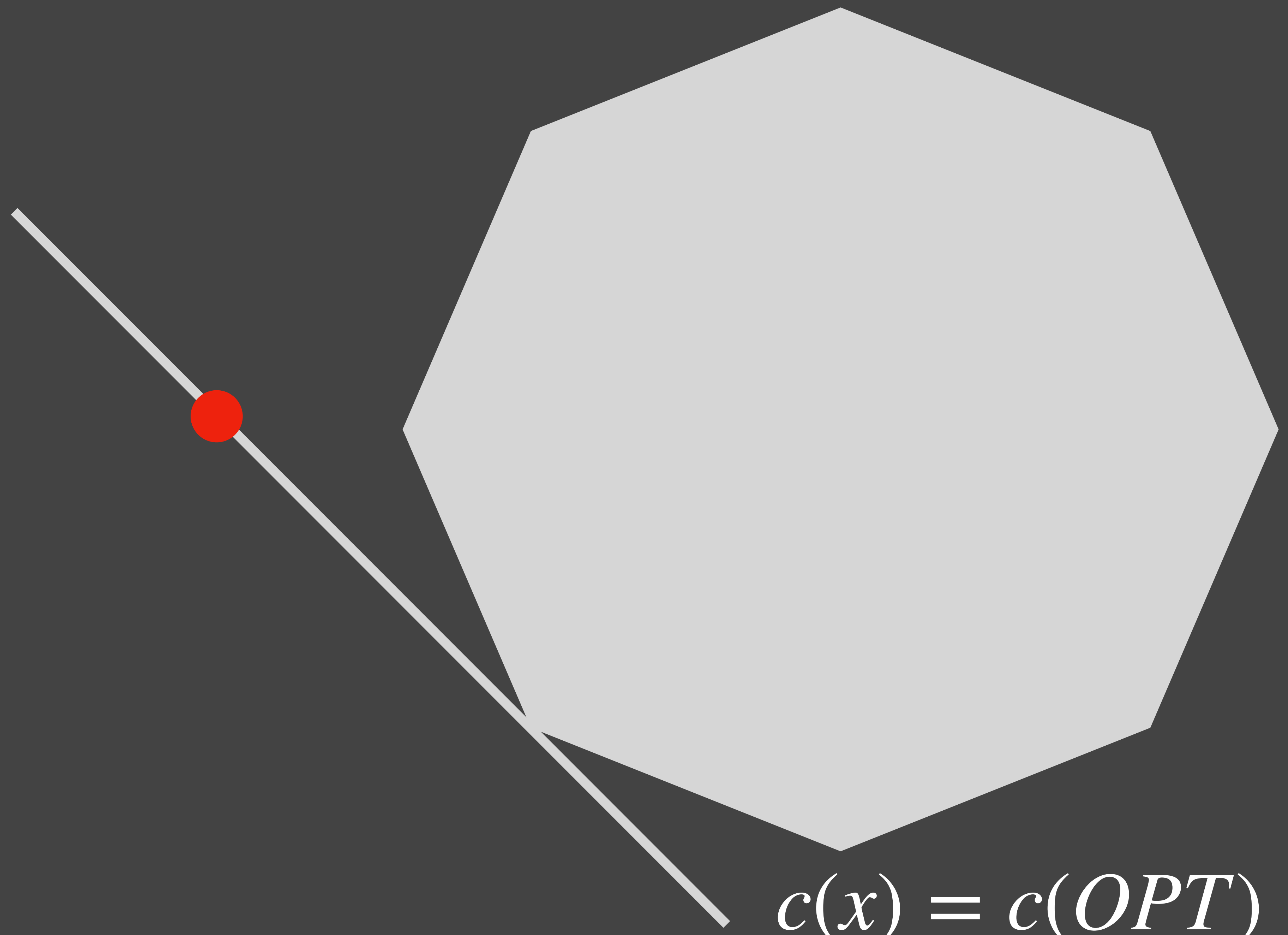
(Goal is to minimize f in smallest # of steps)

$\nabla f|_S(x) =$ # uncovered elements in S

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

(Goal is to minimize f in smallest # of steps)

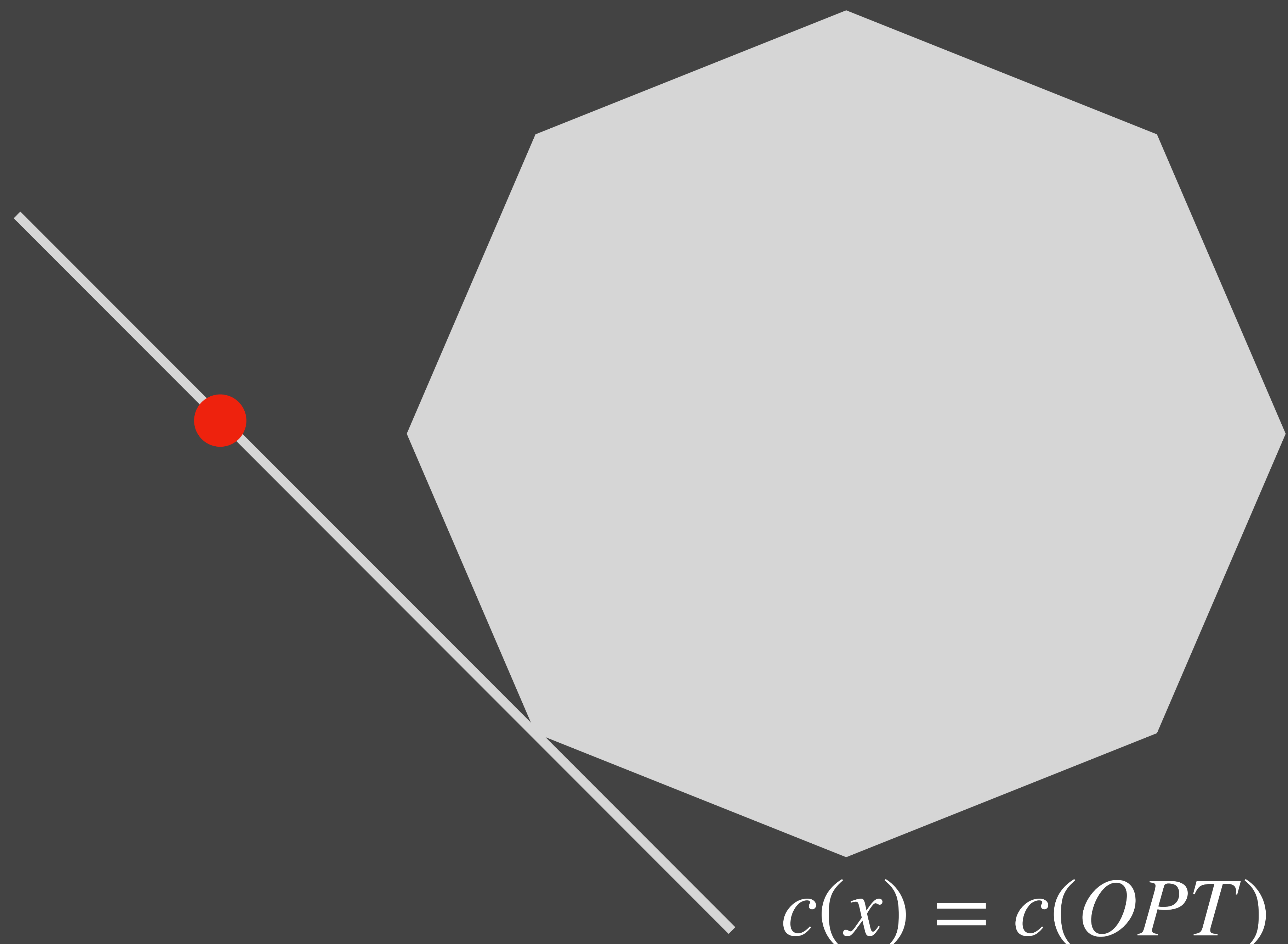
$$\nabla f|_S(x) = \text{ \# uncovered elements in } S$$

$$\propto E[\mathbb{1}\{v \in S \mid v \text{ uncovered}\}]$$

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

Define

$$f(x) := \sum_{v} \max \left(0, 1 - \sum_{S \ni v} x_S \right)$$

(Goal is to minimize f in smallest # of steps)

$$\nabla f|_S(x) = \text{ \# uncovered elements in } S$$

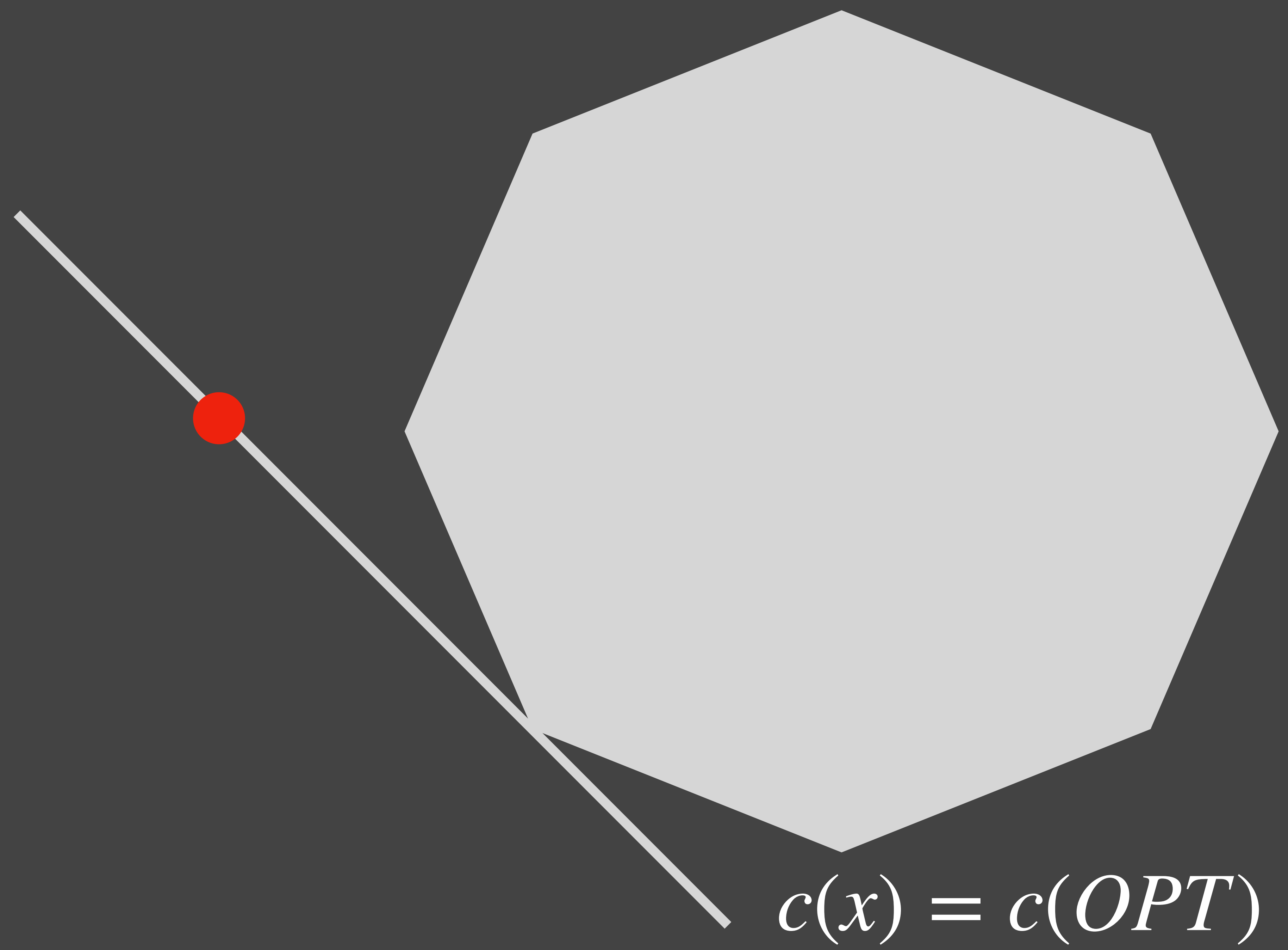
$$\propto E[\mathbb{1}\{v \in S \mid v \text{ uncovered}\}]$$

RO reveals stochastic gradient…

LearnOrCover

(Some philosophy)

Perspective 1:



LearnOrCover

Perspective 2:

Define

$$f(x) := \sum_{v} \max\left(0, 1 - \sum_{S \ni v} x_S\right)$$

(Goal is to minimize f in smallest # of steps)

$$\nabla f|_S(x) = \text{ \# uncovered elements in } S$$
$$\propto E[\mathbb{1}\{v \in S \mid v \text{ uncovered}\}]$$

RO reveals stochastic gradient…
… LearnOrCover is running SGD!

Talk Outline

- Intro
- Secretary
    - LearnOrCover in Exponential Time
    - ➡ LearnOrCover in Poly Time
- (Single Sample) Prophet
- Conclusion & Extensions

Talk Outline

- Intro
- Secretary
  - LearnOrCover in Exponential Time
  - LearnOrCover in Poly Time
- ➡ (Single Sample) Prophet
- Conclusion & Extensions

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets

s_1 ●

s_2 ●

s_3 ●

s_4 ●

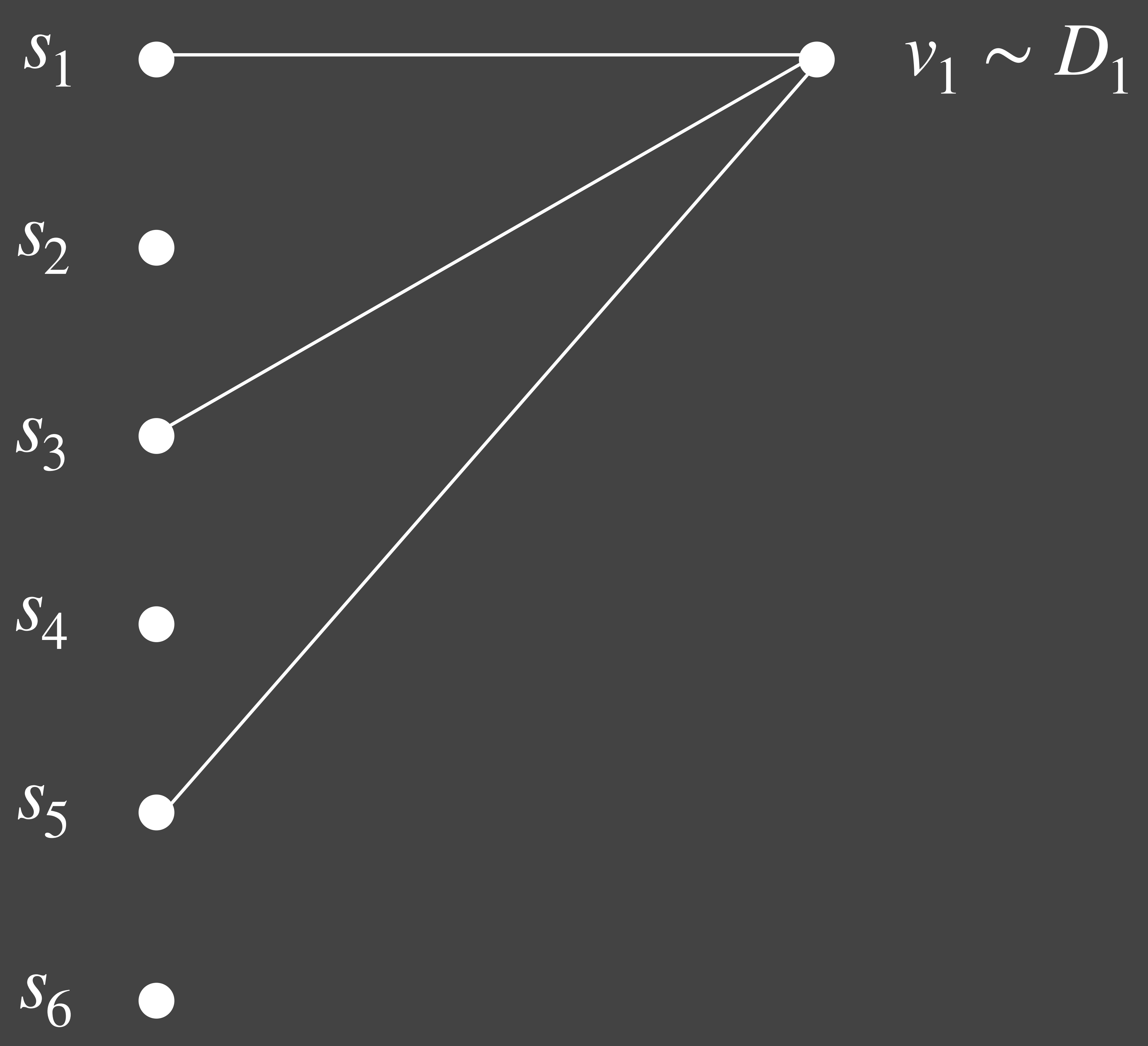
s_5 ●

s_6 ●

$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet



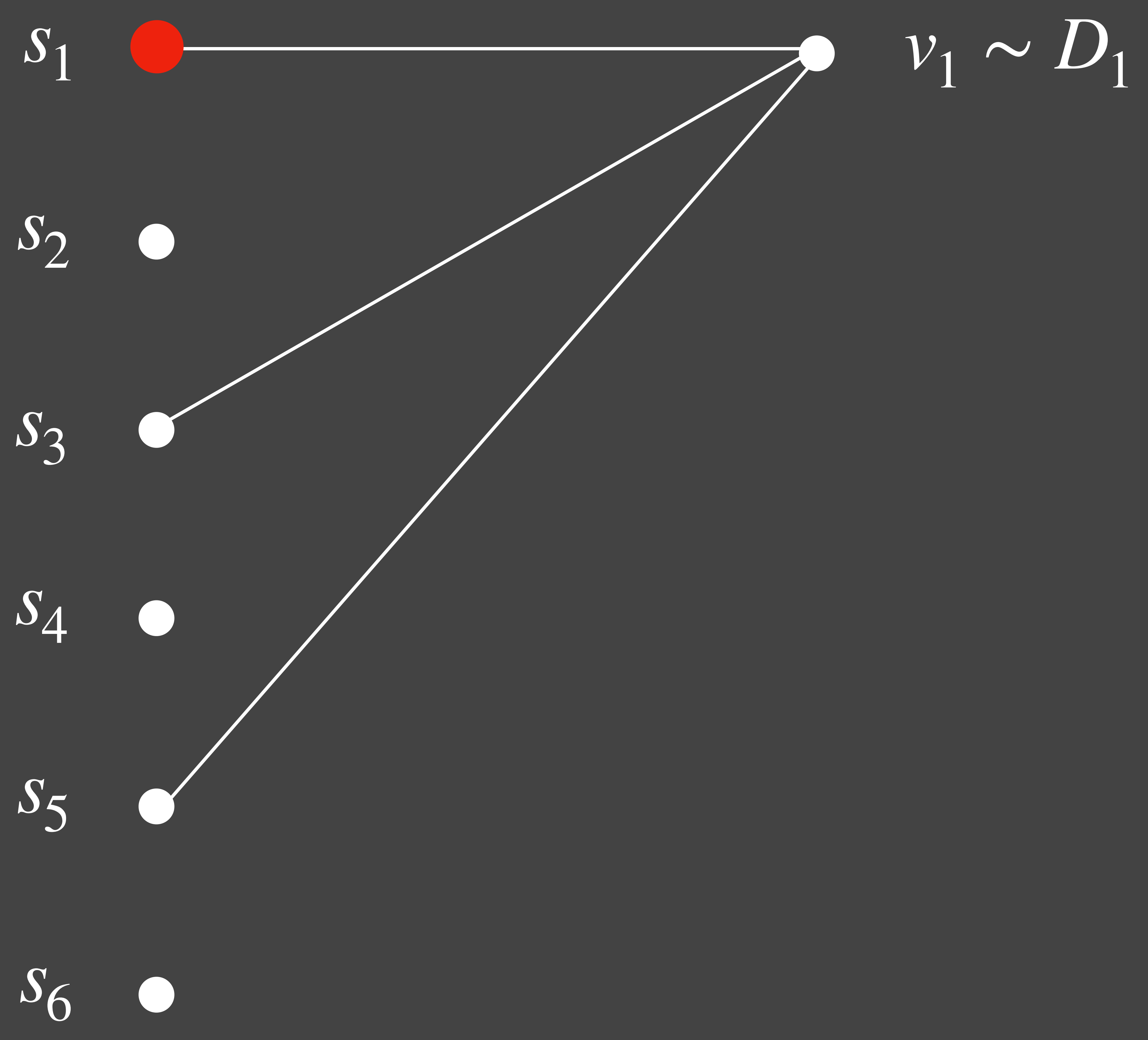
$\mathcal{S}$

m sets

$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet



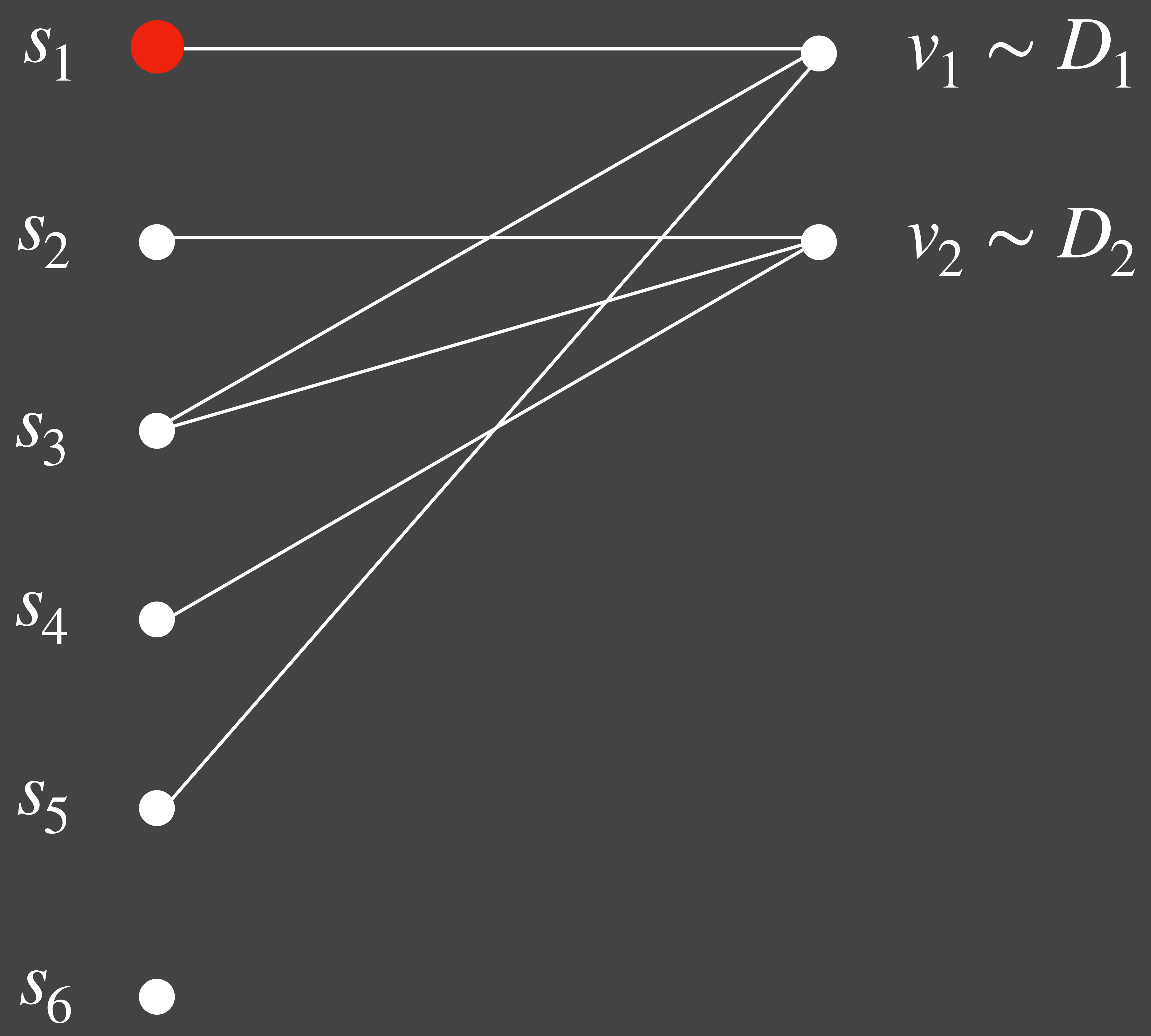
$\mathcal{S}$

m sets

$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet



$\mathcal{S}$

m sets

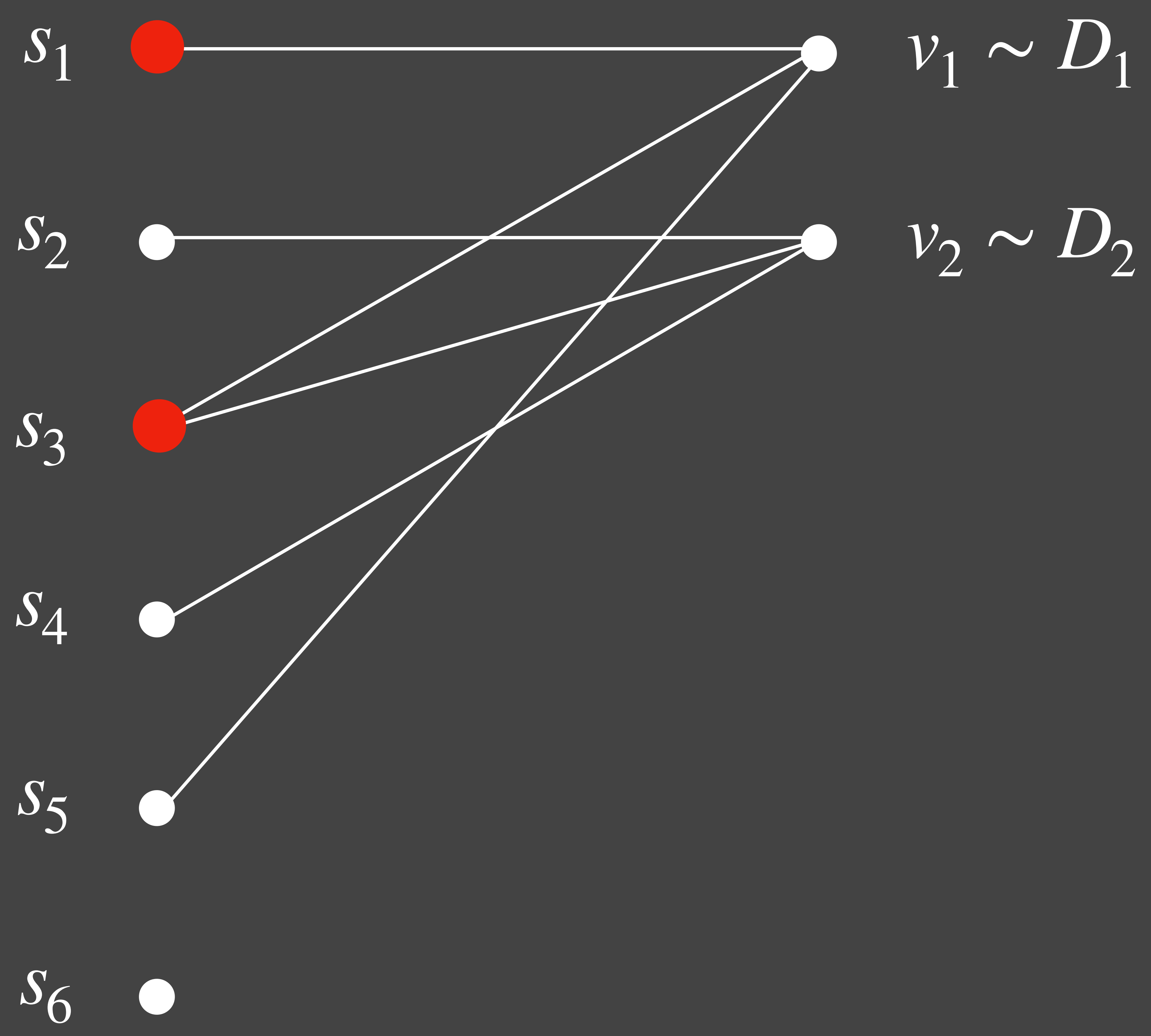
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



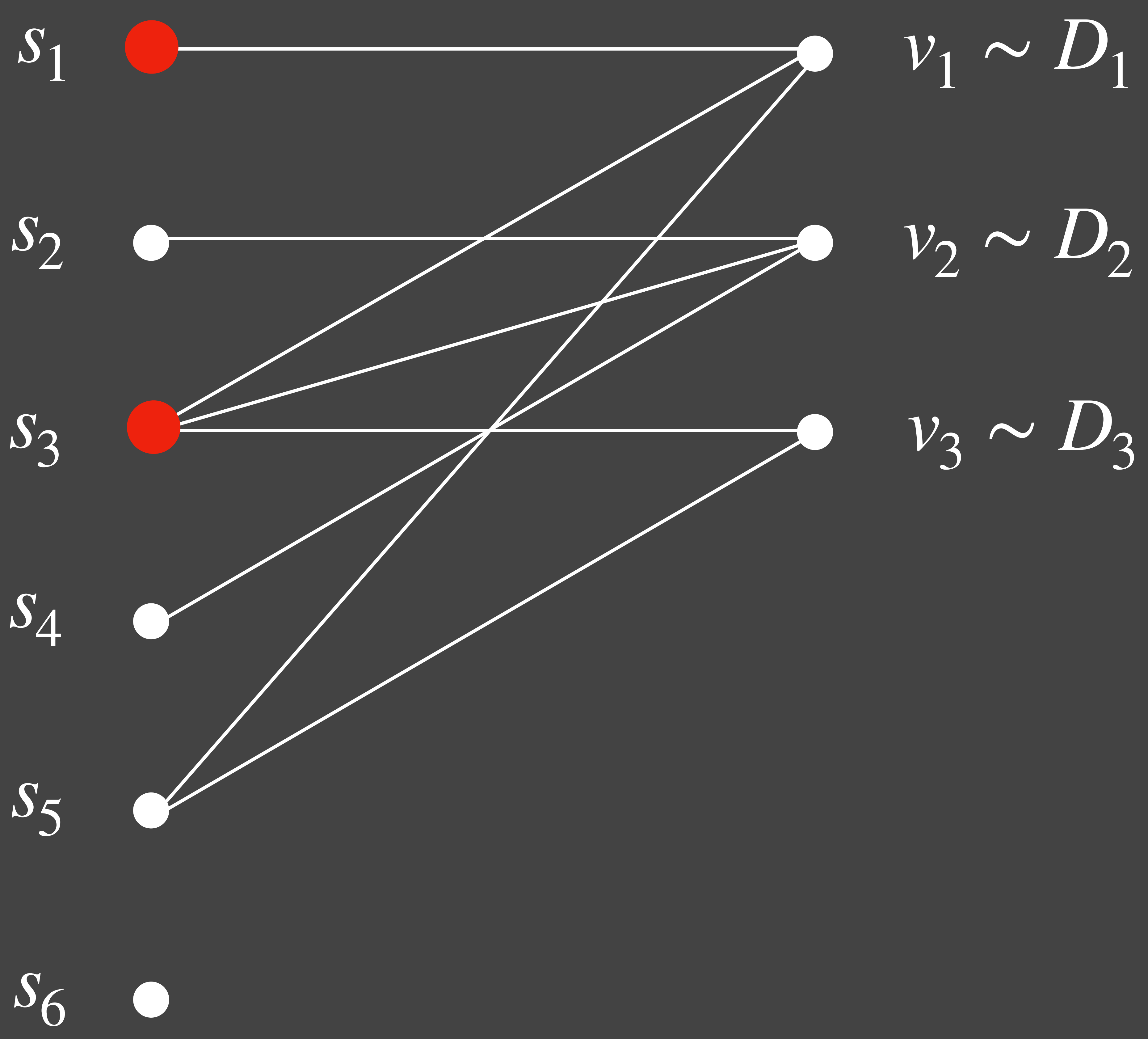
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



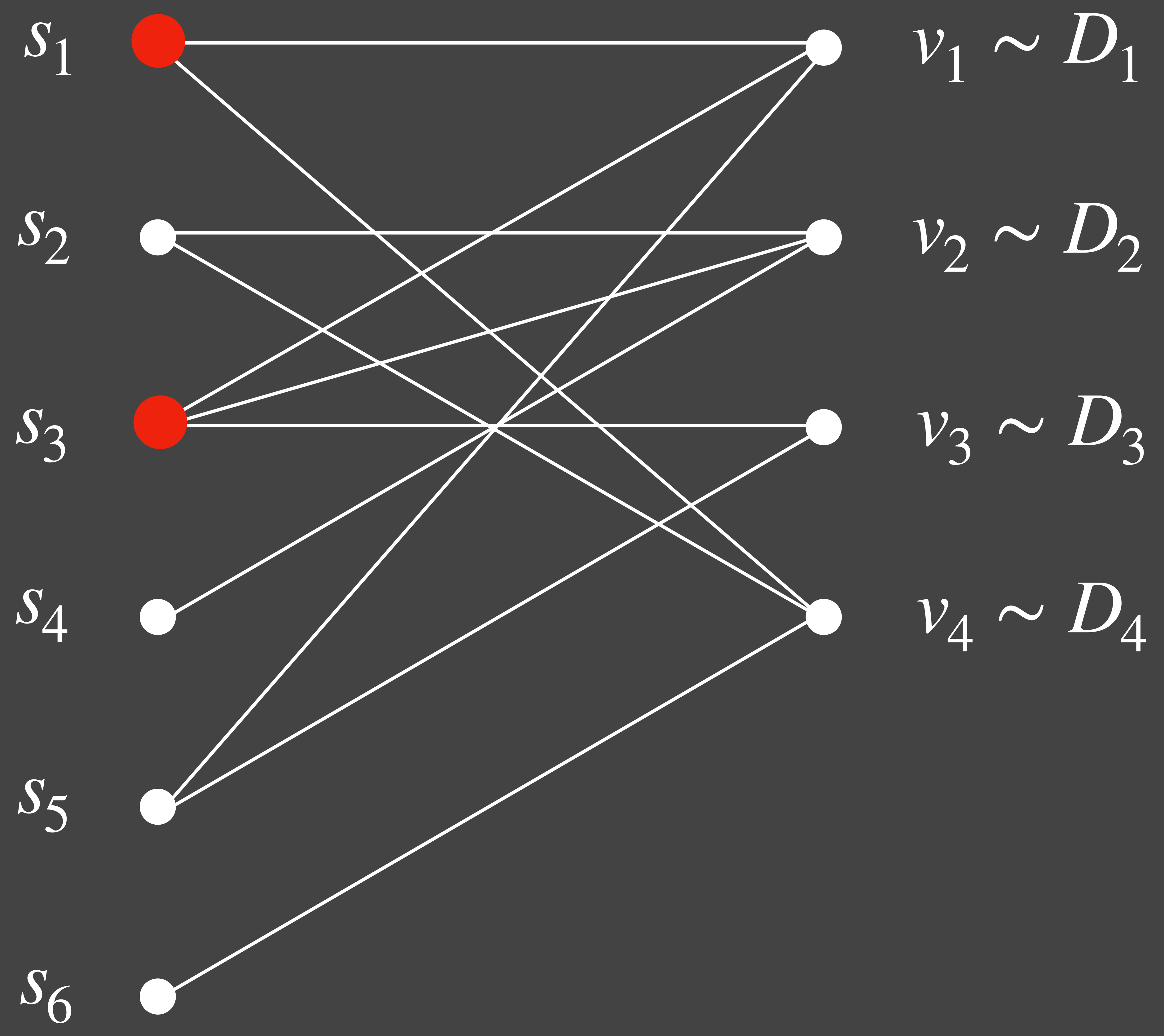
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



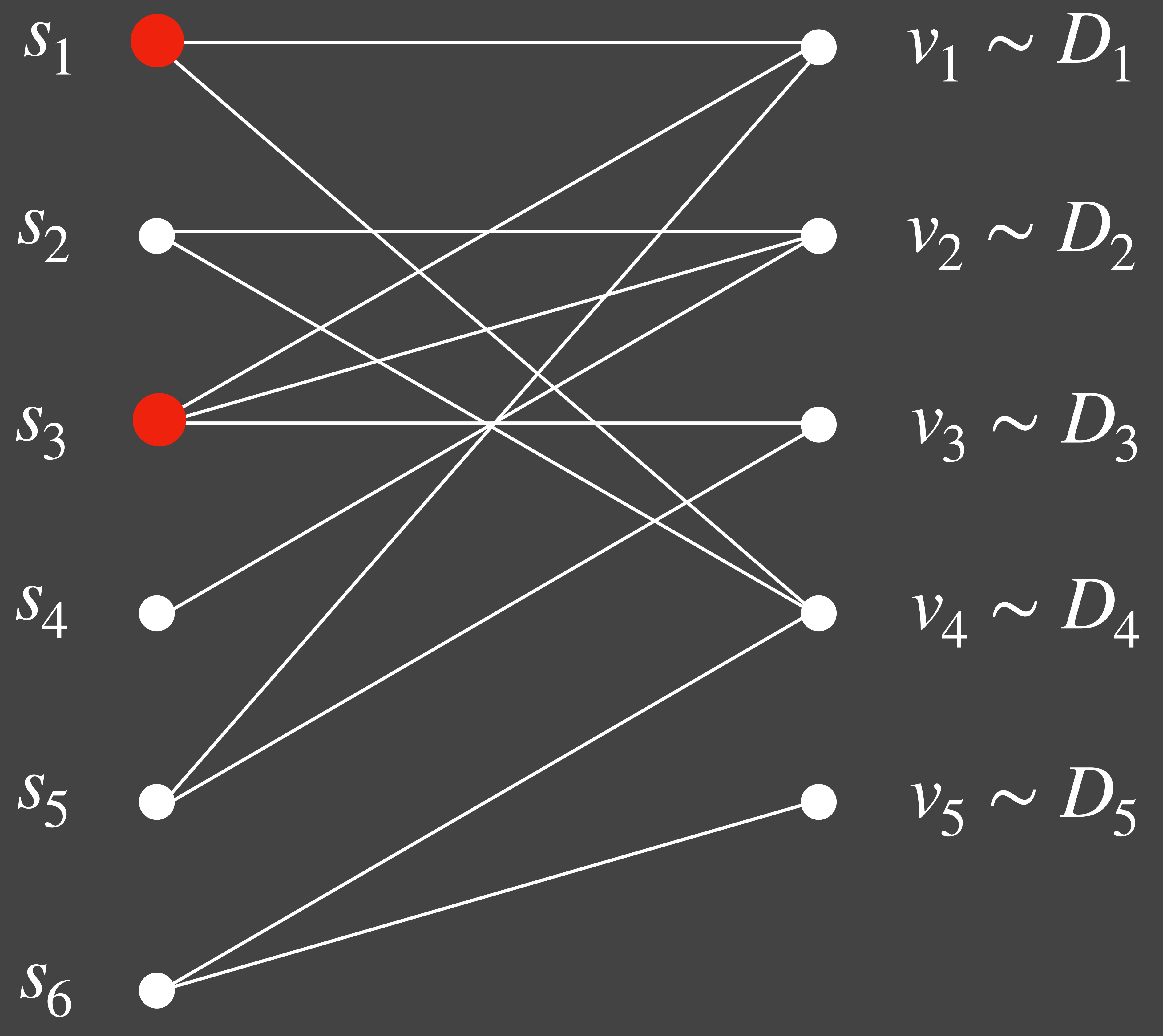
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



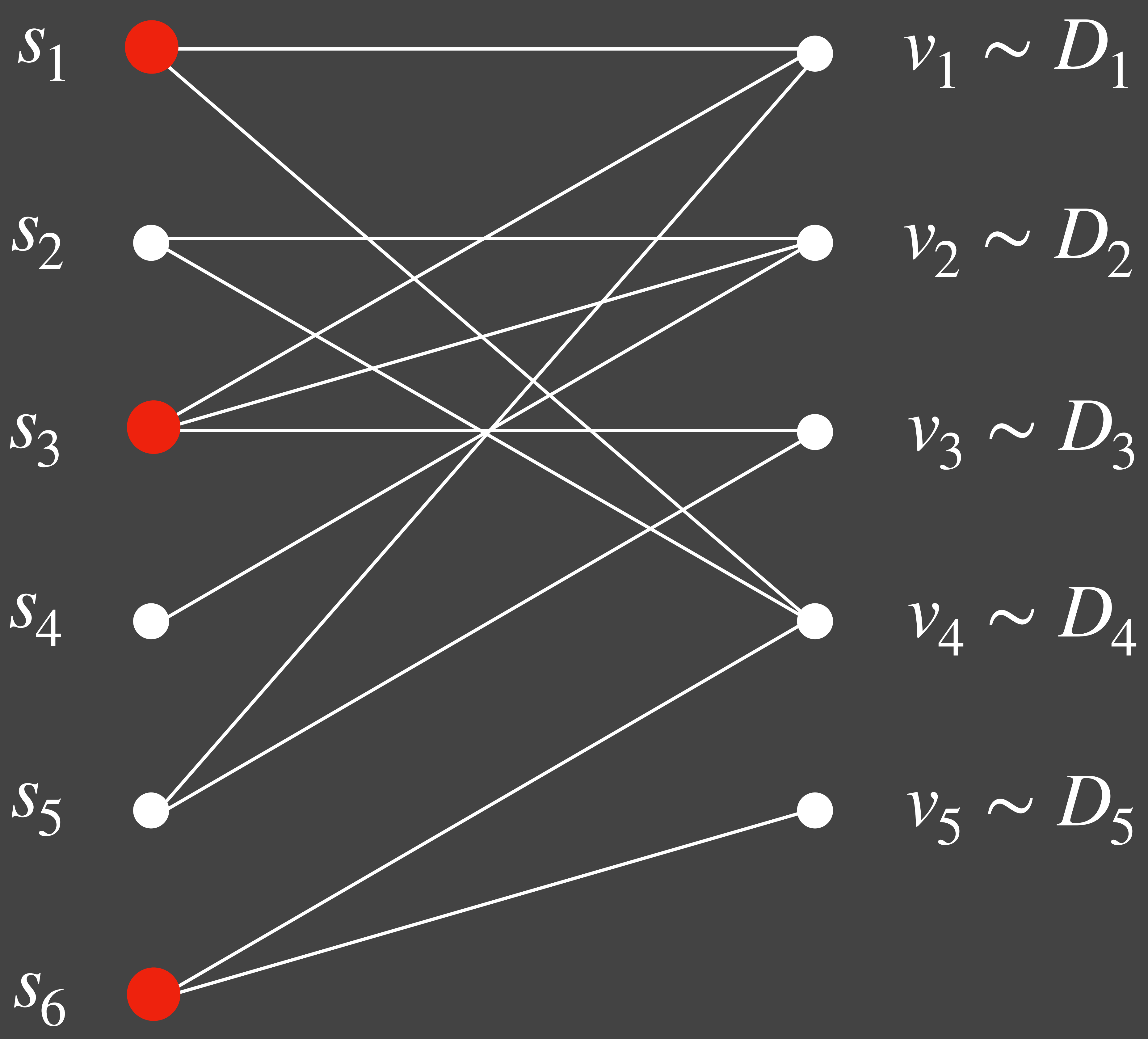
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



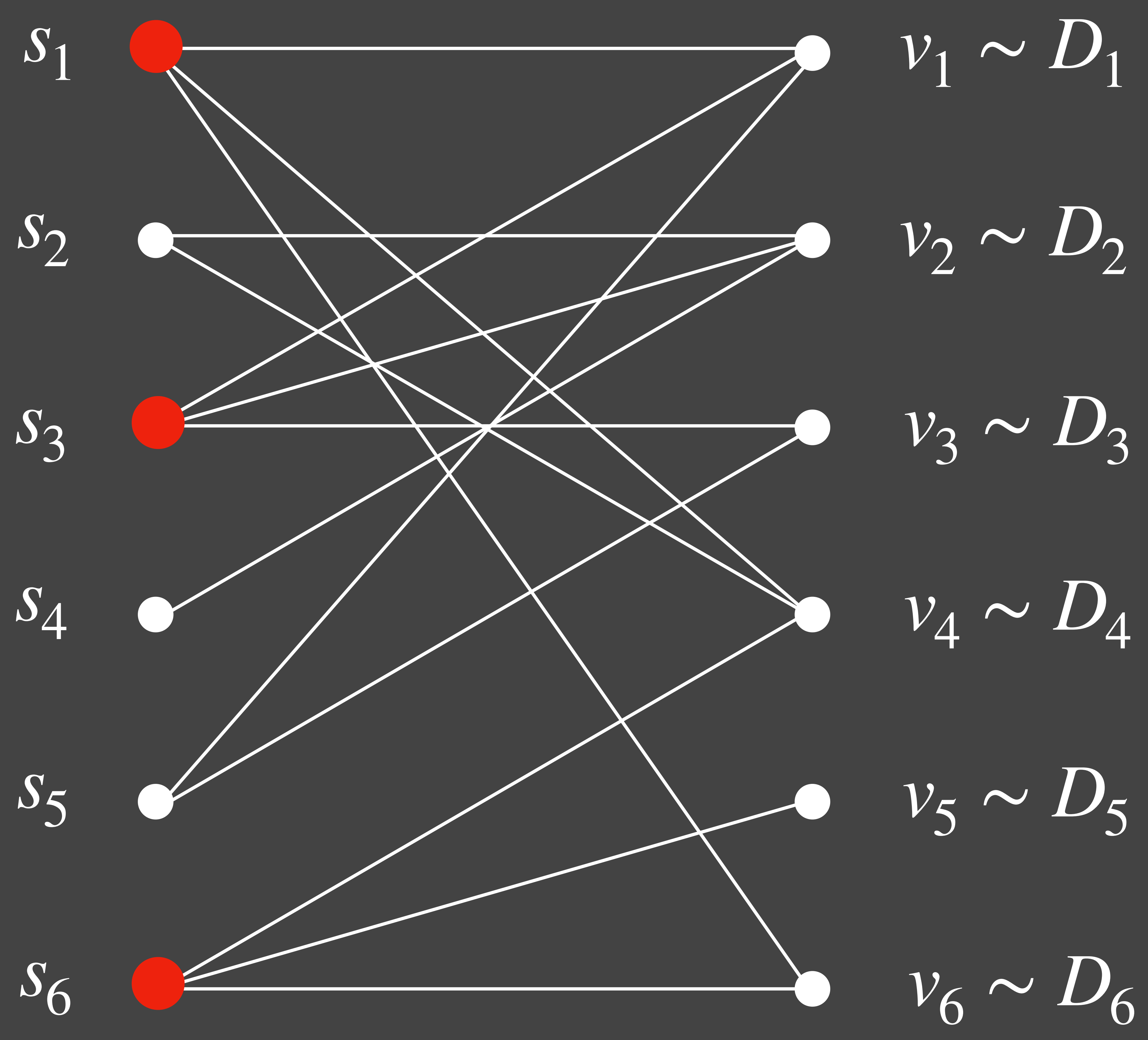
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



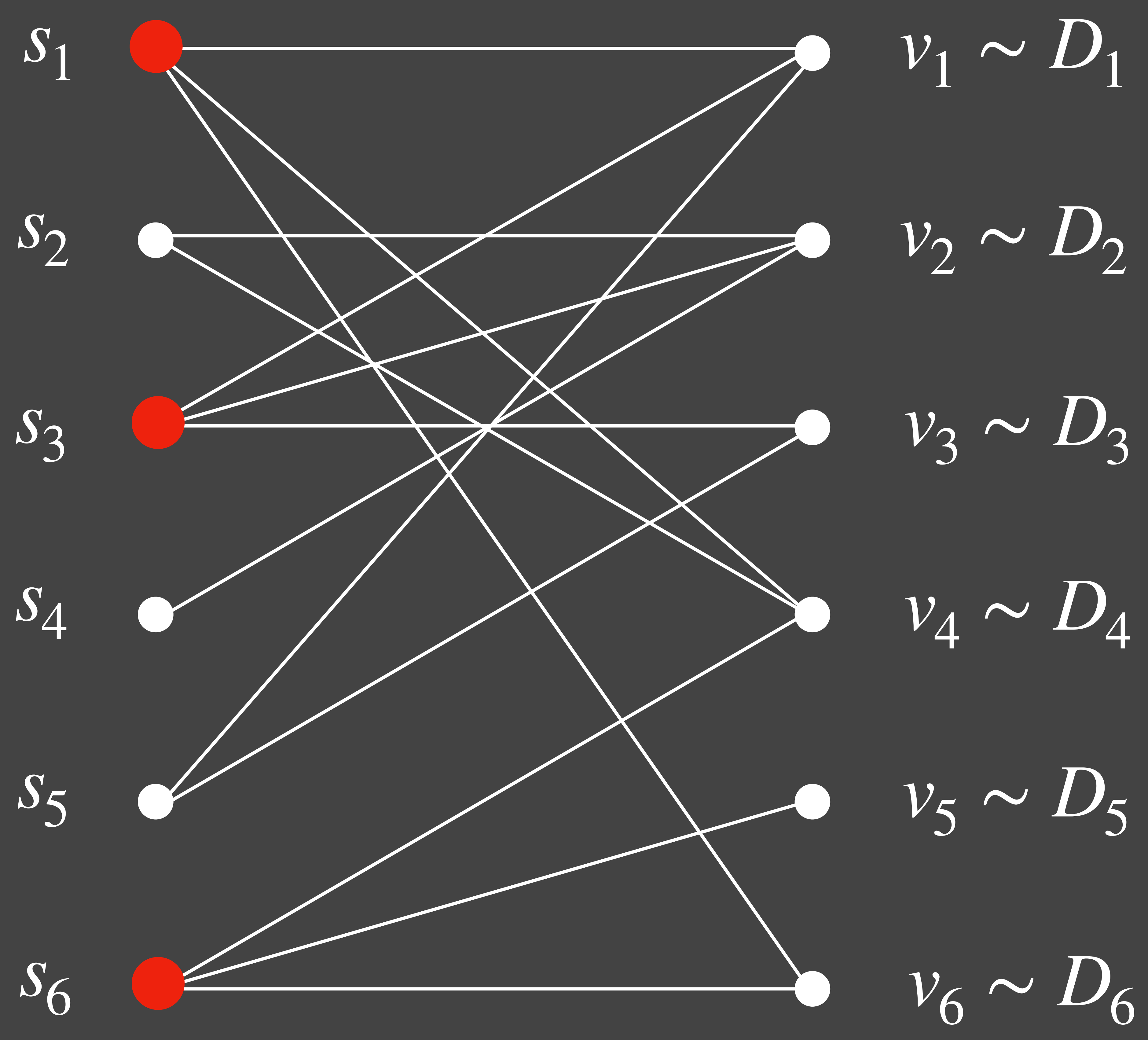
$\mathcal{U}$

n elements

Recall the model: Single-Sample Prophet

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

Only have 1 sample $\widehat{v}_i$ from each D_i.

Reduction to LearnOrCover (LoC)!

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
“Real” draws, $v_1, \ldots, v_n$

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$.

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$.

@ time t, real element v_t arrives:

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
“Real” draws, $v_1, \ldots, v_n$

```
Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$.
@ time $t$, real element $v_t$ arrives:
    If $v_t$ covered, do nothing.
```

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
“Real” draws, $v_1, \ldots, v_n$

```
Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$.
@ time $t$, real element $v_t$ arrives:
    If $v_t$ covered, do nothing.
    Else buy arbitrary set to cover $v_t$.
```

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
“Real” draws, $v_1, \ldots, v_n$

```
Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$.
@ time $t$, real element $v_t$ arrives:
    If $v_t$ covered, do nothing.
    Else buy arbitrary set to cover $v_t$.
```

$$E[c(\text{LoC}(\widehat{v}_1, \ldots, \widehat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

Reduction to LearnOrCover (LoC)!

Samples $\widehat{v}_1, \ldots, \widehat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\widehat{v}_1, \ldots, \widehat{v}_n$. ✓

@ time t, real element v_t arrives:

  If v_t covered, do nothing.

  Else buy arbitrary set to cover v_t.

$$E[c(\text{LoC}(\widehat{v}_1, \ldots, \widehat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

Reduction to LearnOrCover (LoC)!

Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓

@ time t, real element v_t arrives:

If v_t covered, do nothing.

Else buy arbitrary set to cover v_t.

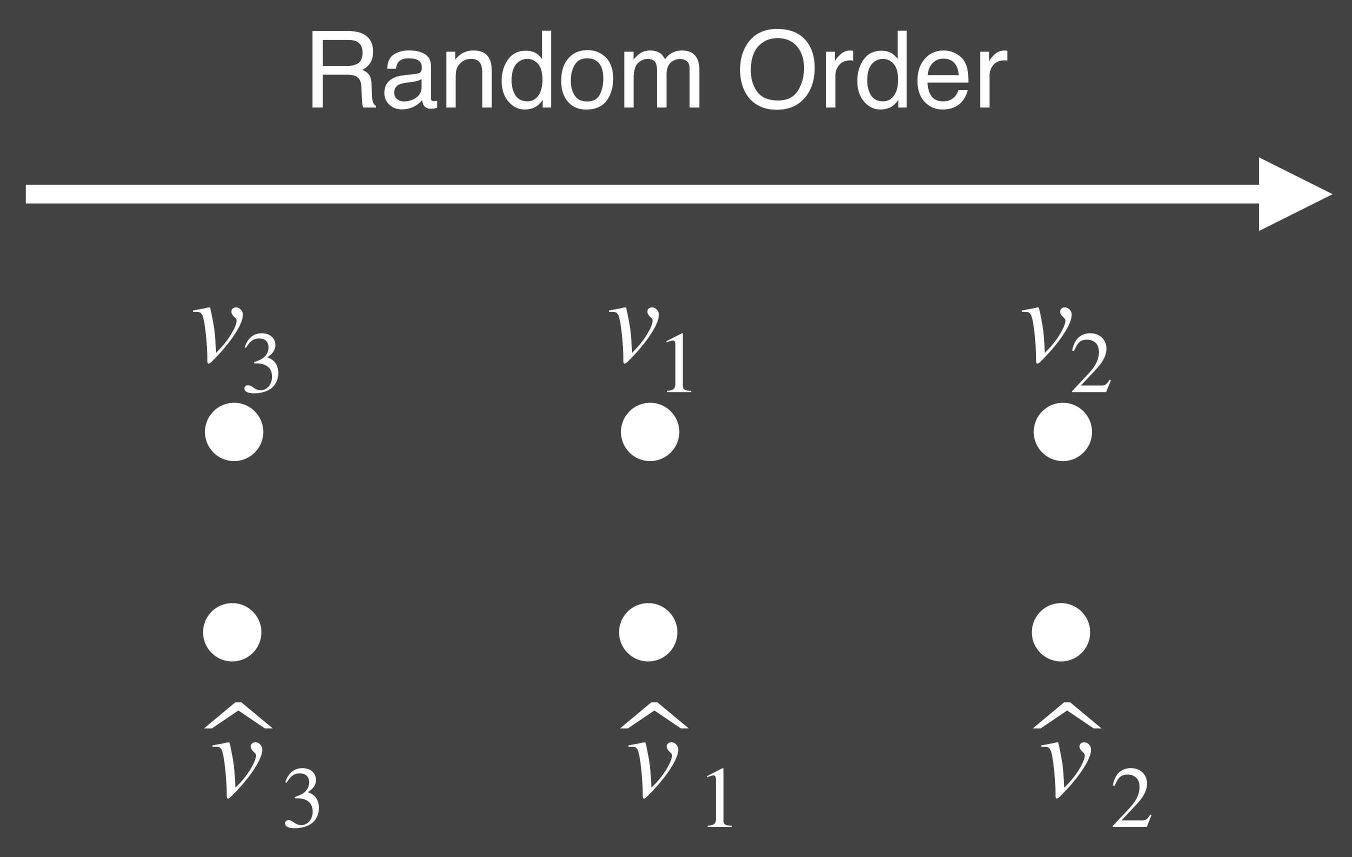
$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

Reduction to LearnOrCover (LoC)!

Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓
@ time t, real element v_t arrives:
    If v_t covered, do nothing.
    Else buy arbitrary set to cover v_t.


Random Order

$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

Reduction to LearnOrCover (LoC)!

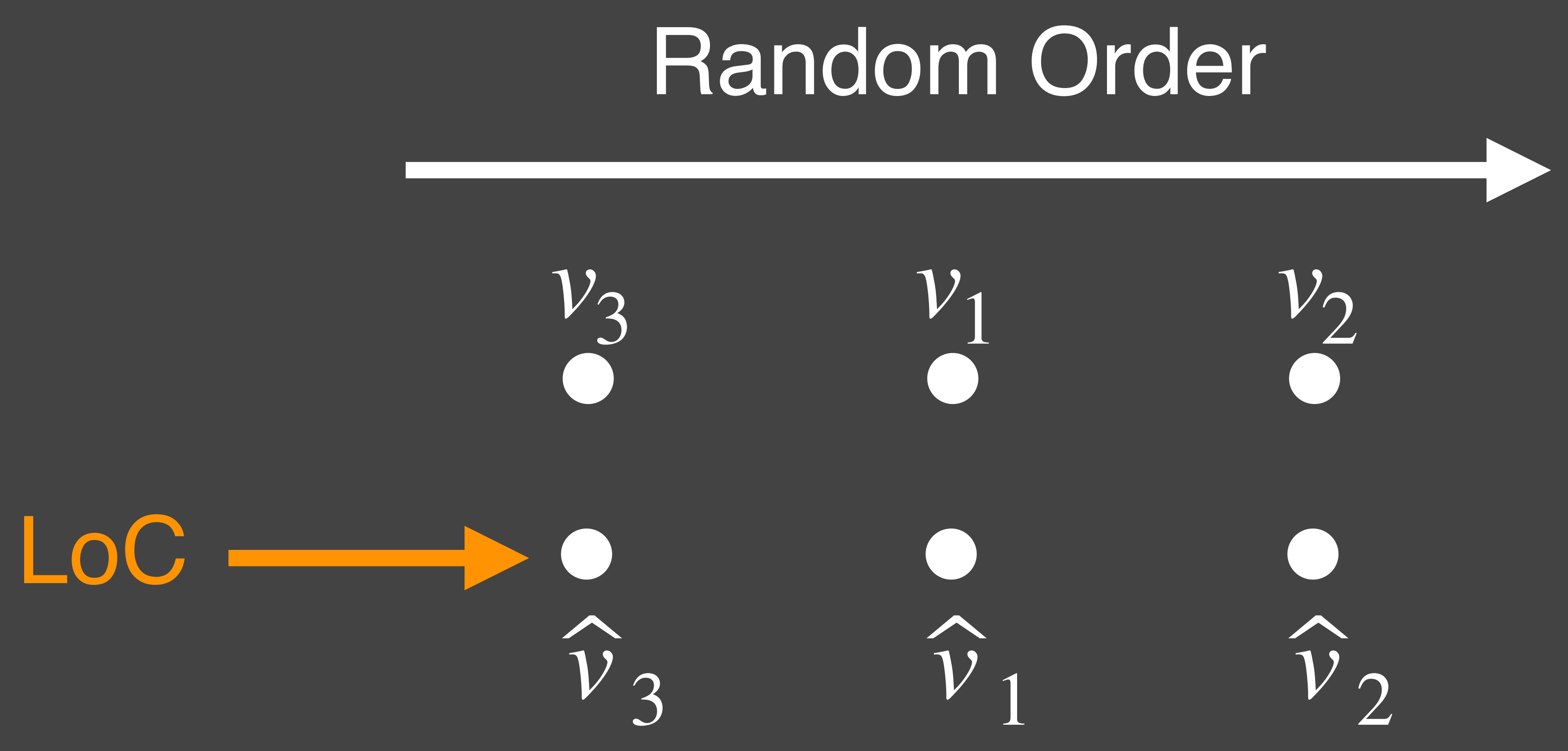
Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓

@ time t, real element v_t arrives:

If v_t covered, do nothing.

Else buy arbitrary set to cover v_t.



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

Reduction to LearnOrCover (LoC)!

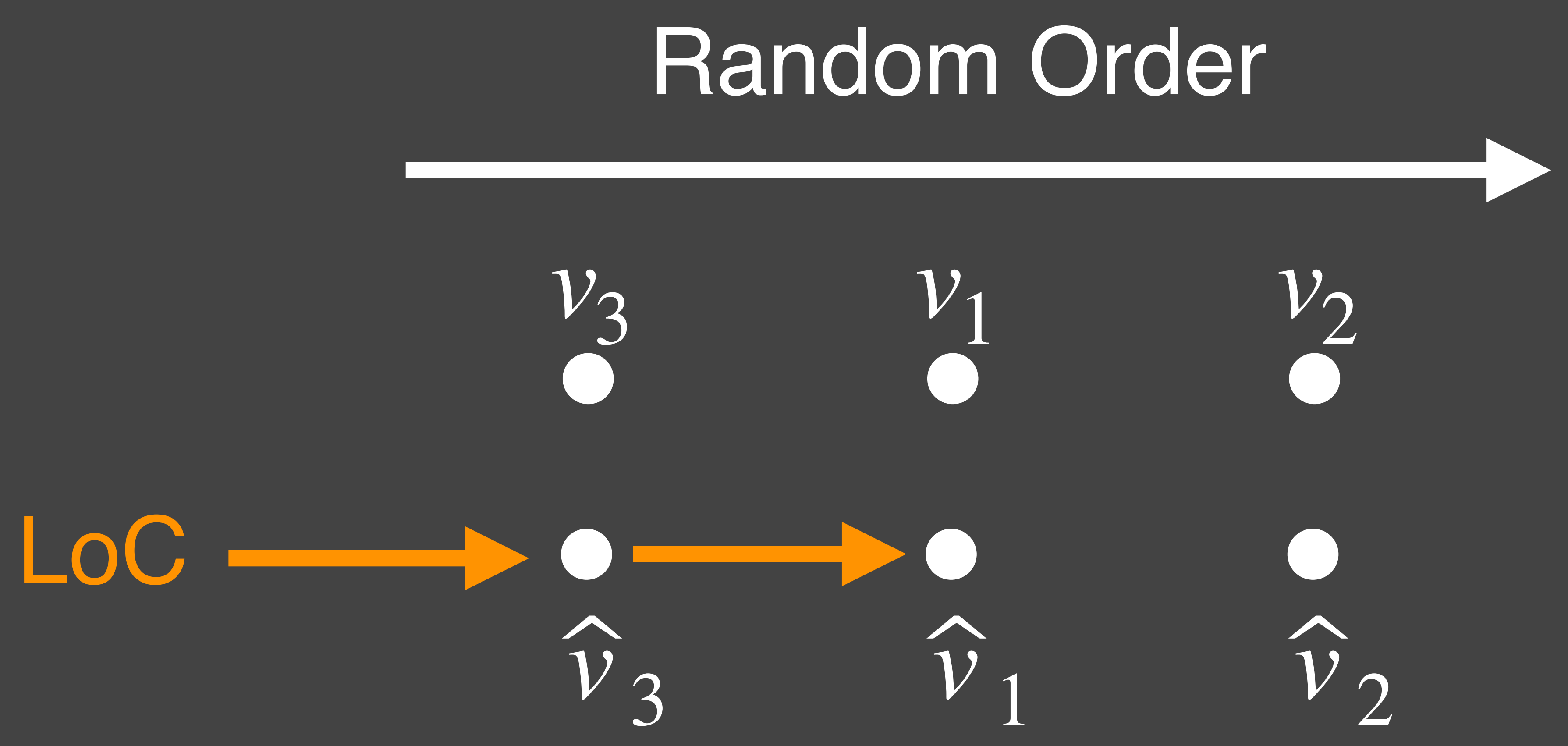
Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓

@ time t, real element v_t arrives:

If v_t covered, do nothing.

Else buy arbitrary set to cover v_t.



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

Reduction to LearnOrCover (LoC)!

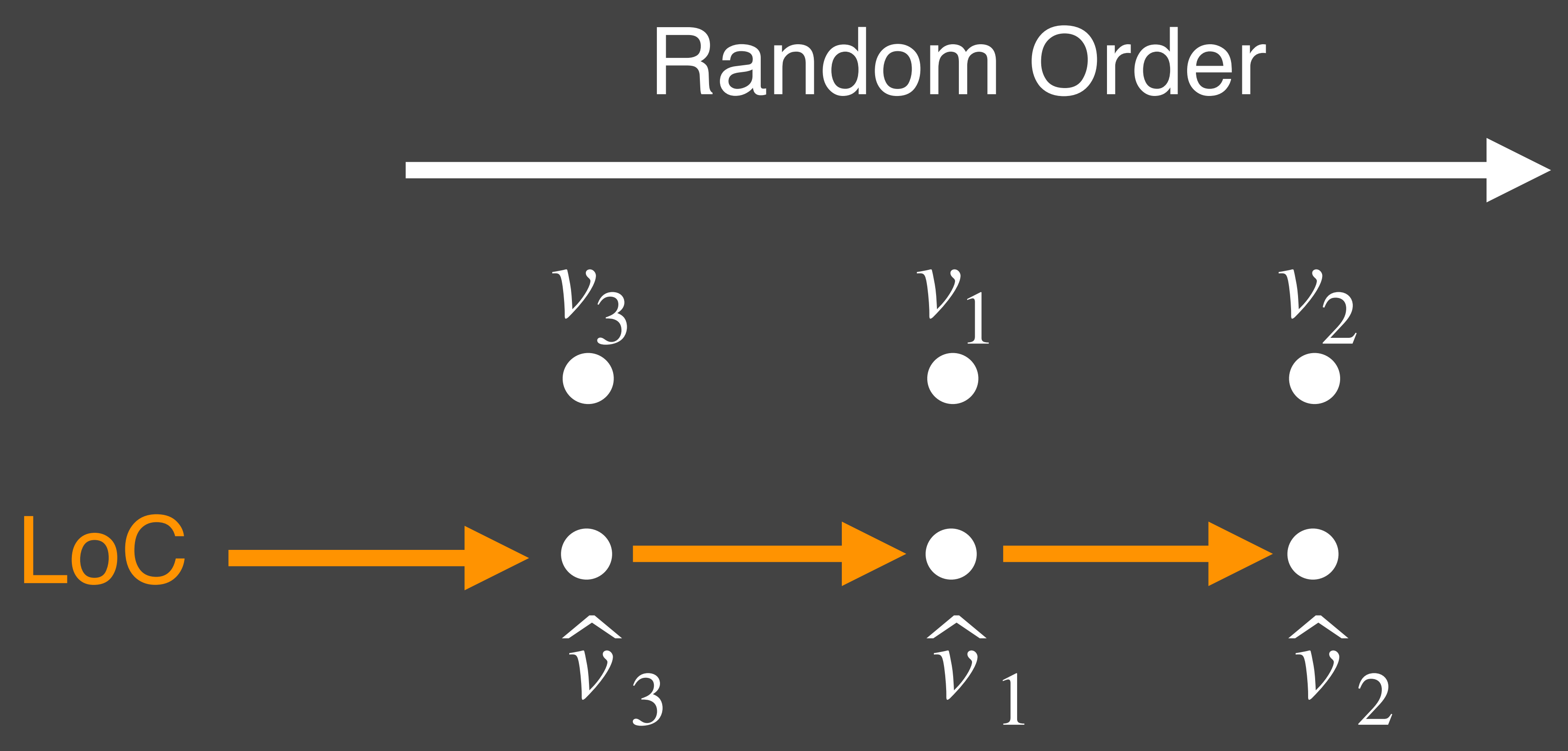
Samples $\hat{v}_1, \ldots, \hat{v}_n$

"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓

@ time t, real element v_t arrives:

- If v_t covered, do nothing.
- Else buy arbitrary set to cover v_t.



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

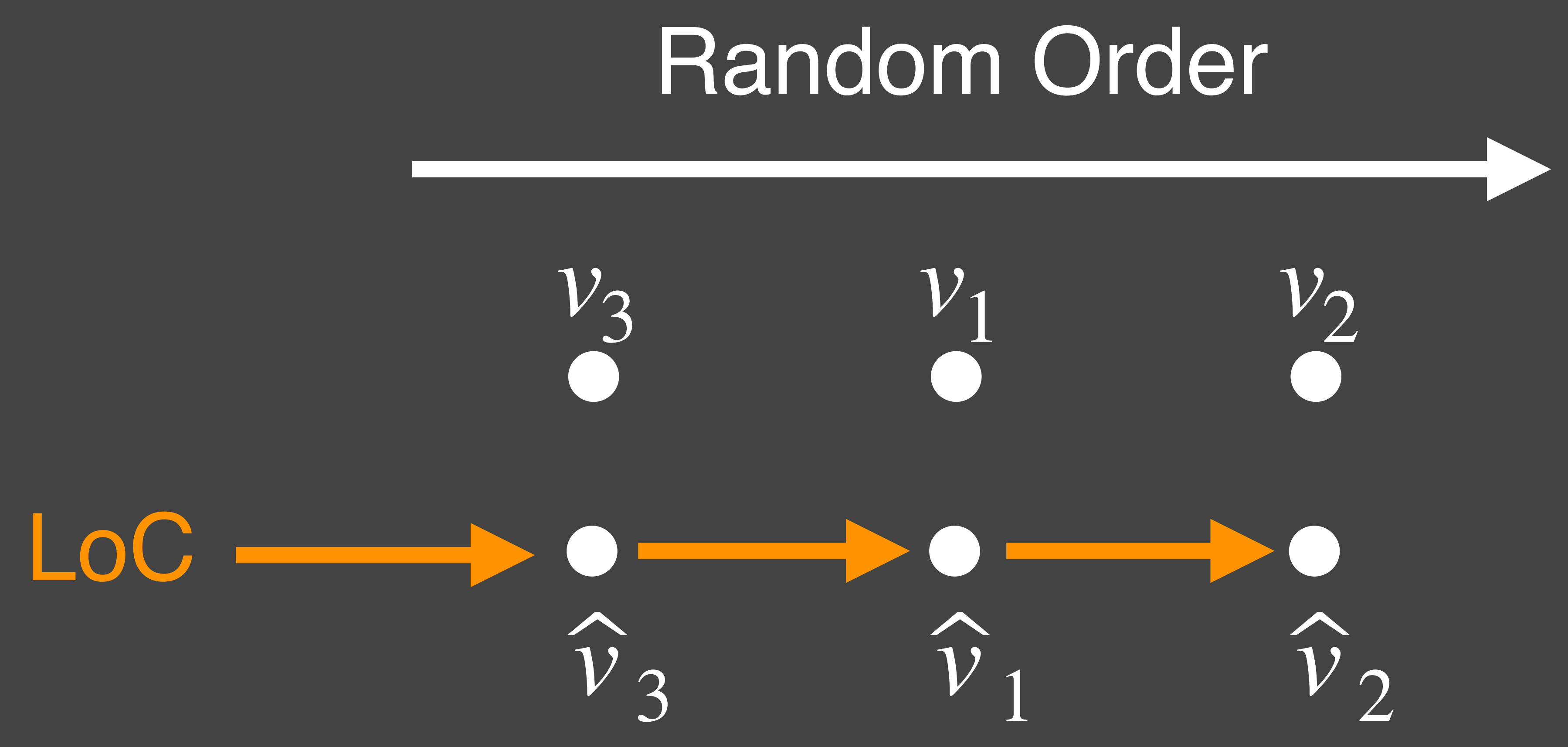
Reduction to LearnOrCover (LoC)!

Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓

@ time t, real element v_t arrives:

- If v_t covered, do nothing.
- Else buy arbitrary set to cover v_t.



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

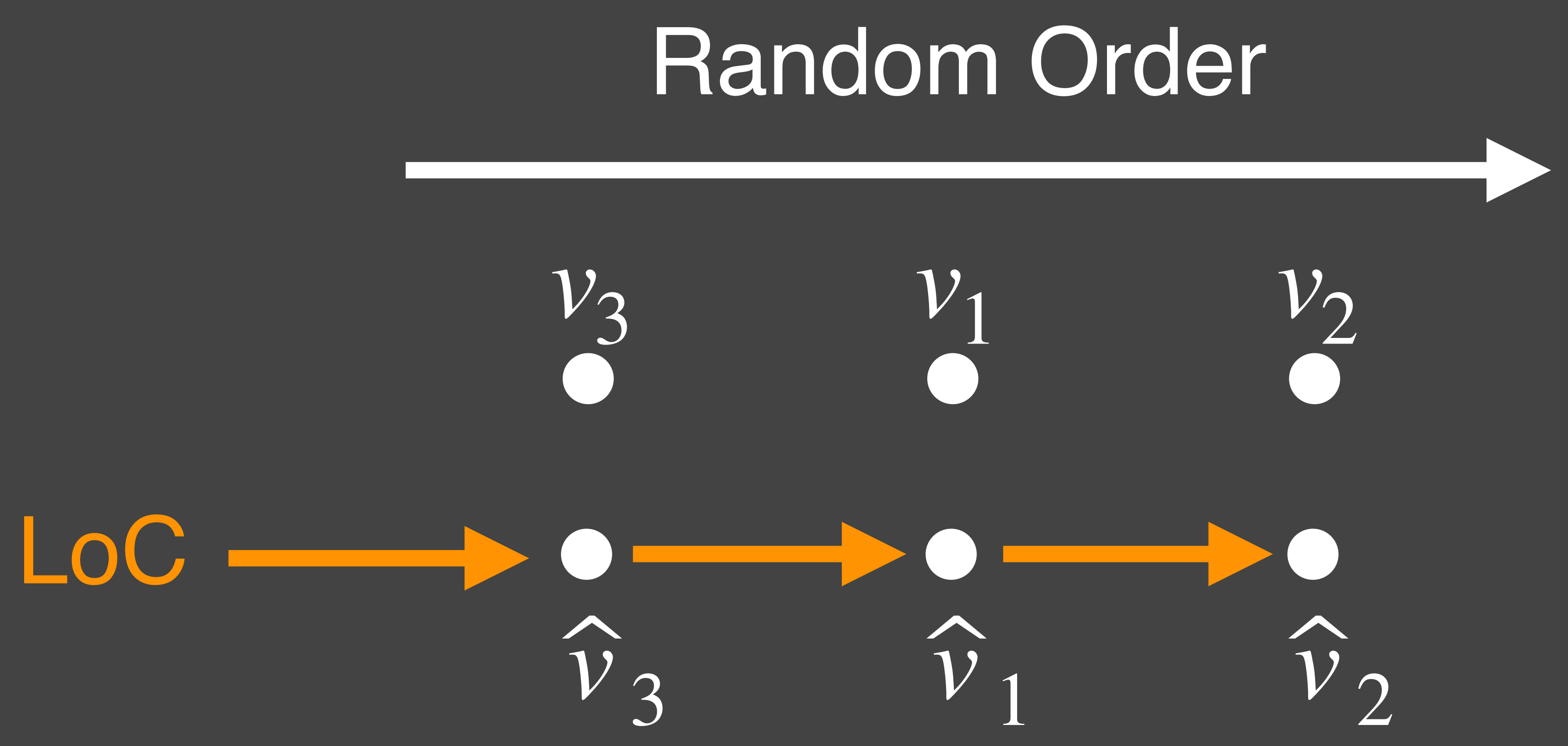
$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

$$\sum_i E[c(\text{backup}(v_i)] \leq \sum_i E[c(\text{backup}(\hat{v}_i))] \leq E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))]$$

Reduction to LearnOrCover (LoC)!

Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓
@ time t, real element v_t arrives:
  If v_t covered, do nothing.
  Else buy arbitrary set to cover v_t. ✓



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

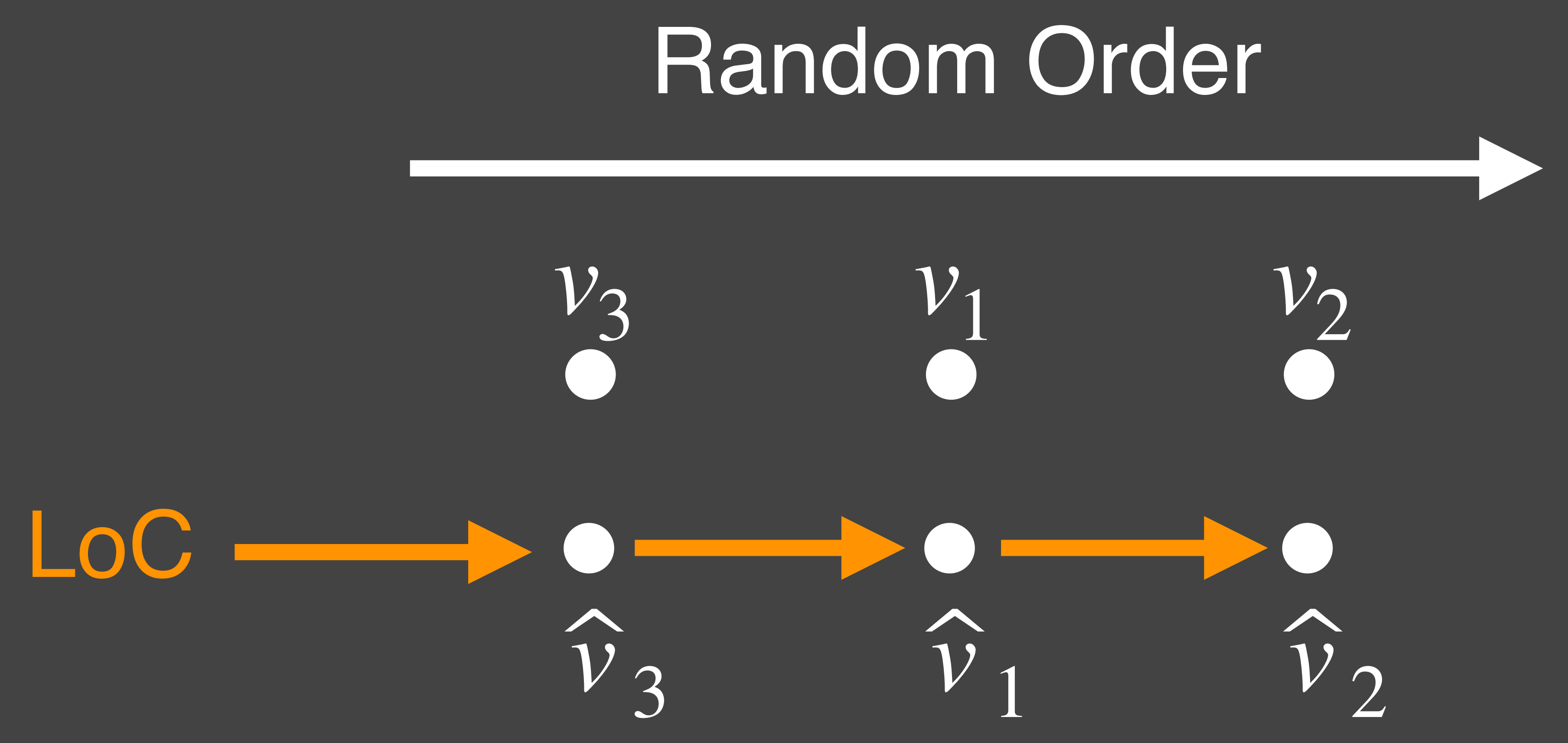
$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

$$\sum_i E[c(\text{backup}(v_i)] \leq \sum_i E[c(\text{backup}(\hat{v}_i))] \leq E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))]$$

Reduction to LearnOrCover (LoC)!

Samples $\hat{v}_1, \ldots, \hat{v}_n$
"Real" draws, $v_1, \ldots, v_n$

Run LoC on $\hat{v}_1, \ldots, \hat{v}_n$. ✓
@ time t, real element v_t arrives:
    If v_t covered, do nothing.
    Else buy arbitrary set to cover v_t. ✓



$$E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))] = E[c(\text{LoC}(v_1, \ldots, v_n))] = O(\log(mn)) \cdot OPT$$

$$E[c(\text{backup}(v_i)] \leq E[c(\text{backup}(\hat{v}_i))]$$

$$\sum_i E[c(\text{backup}(v_i)] \leq \sum_i E[c(\text{backup}(\hat{v}_i))] \leq E[c(\text{LoC}(\hat{v}_1, \ldots, \hat{v}_n))]$$

🥳

Universality

Universality

Can build map $f : \mathscr{U} \to \mathscr{S}$ after only seeing $\widehat{v}_1, \ldots, \widehat{v}_n$ …

… when $v \in \mathscr{U}$ arrives, commit to buying $f(v)$!

Universality

Can build map $f : \mathcal{U} \to \mathcal{S}$ after only seeing $\widehat{v}_1, \ldots, \widehat{v}_n$ …

… when $v \in \mathcal{U}$ arrives, commit to buying $f(v)$!

I.e. build f ***before*** seeing “real” elements!

Universality

Can build map $f : \mathscr{U} \to \mathscr{S}$ after only seeing $\widehat{v}_1, \ldots, \widehat{v}_n$ …

… when $v \in \mathscr{U}$ arrives, commit to buying $f(v)$!

I.e. build f ***before*** seeing “real” elements!

⇒ Only need $O(n)$ samples to build this map.

Universality

Can build map $f : \mathscr{U} \to \mathscr{S}$ after only seeing $\widehat{v}_1, \ldots, \widehat{v}_n$ …

… when $v \in \mathscr{U}$ arrives, commit to buying $f(v)$!

I.e. build f ***before*** seeing "real" elements!

⇒ Only need $O(n)$ samples to build this map.

Previously only known with full knowledge of D_i, and only for iid case [GGLMSS 08].

Talk Outline

Intro

Secretary

- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

➡ (Single Sample) Prophet

Conclusion & Extensions

Talk Outline

Intro

Secretary
- LearnOrCover in Exponential Time
- LearnOrCover in Poly Time

(Single Sample) Prophet

➡ Conclusion & Extensions

LearnOrCover gives

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

+ Streaming!

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

+ Streaming!

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

+ Streaming!

+ Single-Sample!
+ Universal!

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

+ Streaming!

+ Single-Sample!
+ Universal!

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

+ Streaming!

+ Single-Sample!
+ Universal!

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

+ Streaming!

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

+ Single-Sample!
+ Universal!

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

Theorem: Same results for 2-stage variant.

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

Theorem: Same results for 2-stage variant.

+ Streaming!

+ Single-Sample!
+ Universal!

Open Questions:

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

Theorem: Same results for 2-stage variant.

+ Streaming!

+ Single-Sample!
+ Universal!

Open Questions:

Does the LearnOrCover idea lend itself to other problems?

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

Theorem: Same results for 2-stage variant.

+ Streaming!

+ Single-Sample!
+ Universal!

Open Questions:

Does the LearnOrCover idea lend itself to other problems?
Harder covering problems? Covering IPs w/ box constraints?

LearnOrCover gives

Theorem: $O(\log mn)$-comp. algo for RO Covering IPs.

Theorem: $O(\log mn)$-comp. algo for Prophet Covering IPs.

Theorem: Same results for Non-metric facility location.

Theorem: Same results for with-a-sample variant.

Theorem: Same results for 2-stage variant.

+ Streaming!

+ Single-Sample!
+ Universal!

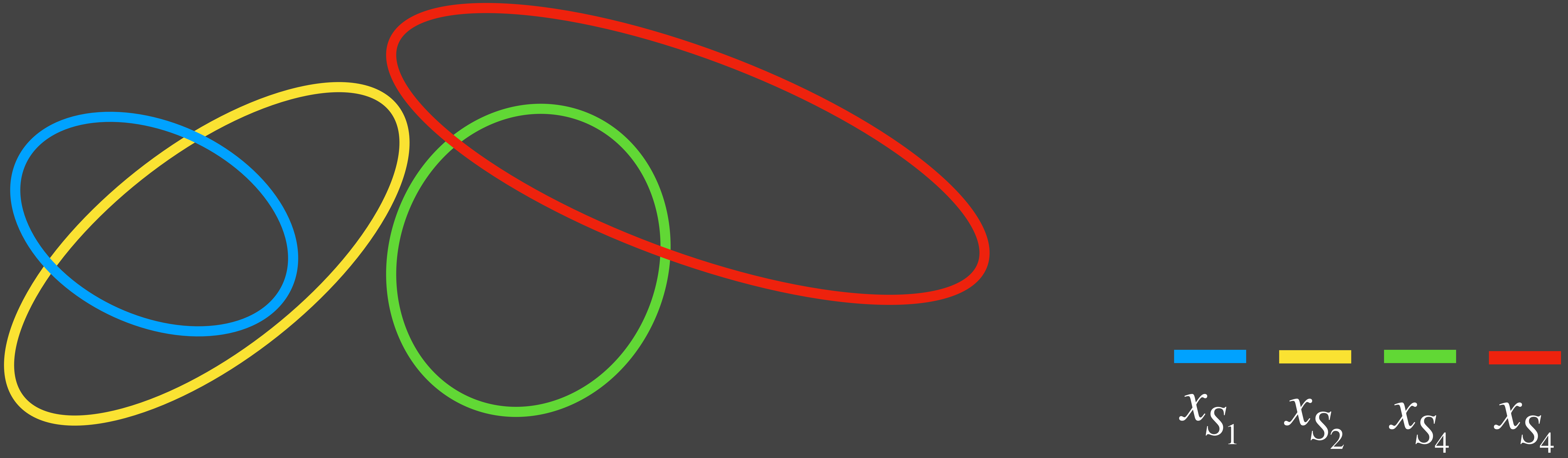
Open Questions:

Does the LearnOrCover idea lend itself to other problems?
Harder covering problems? Covering IPs w/ box constraints?
Unified theory? Reinterpret old RO results as LearnOrCover?

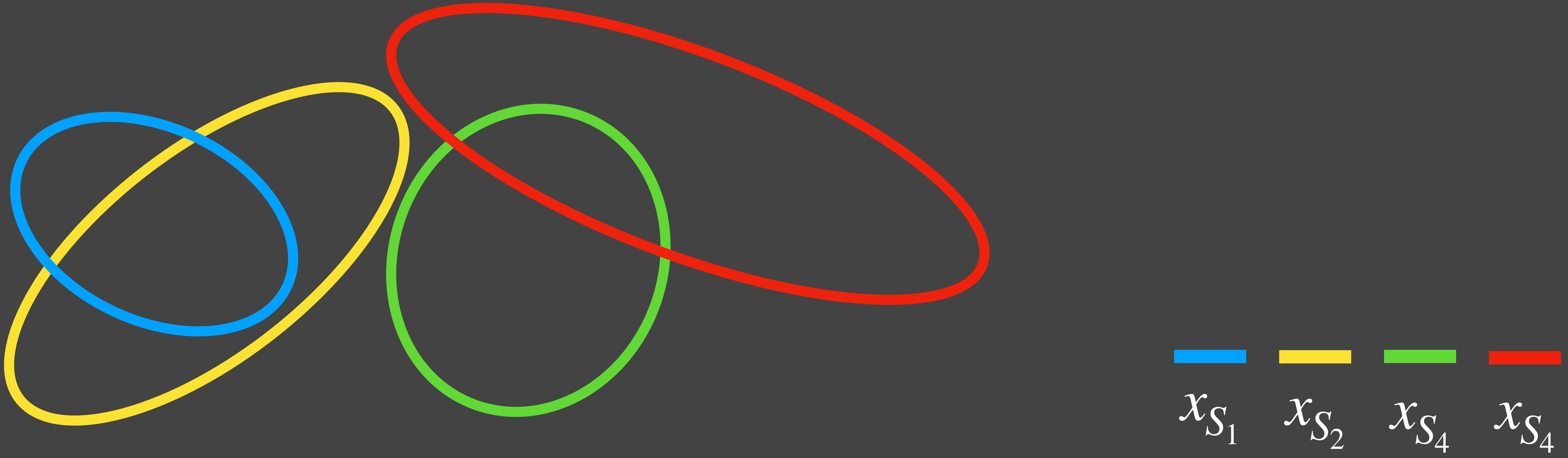
Thanks!

Backup Slides

Online LP Solver of [Alon+ 03]



Online LP Solver of [Alon+ 03]

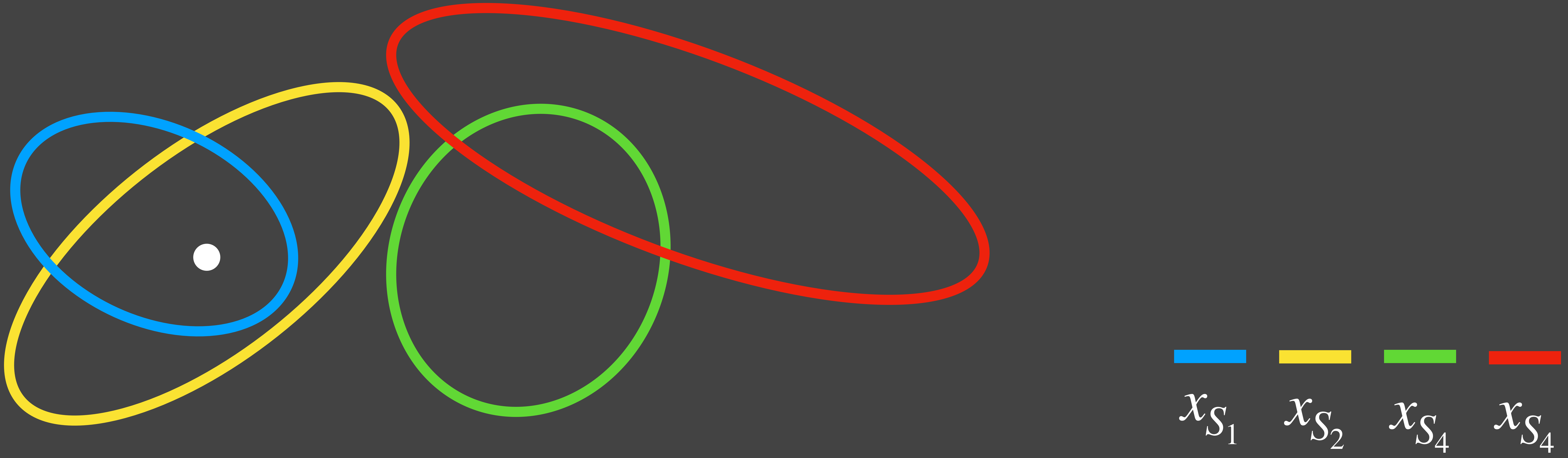


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

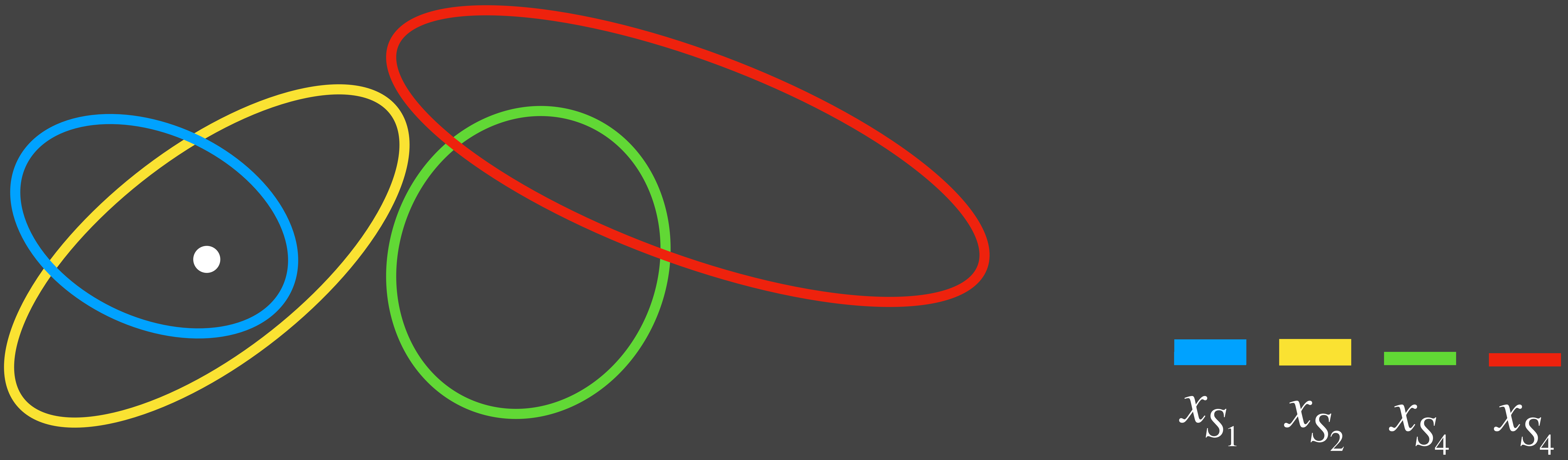


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

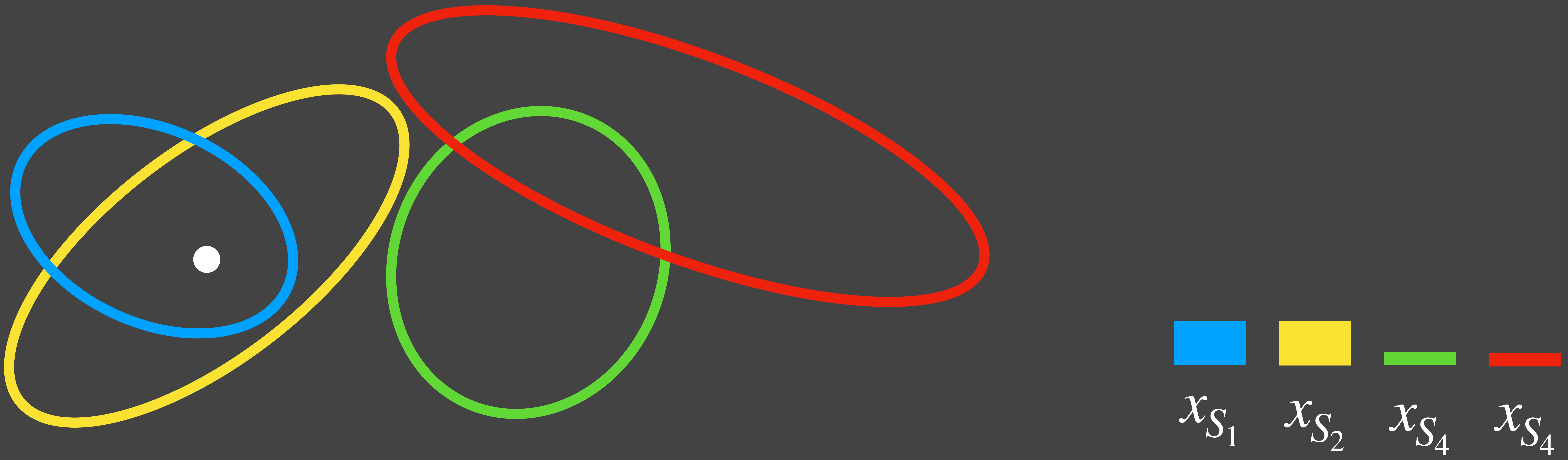


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

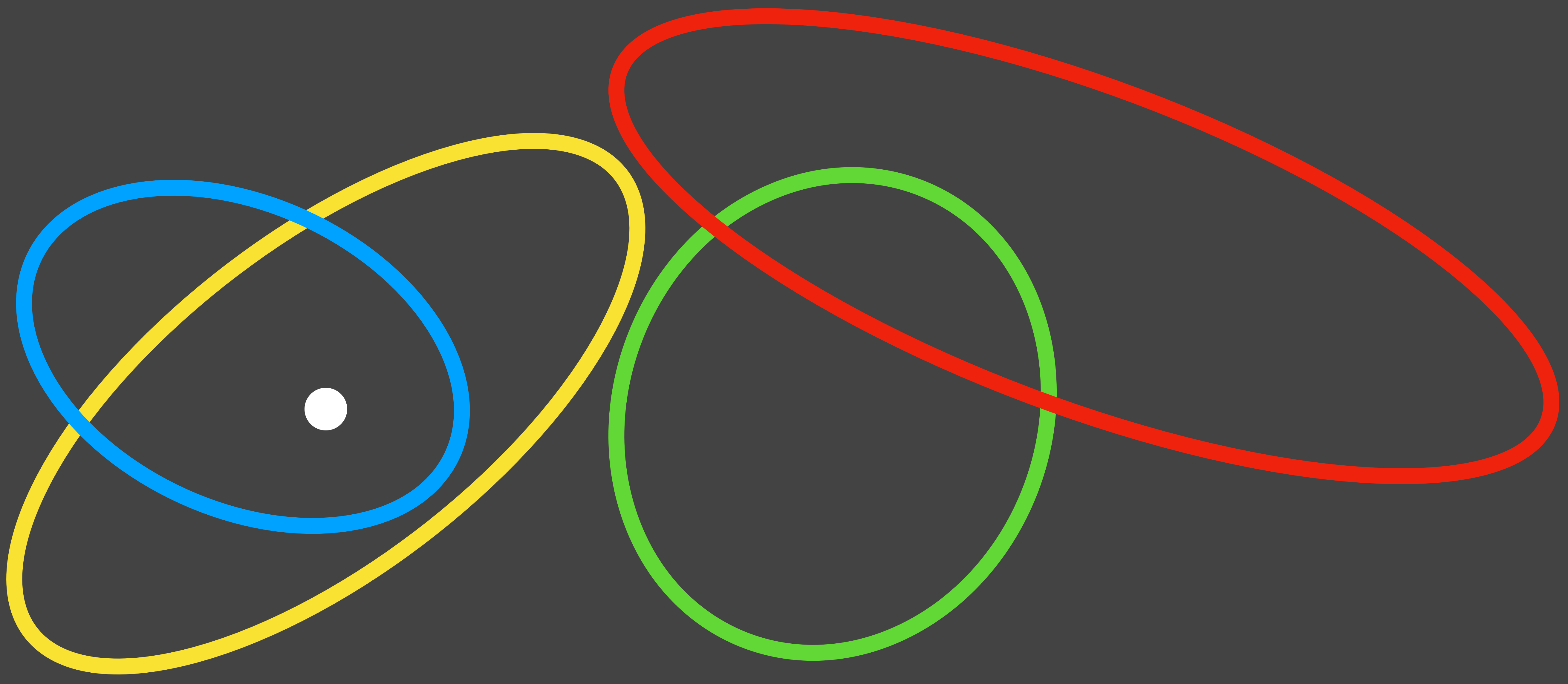


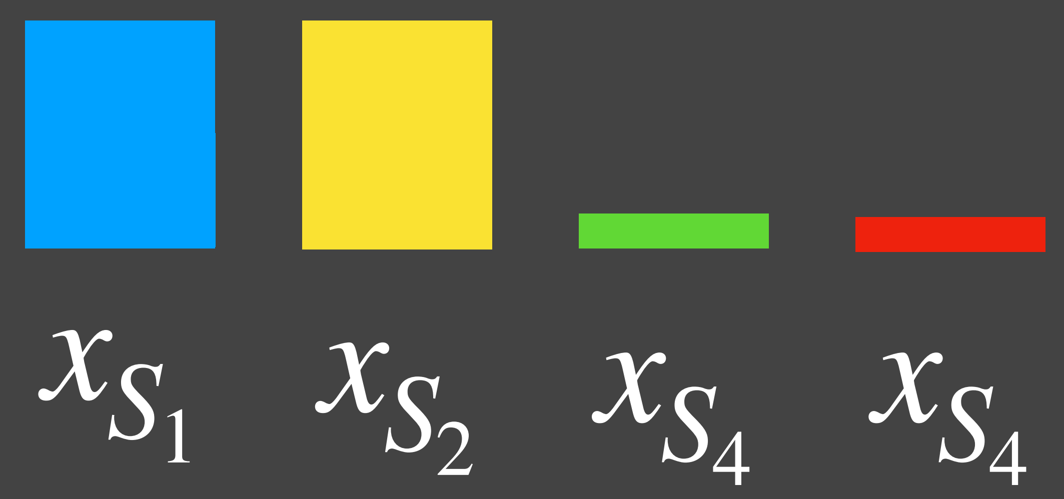
Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]



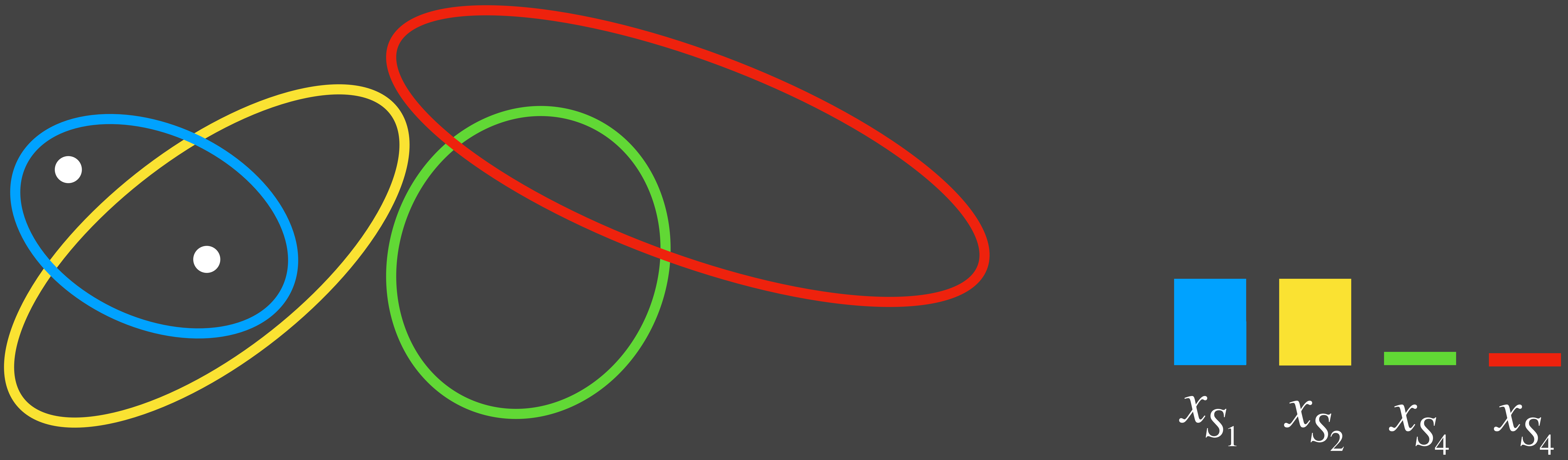


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

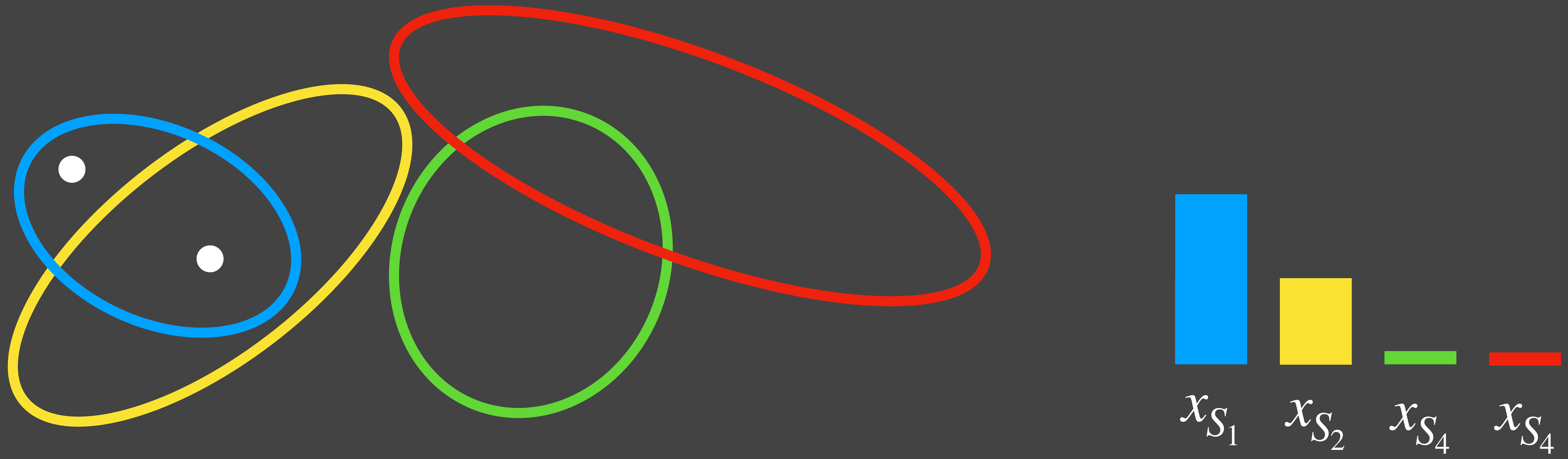


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

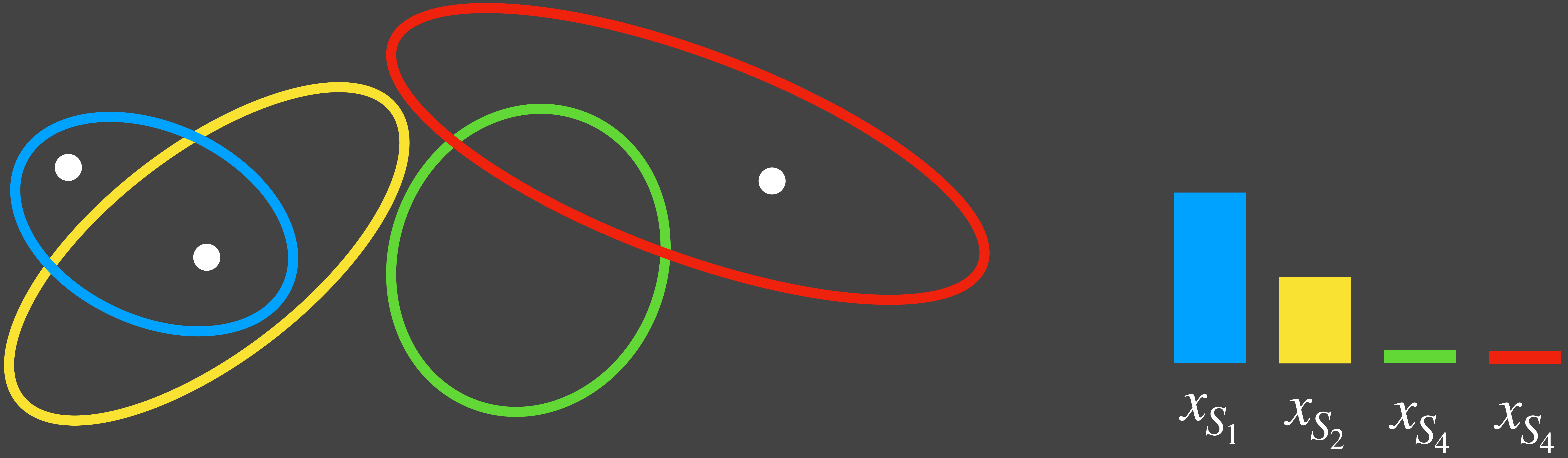


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

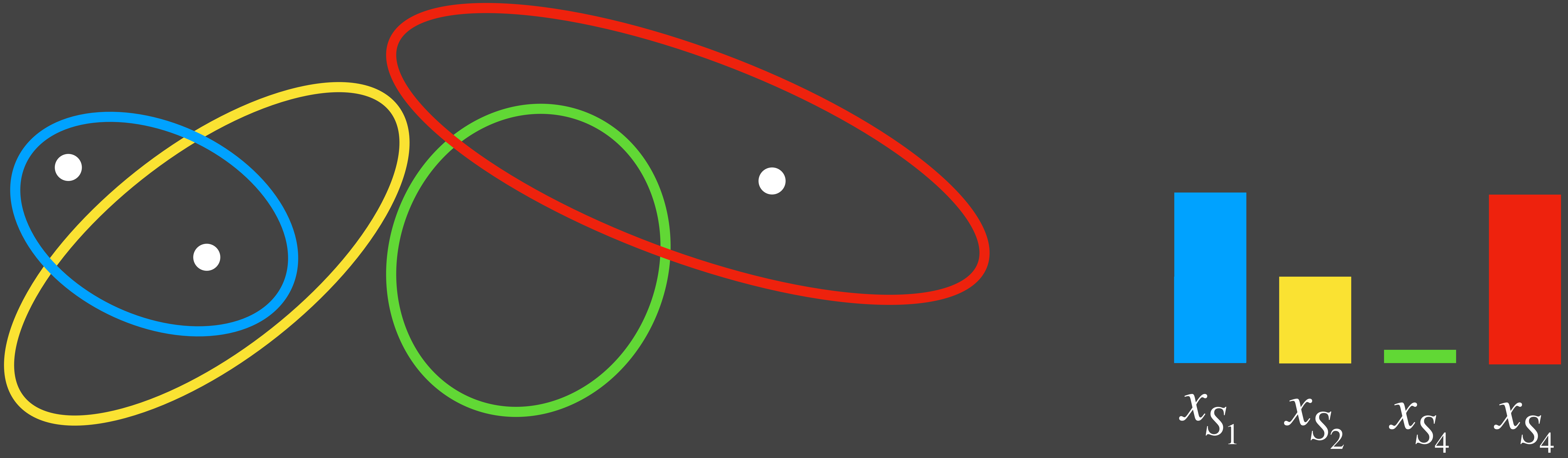


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

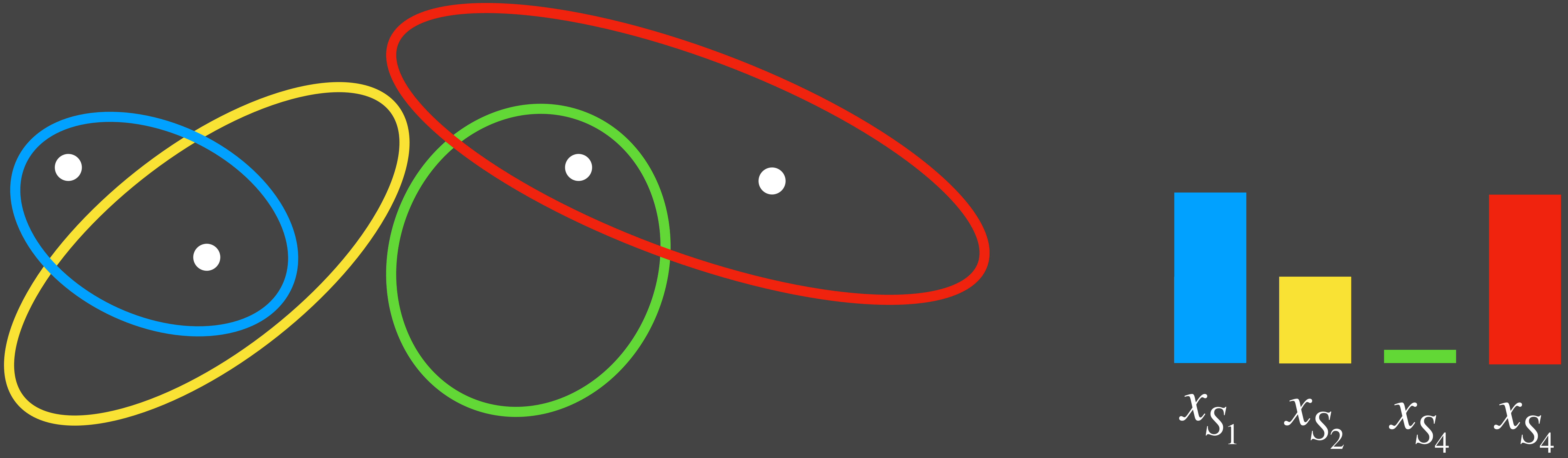


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]

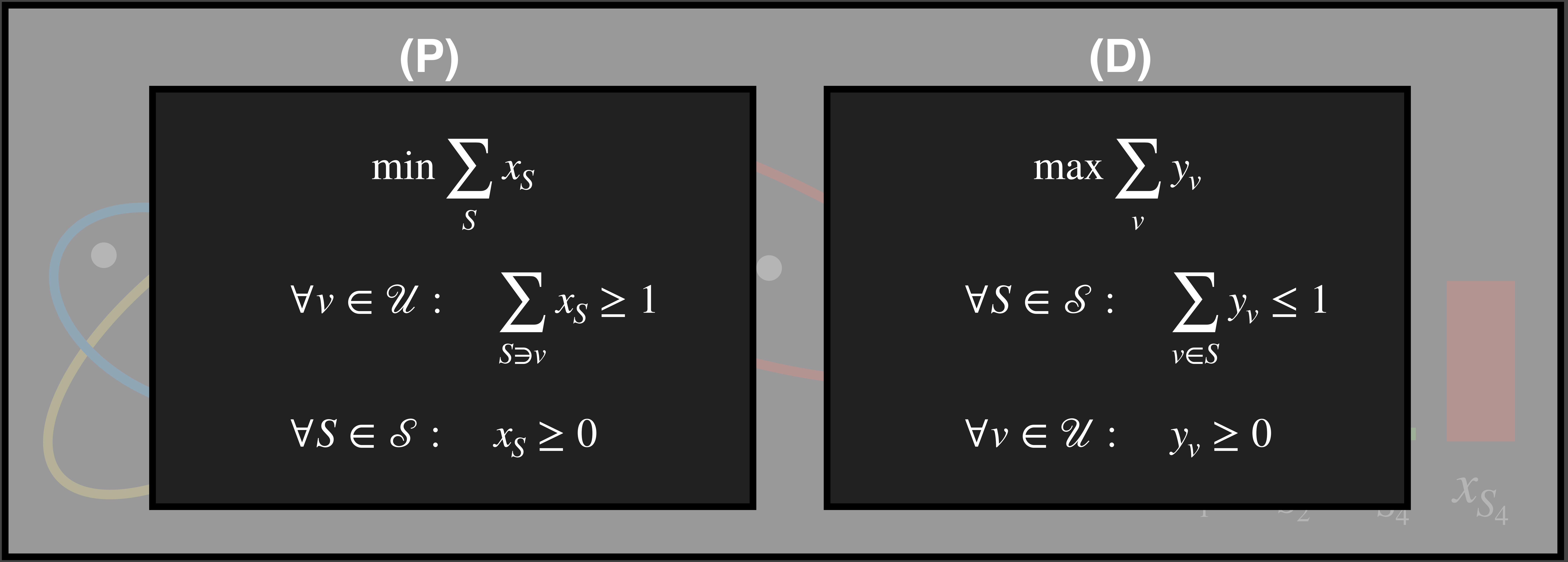


Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]



(P)

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(D)

$$\max \sum_{v} y_v$$

$$\forall S \in \mathcal{S} : \quad \sum_{v \in S} y_v \leq 1$$

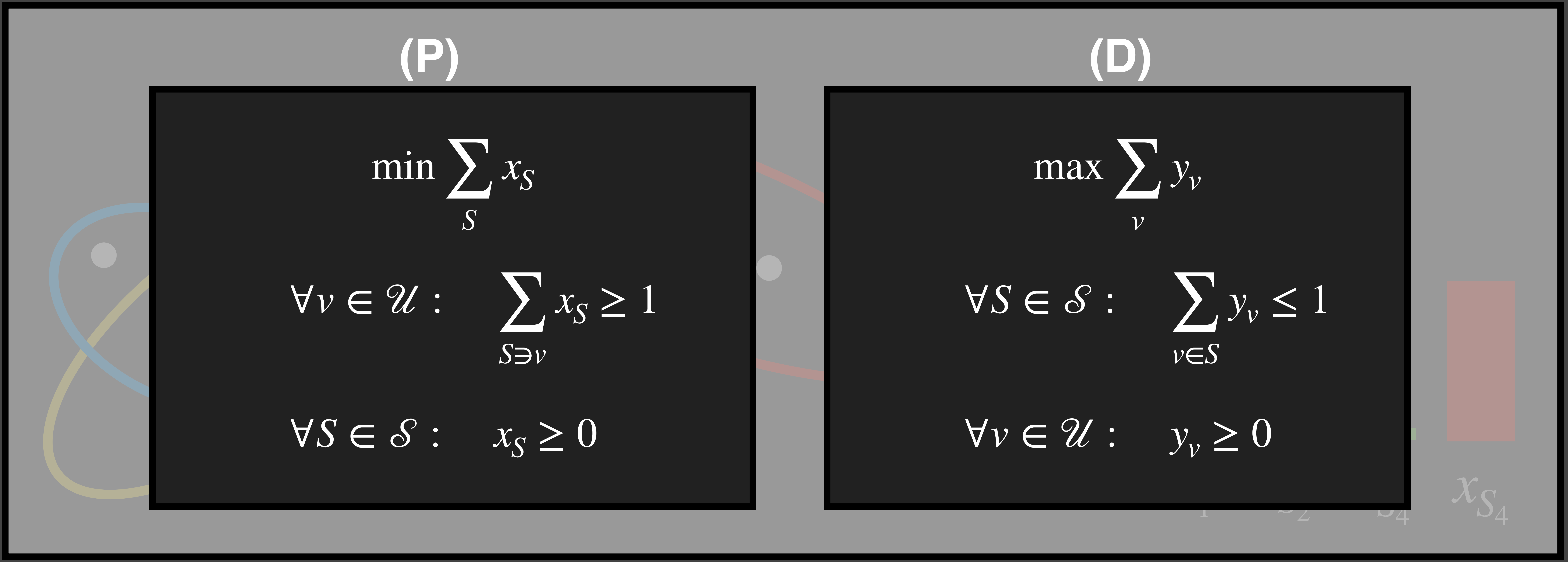
$$\forall v \in \mathcal{U} : \quad y_v \geq 0$$

Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.

Online LP Solver of [Alon+ 03]



(P)

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(D)

$$\max \sum_{v} y_v$$

$$\forall S \in \mathcal{S} : \quad \sum_{v \in S} y_v \leq 1$$

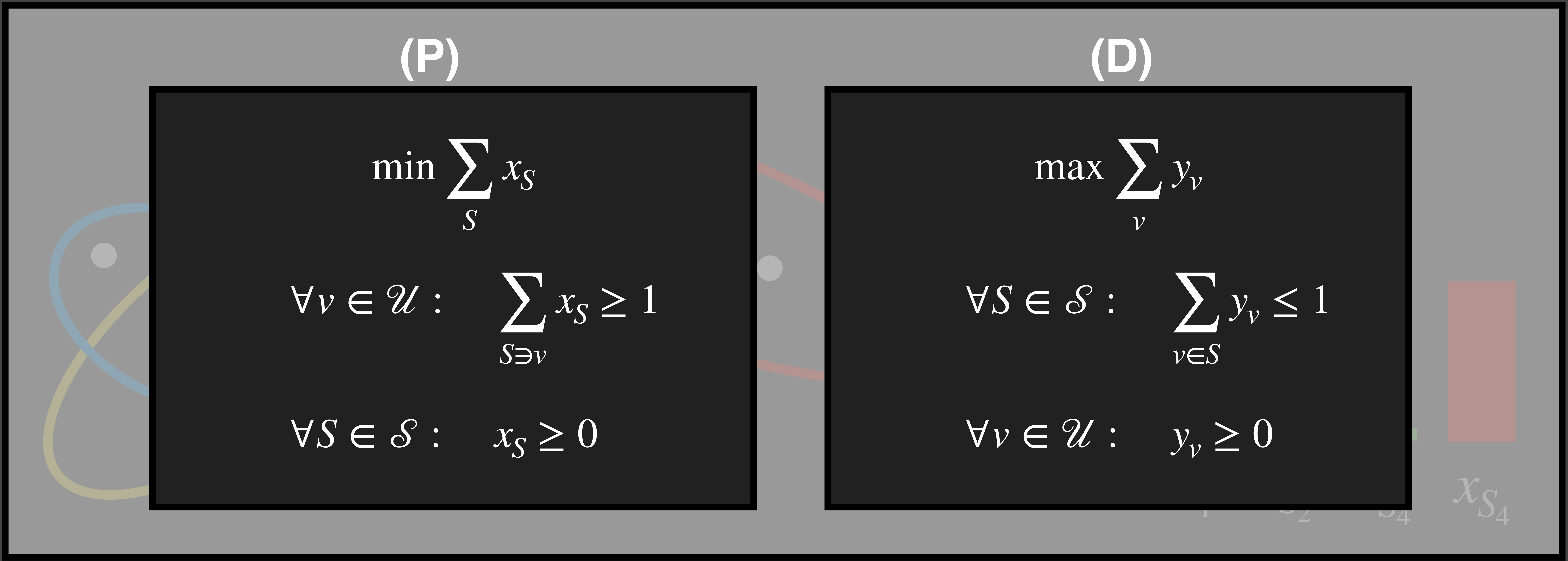
$$\forall v \in \mathcal{U} : \quad y_v \geq 0$$

Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.
- $+1$ to y_v.

Online LP Solver of [Alon+ 03]



(P)

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(D)

$$\max \sum_{v} y_v$$

$$\forall S \in \mathcal{S} : \quad \sum_{v \in S} y_v \leq 1$$

$$\forall v \in \mathcal{U} : \quad y_v \geq 0$$

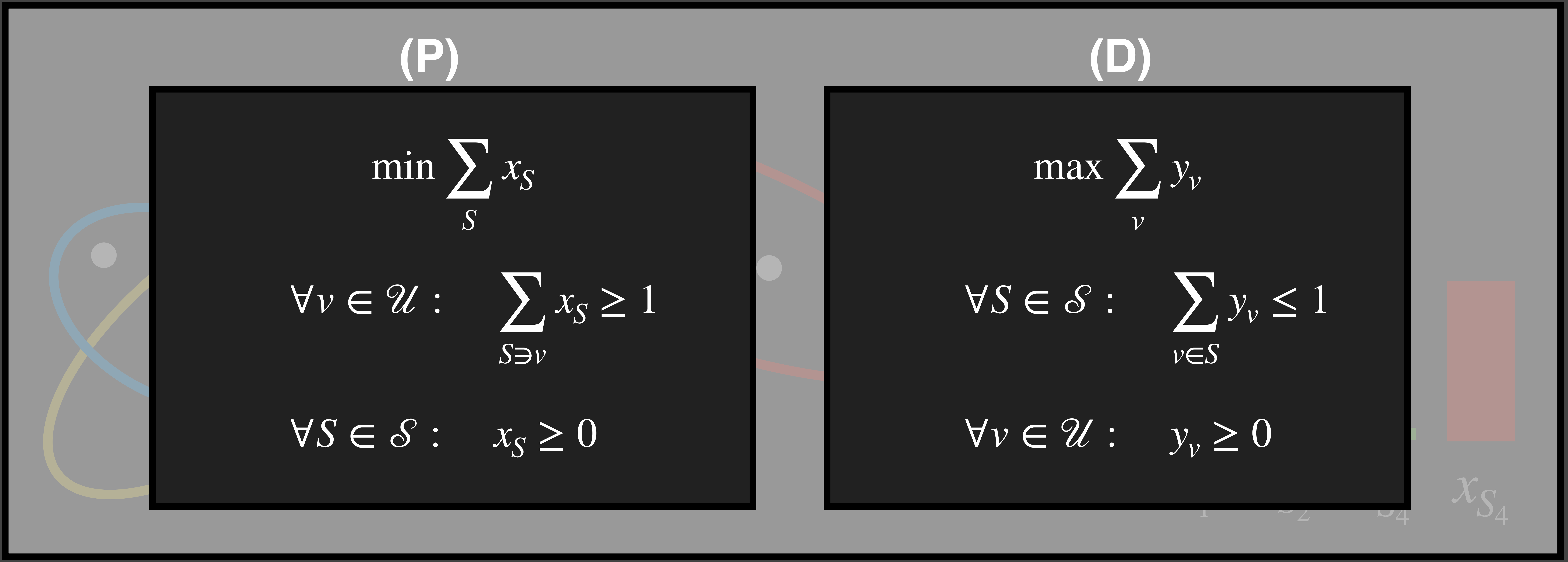
Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.
- $+1$ to y_v.

Claim 1: x feasible for (**P**).

Online LP Solver of [Alon+ 03]



(P)

$$\min \sum_{S} x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(D)

$$\max \sum_{v} y_v$$

$$\forall S \in \mathcal{S} : \quad \sum_{v \in S} y_v \leq 1$$

$$\forall v \in \mathcal{U} : \quad y_v \geq 0$$

Init $x \leftarrow 1/m$.

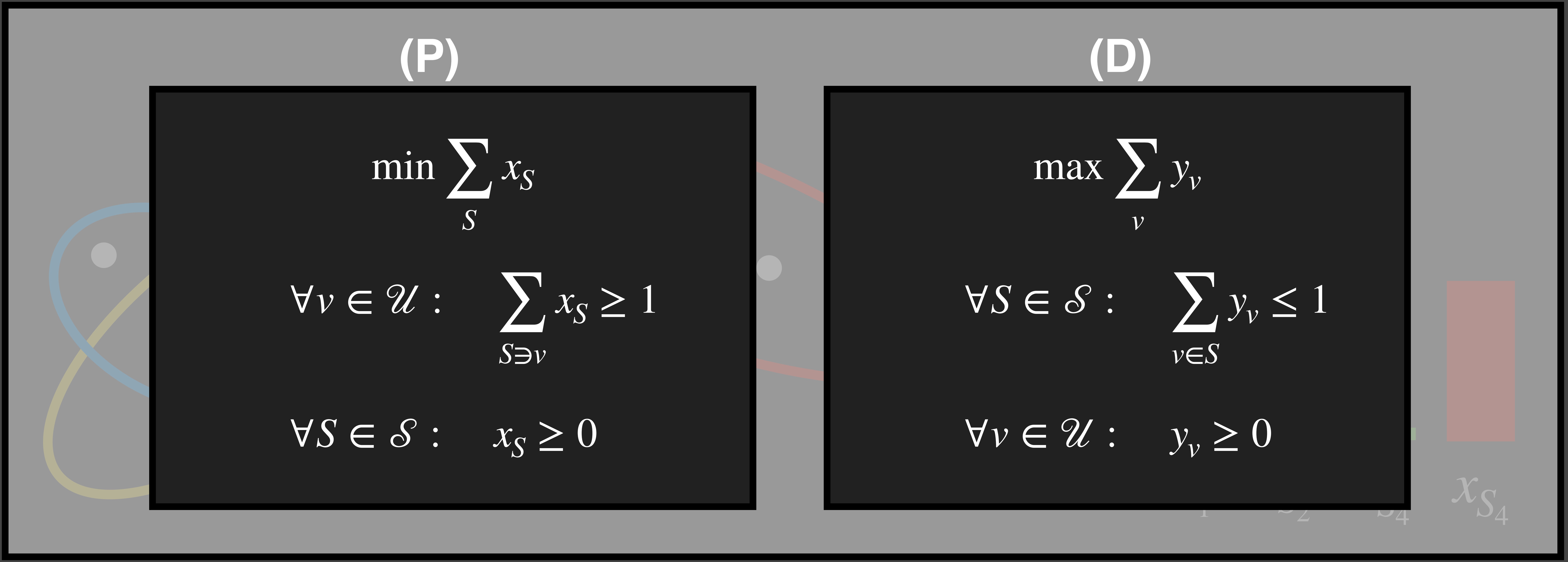
While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.
- $+1$ to y_v.

Claim 1: x feasible for (**P**).

Claim 2: $c(x) \leq c(y)$

Online LP Solver of [Alon+ 03]



(P)

$$\min \sum_S x_S$$

$$\forall v \in \mathcal{U} : \quad \sum_{S \ni v} x_S \geq 1$$

$$\forall S \in \mathcal{S} : \quad x_S \geq 0$$

(D)

$$\max \sum_v y_v$$

$$\forall S \in \mathcal{S} : \quad \sum_{v \in S} y_v \leq 1$$

$$\forall v \in \mathcal{U} : \quad y_v \geq 0$$

Init $x \leftarrow 1/m$.

While v (fractionally) uncovered:

- $\times 2$ to x_S for all $S \ni v$.
- $+1$ to y_v.

Claim 1: x feasible for (**P**).

Claim 2: $c(x) \leq c(y)$

Claim 3: $y/\log m$ feasible for (**D**).

LearnOrCover for non-unit costs

LearnOrCover for non-unit costs

Main issue: # uncovered elements <u>not</u> good proxy for cost.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

LearnOrCover

```
Init. $x_S \leftarrow 1/m$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
      (I) Buy every set $R$ w.p. $x_R$.
      (II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.
      Renormalize $x = x/||x||_1$.
       Buy arbitrary set to cover $v$.
```

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
         Buy cheapest set to cover $v$.
```

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
         Buy cheapest set to cover $v$.
```

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

LearnOrCover

```
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
        Buy cheapest set to cover $v$.
```

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
        Buy cheapest set to cover $v$.
```

Claim 1: $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v / c_S} \cdot x_S$.
        Renormalize $x \leftarrow x / \langle c, x \rangle$.
        Buy cheapest set to cover $v$.
```

Claim 1: $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
         Buy cheapest set to cover $v$.
```

Claim 1: $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

Claim 3: $E[\Delta\text{cost}(\text{ALG})] = O(\kappa_v)$.

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

LearnOrCover

```
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v/c_S} \cdot x_S$.
        Renormalize $x \leftarrow x/\langle c, x \rangle$.
        Buy cheapest set to cover $v$.
```

Claim 1: $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

Claim 3: $E[\Delta\text{cost(ALG)}] = O(\kappa_v)$.

$$\Rightarrow \quad E[\Delta\Phi + \Delta\text{cost(ALG)}] = 0.$$

LearnOrCover for non-unit costs

Main issue: # uncovered elements not good proxy for cost.

(Assuming WLOG $c(OPT) = 1$)

κ_v := cost of cheapest set covering v

Main Idea: tune learning & sampling rates as a function of κ_v.

```
LearnOrCover
Init. $x_S \leftarrow 1/(c_S \cdot m)$.
@ time $t$, element $v$ arrives:
    If $v$ covered, do nothing.
    Else:
        (I) Buy every set $R$ w.p. $\kappa_v x_R$.
        (II) $\forall S \ni v$, set $x_S \leftarrow e^{\kappa_v / c_S} \cdot x_S$.
        Renormalize $x \leftarrow x / \langle c, x \rangle$.
        Buy cheapest set to cover $v$.
```

Claim 1: $\Phi(0) = c(\text{OPT}) \cdot O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: $E[\Delta\Phi] = -\Omega(\kappa_v)$.

Claim 3: $E[\Delta\text{cost(ALG)}] = O(\kappa_v)$.

$$\Rightarrow \quad E[\Delta\Phi + \Delta\text{cost(ALG)}] = 0.$$

$$E[\text{cost(ALG)}] \leq \Phi(0).$$