Submodular Optimization Under Uncertainty

Online, Dynamic and Streaming Algorithms

Carnegie Mellon University

Roie Levin

Committee: Anupam Gupta, R. Ravi, David Woodruff, Chandra Chekuri, Seffi Naor

Intro

Classical Approximation Algorithms

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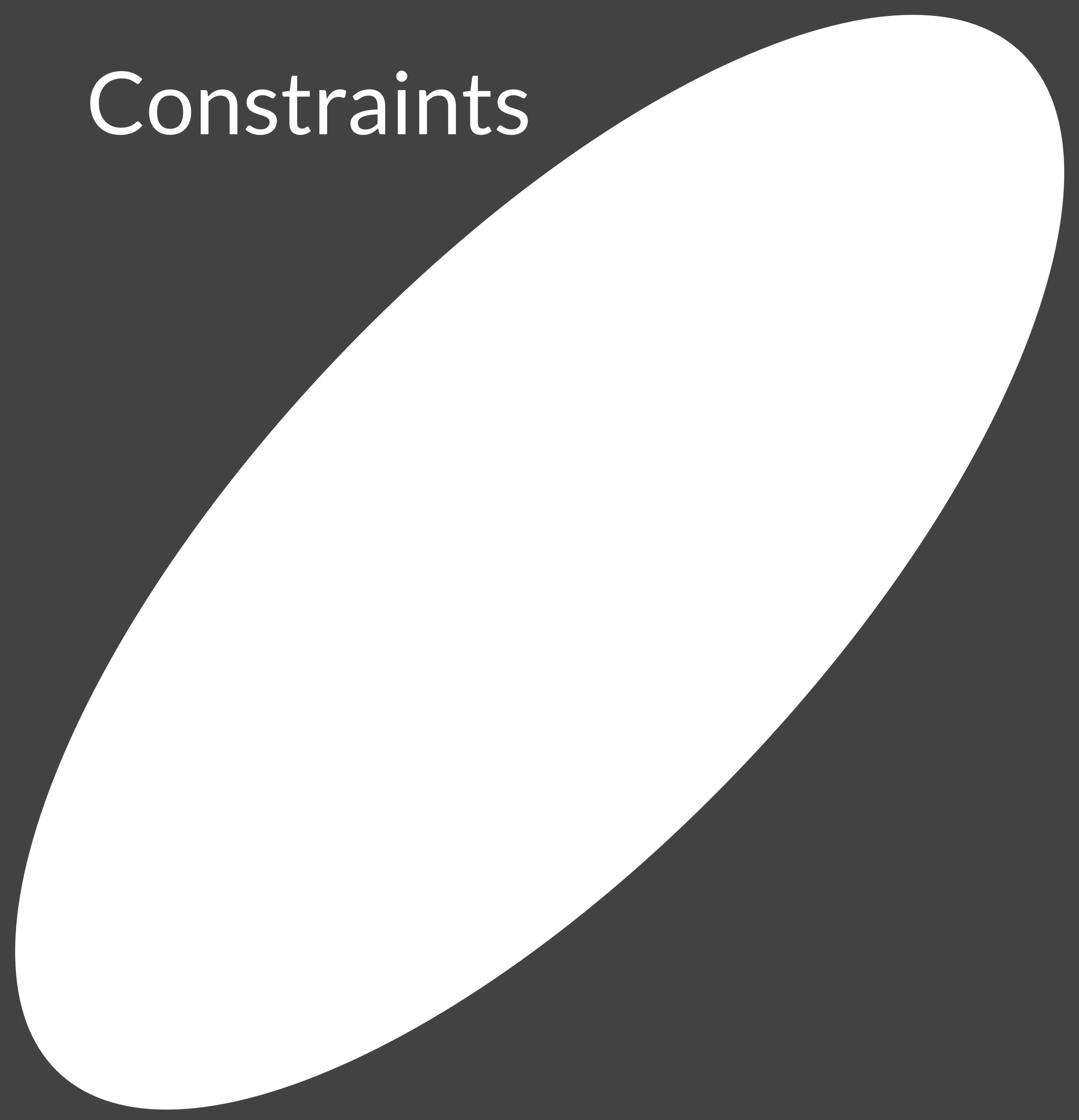

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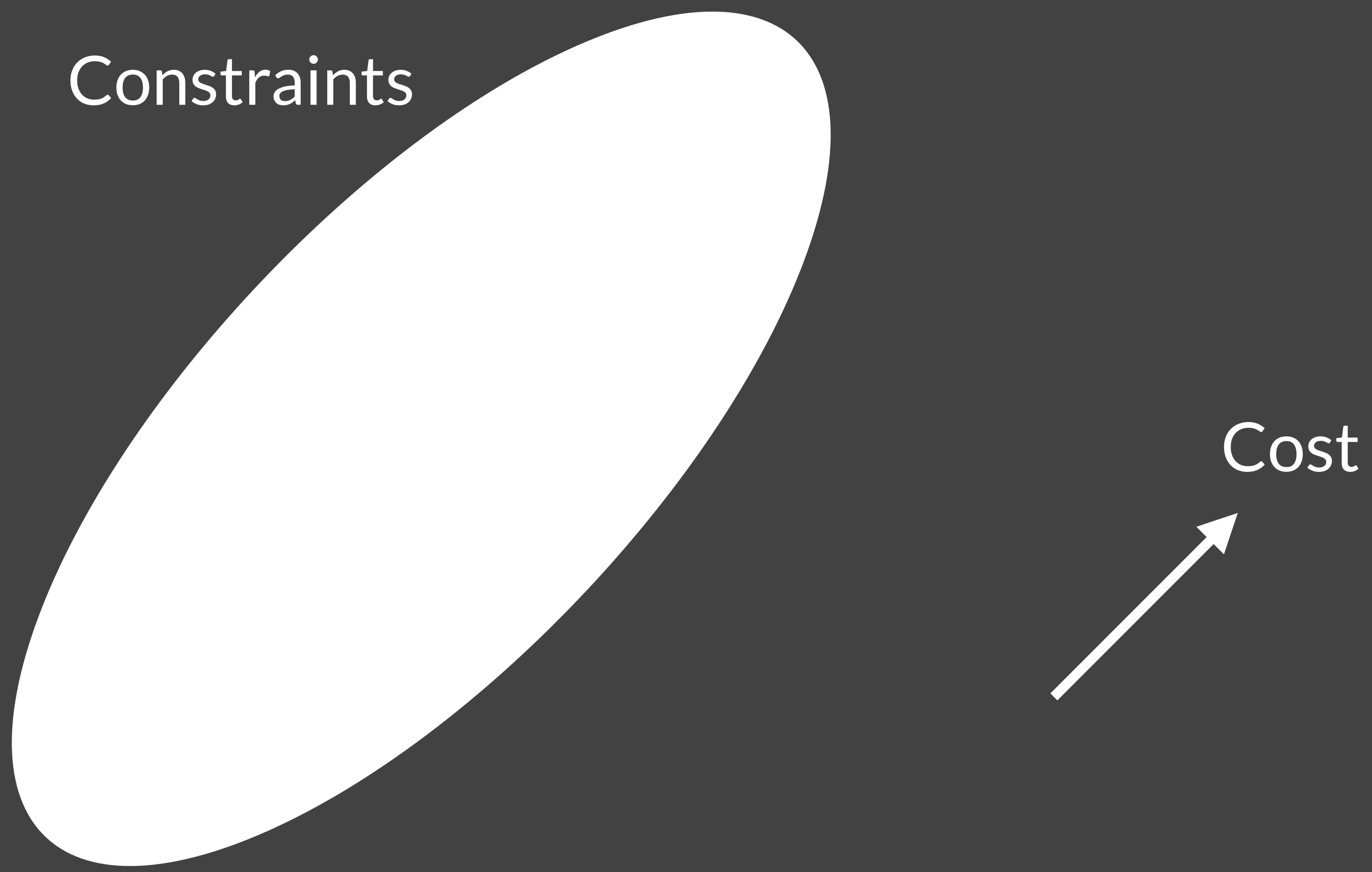

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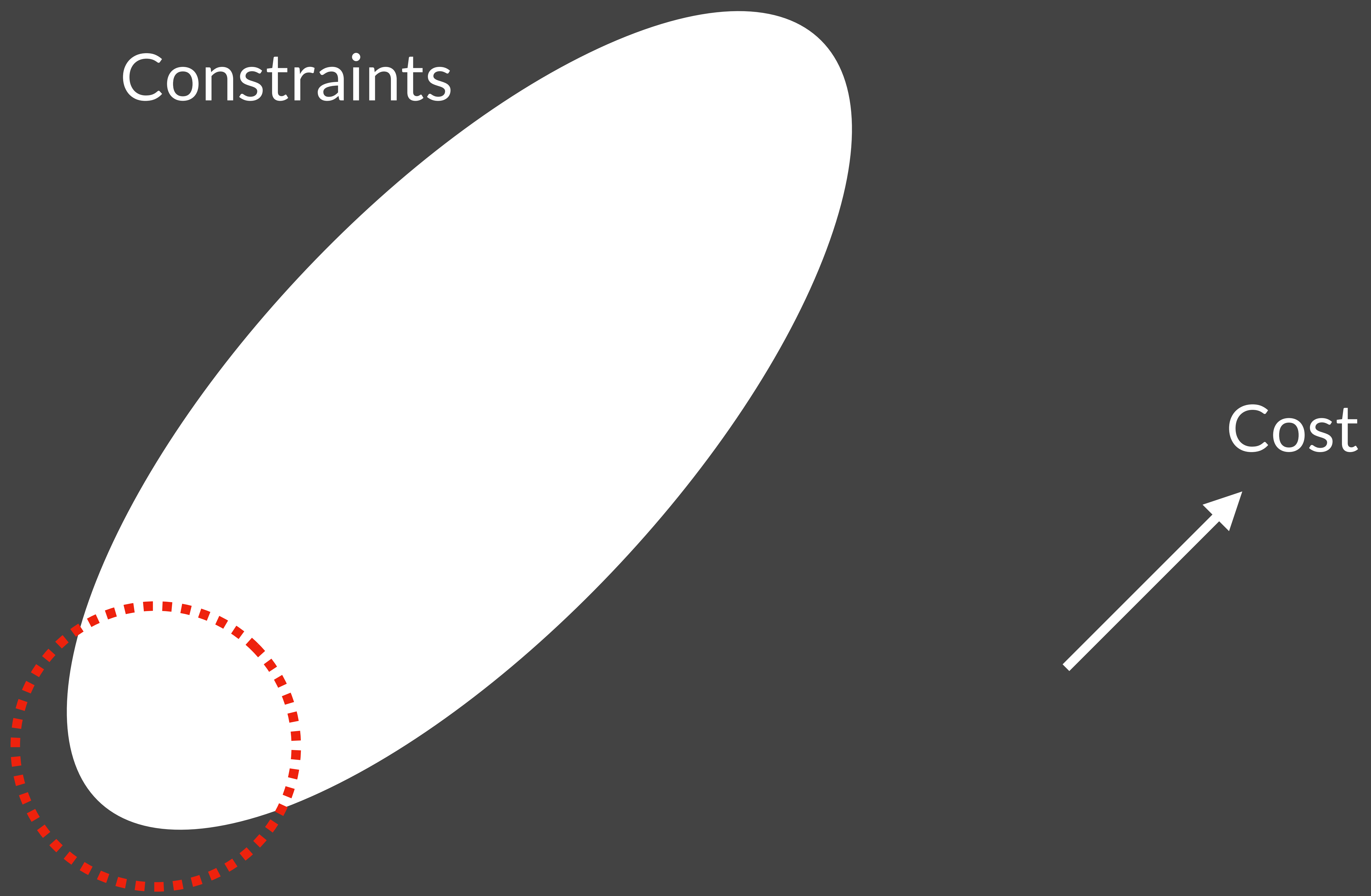

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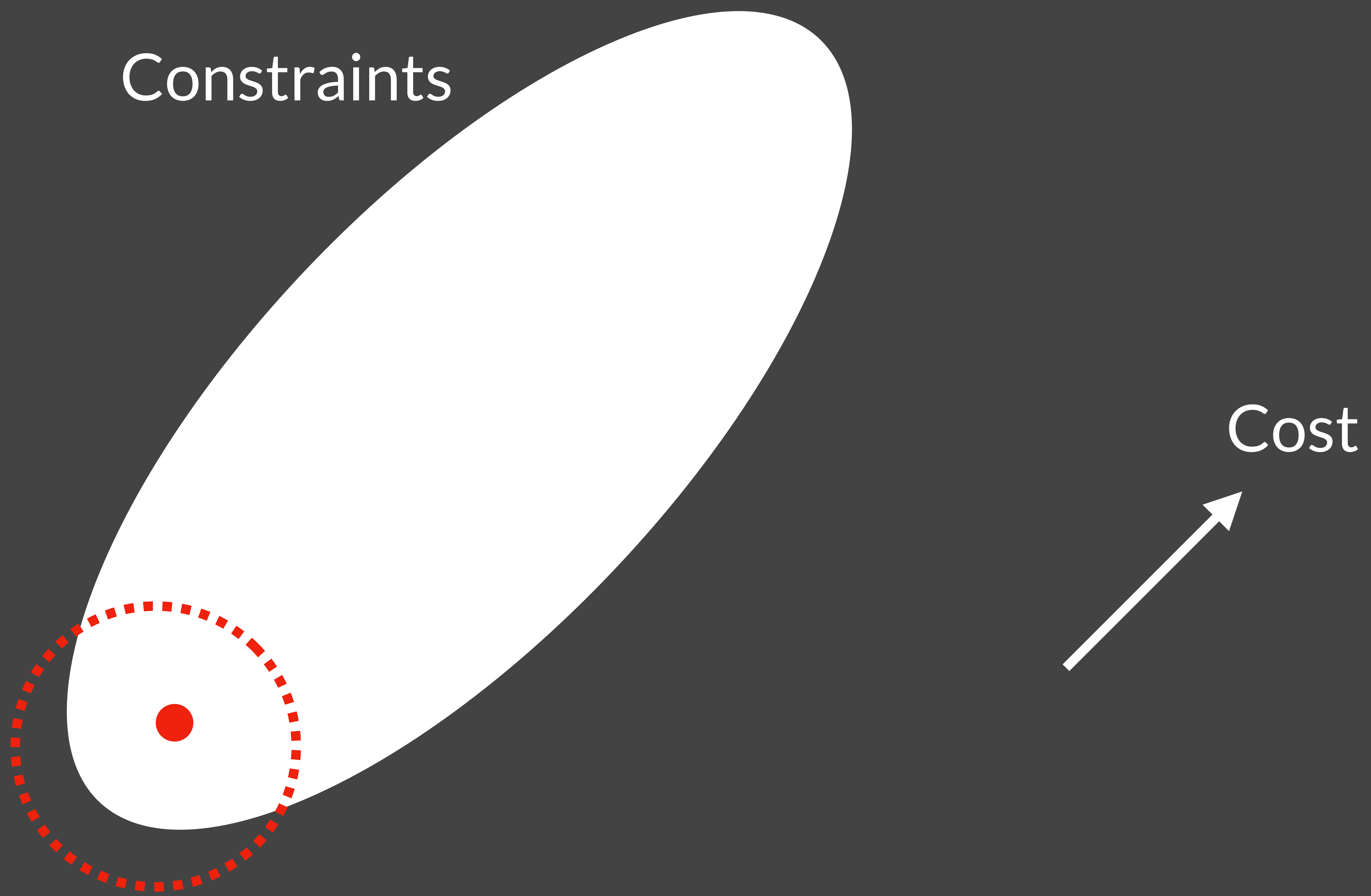

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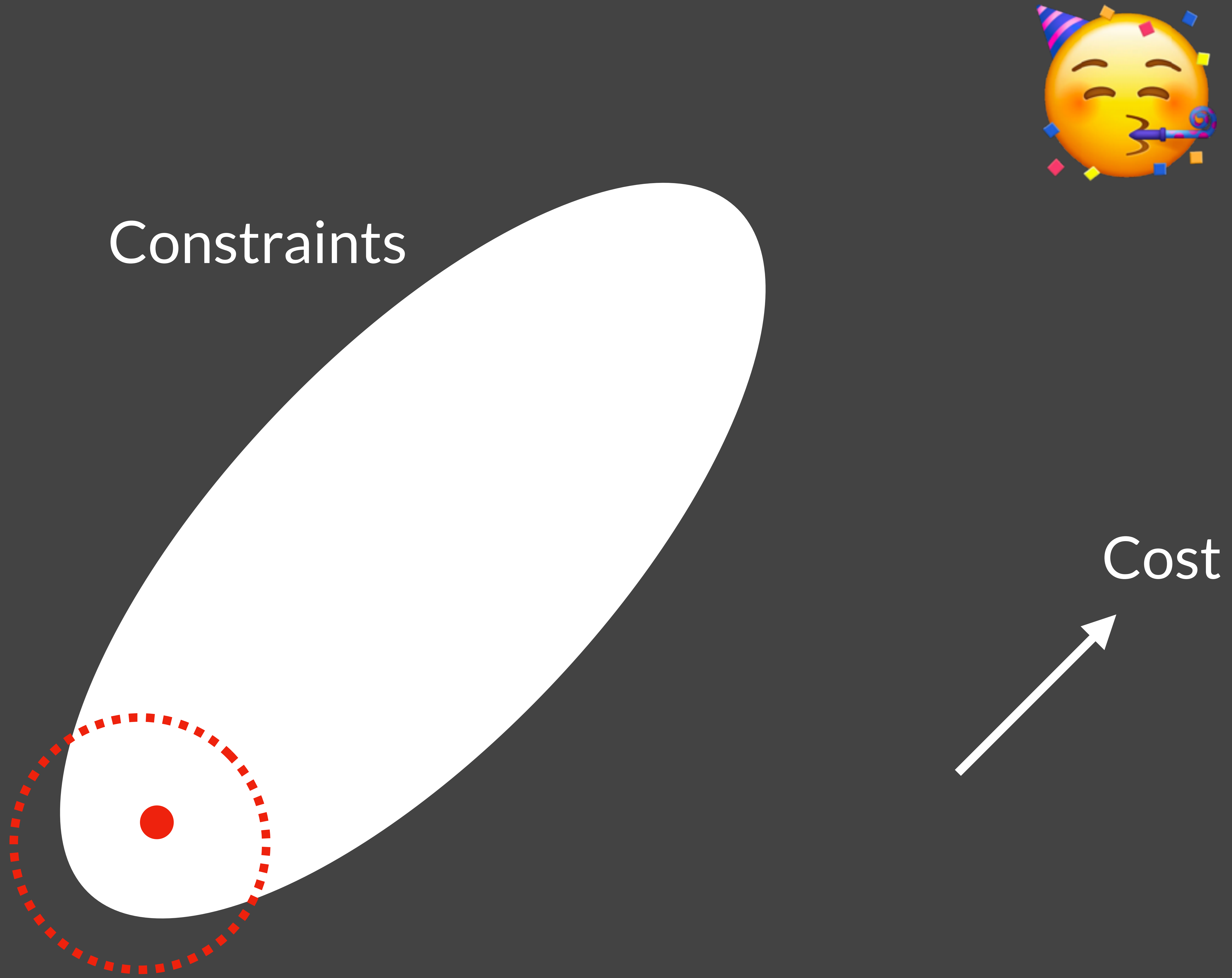

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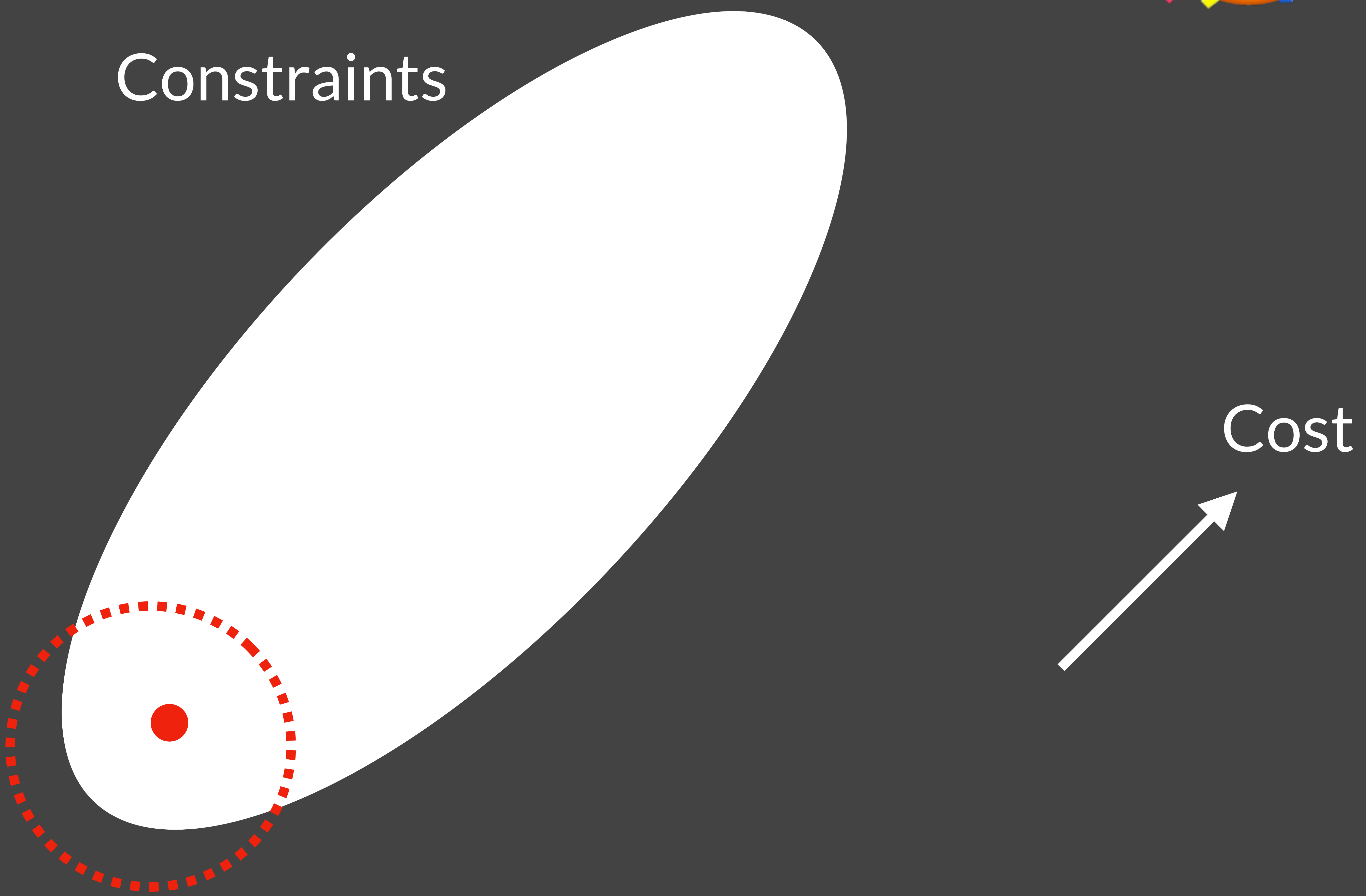

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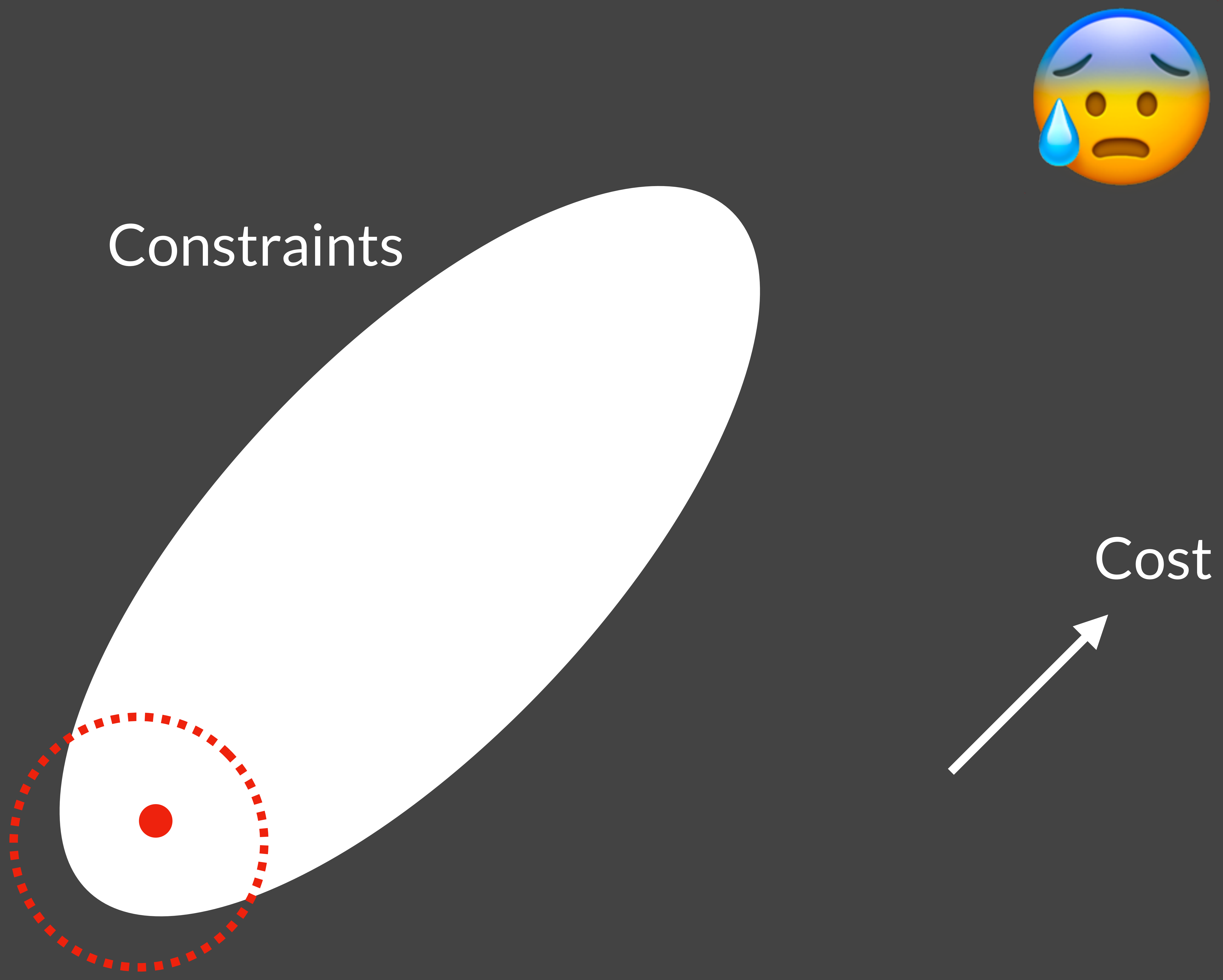

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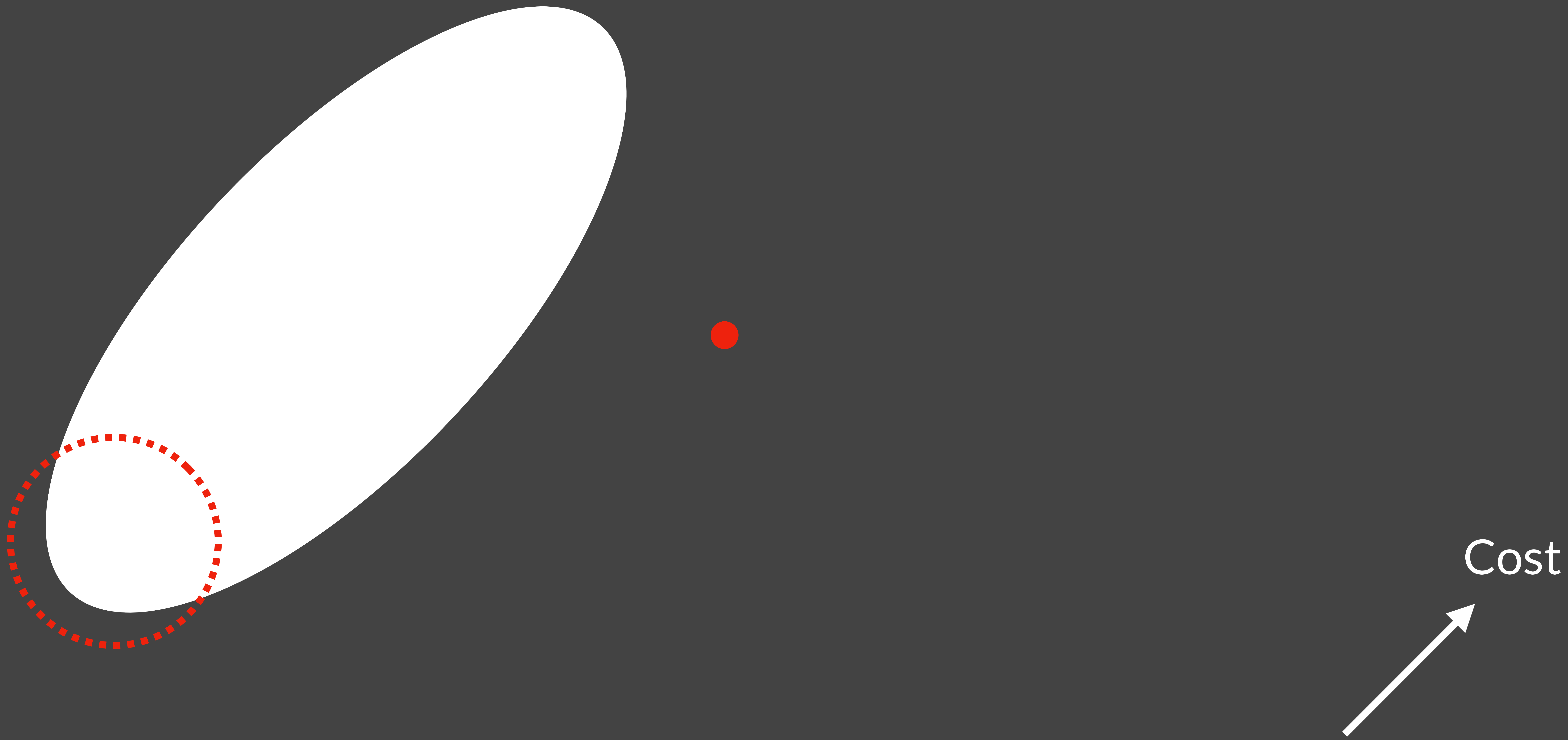

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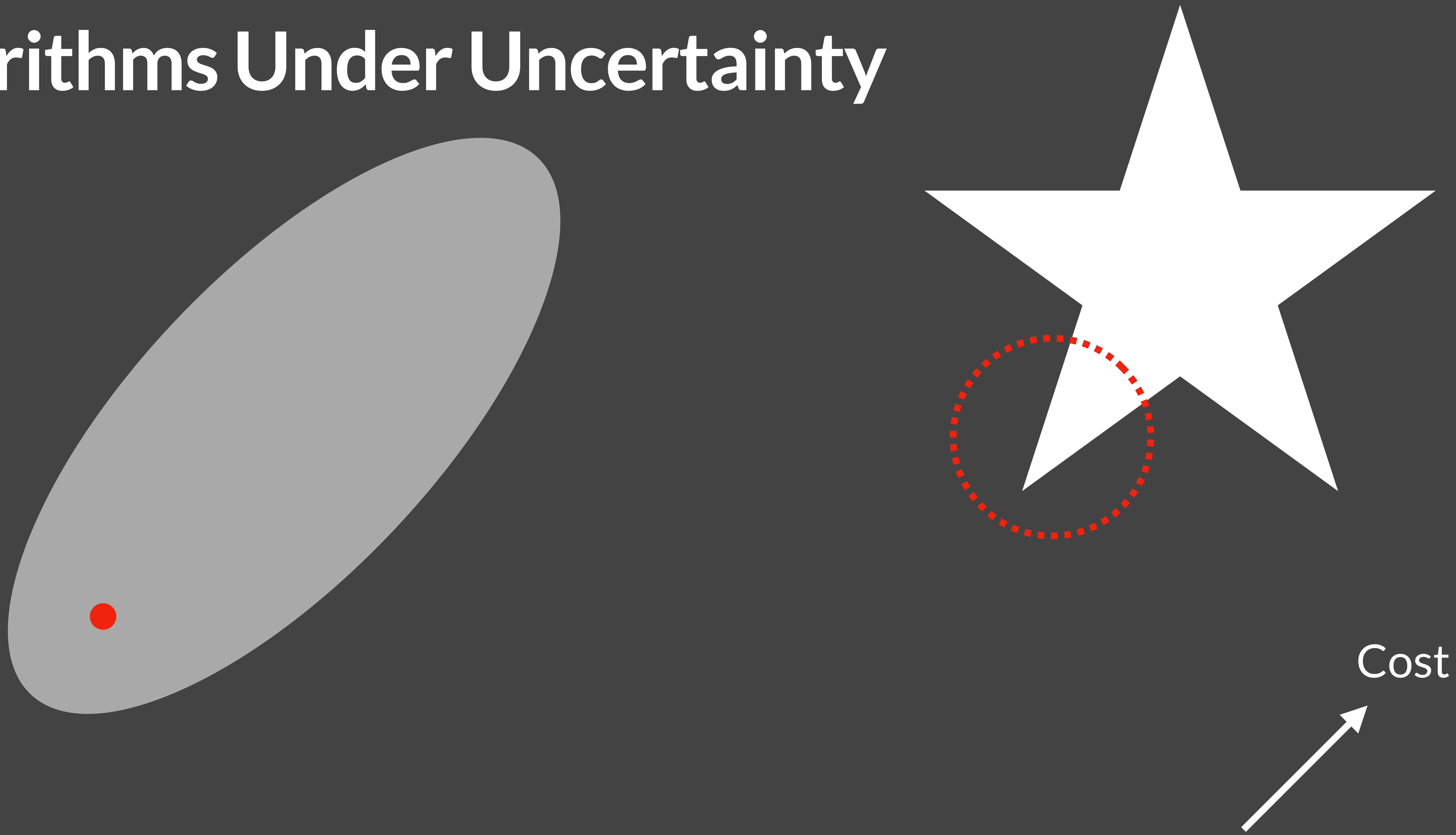

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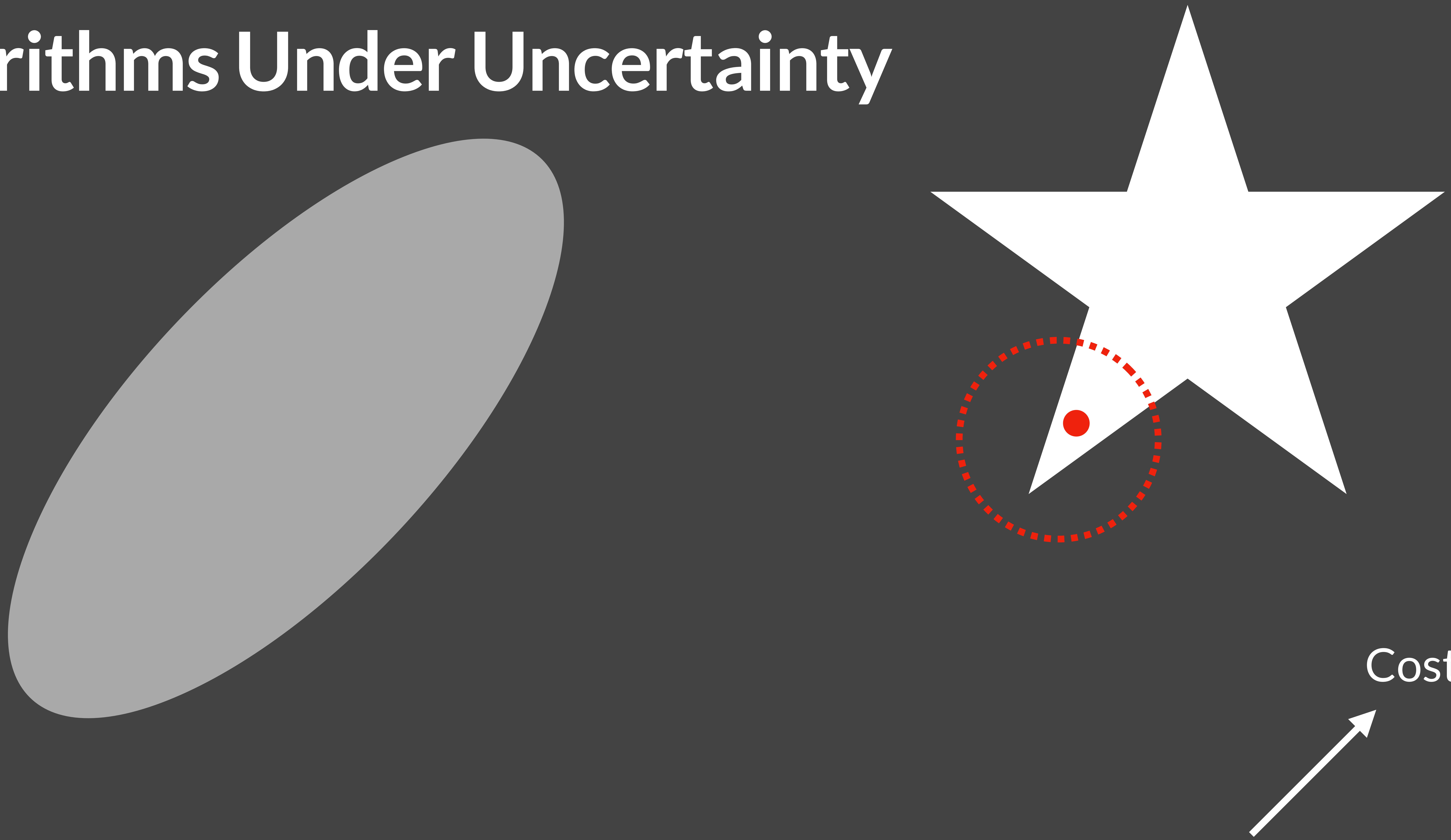

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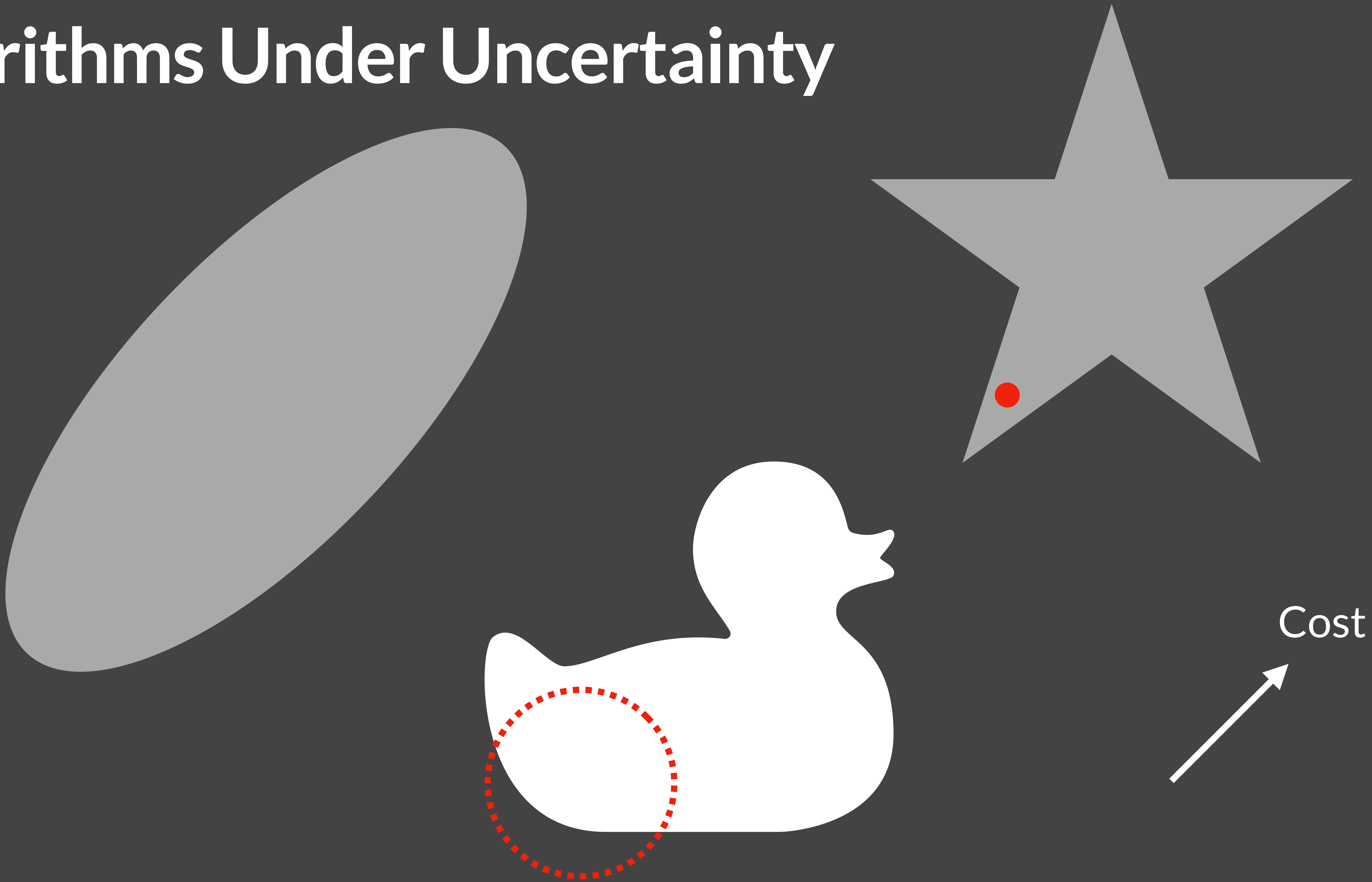

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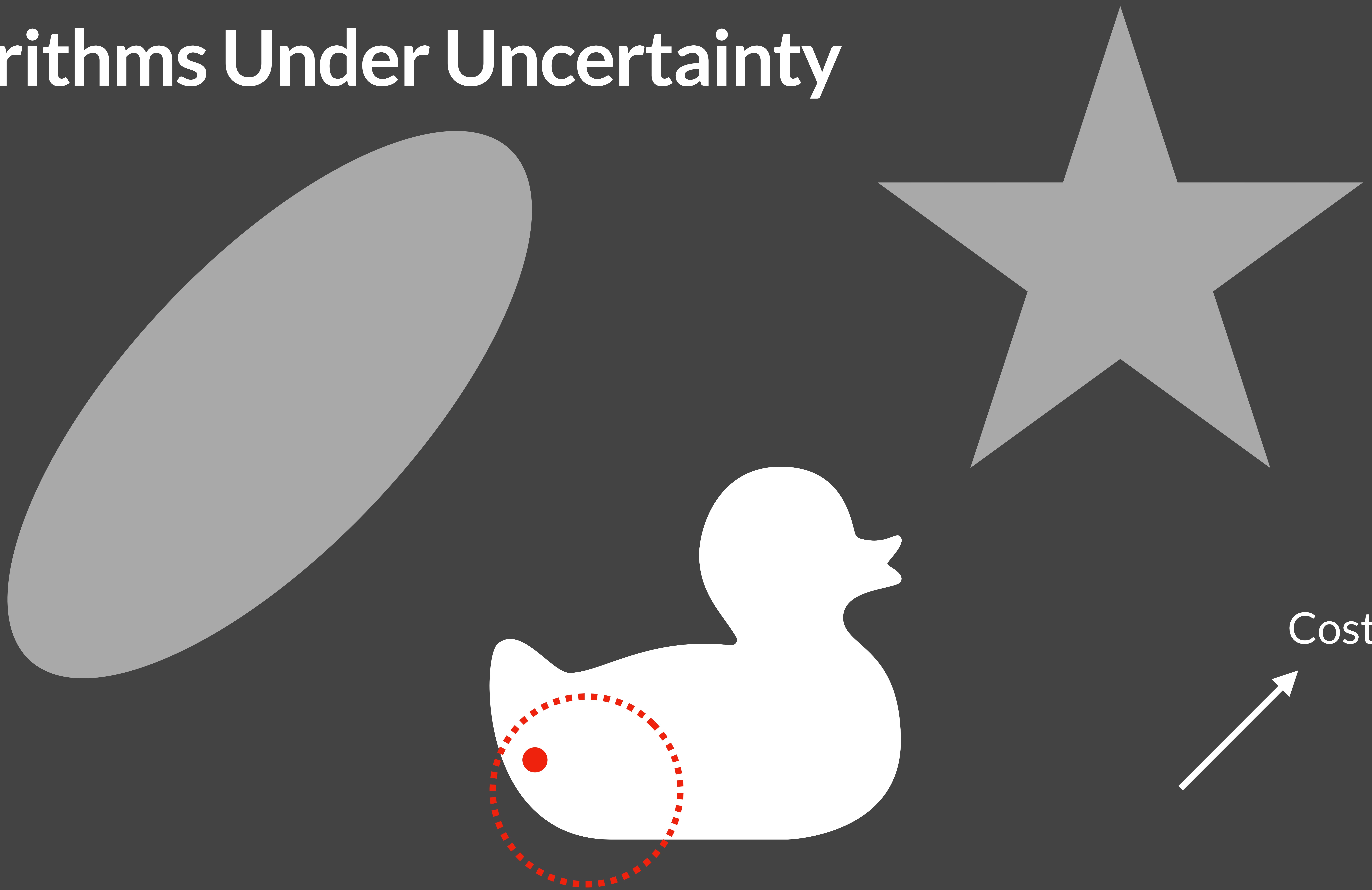

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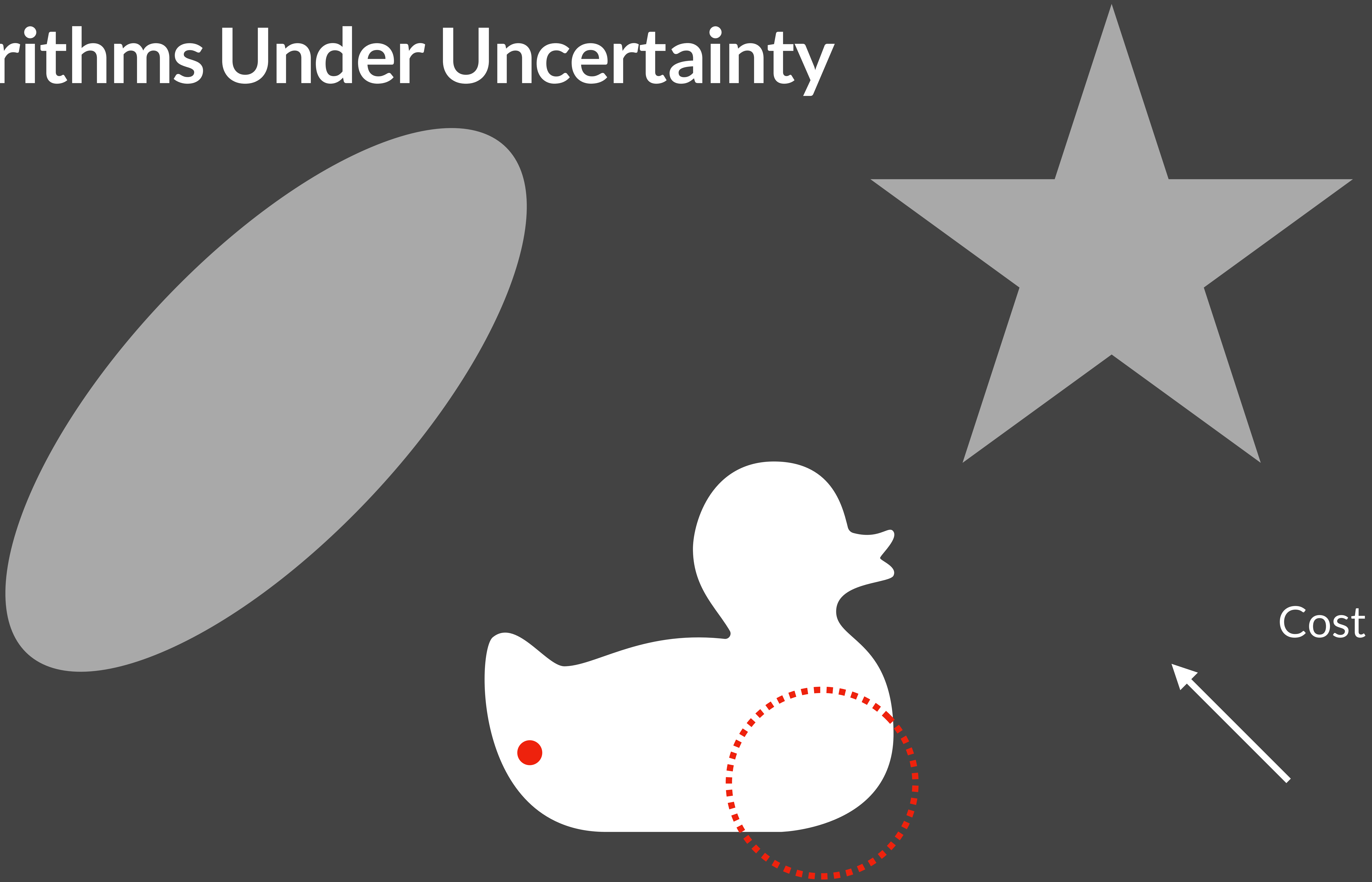

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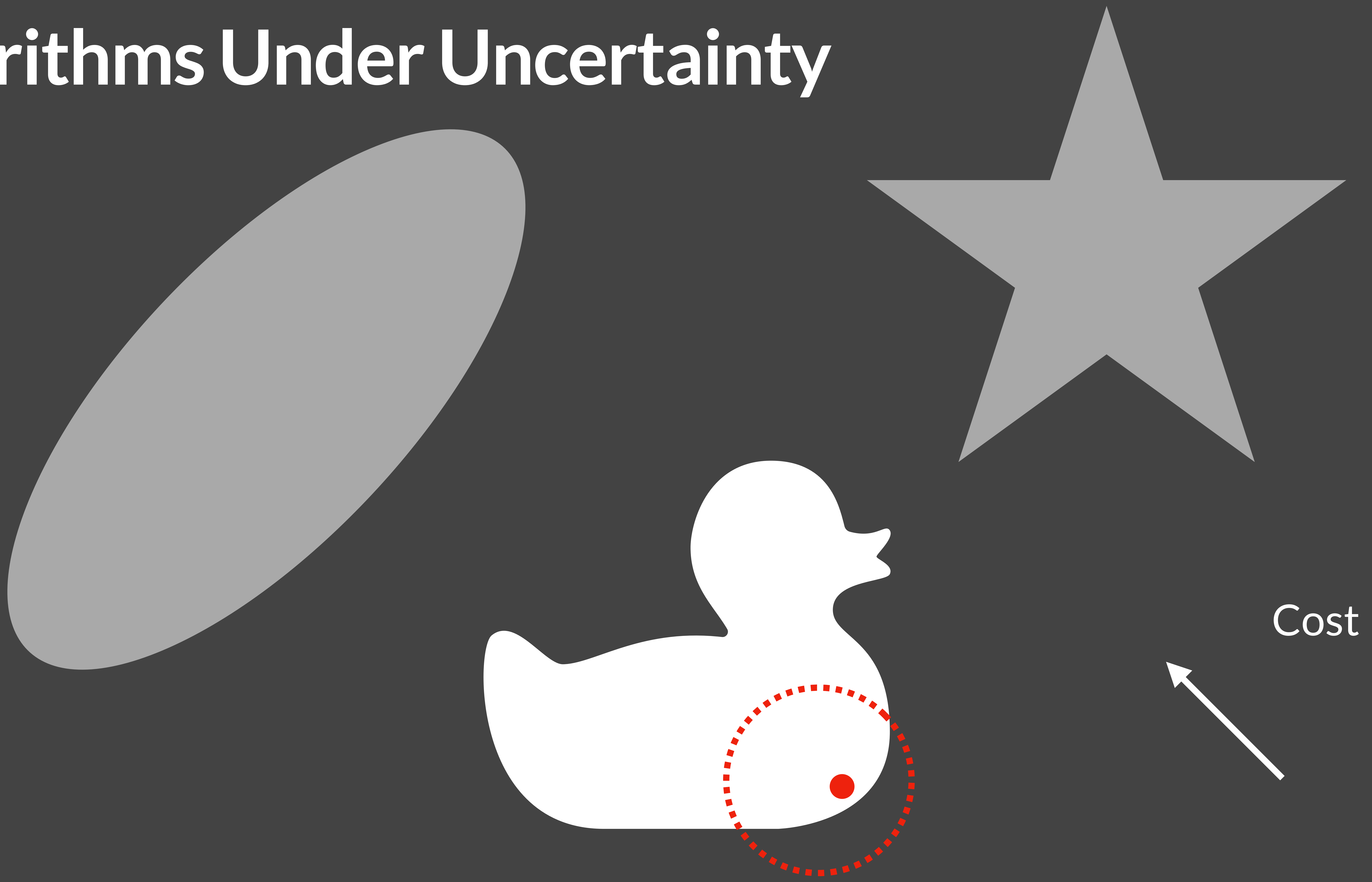

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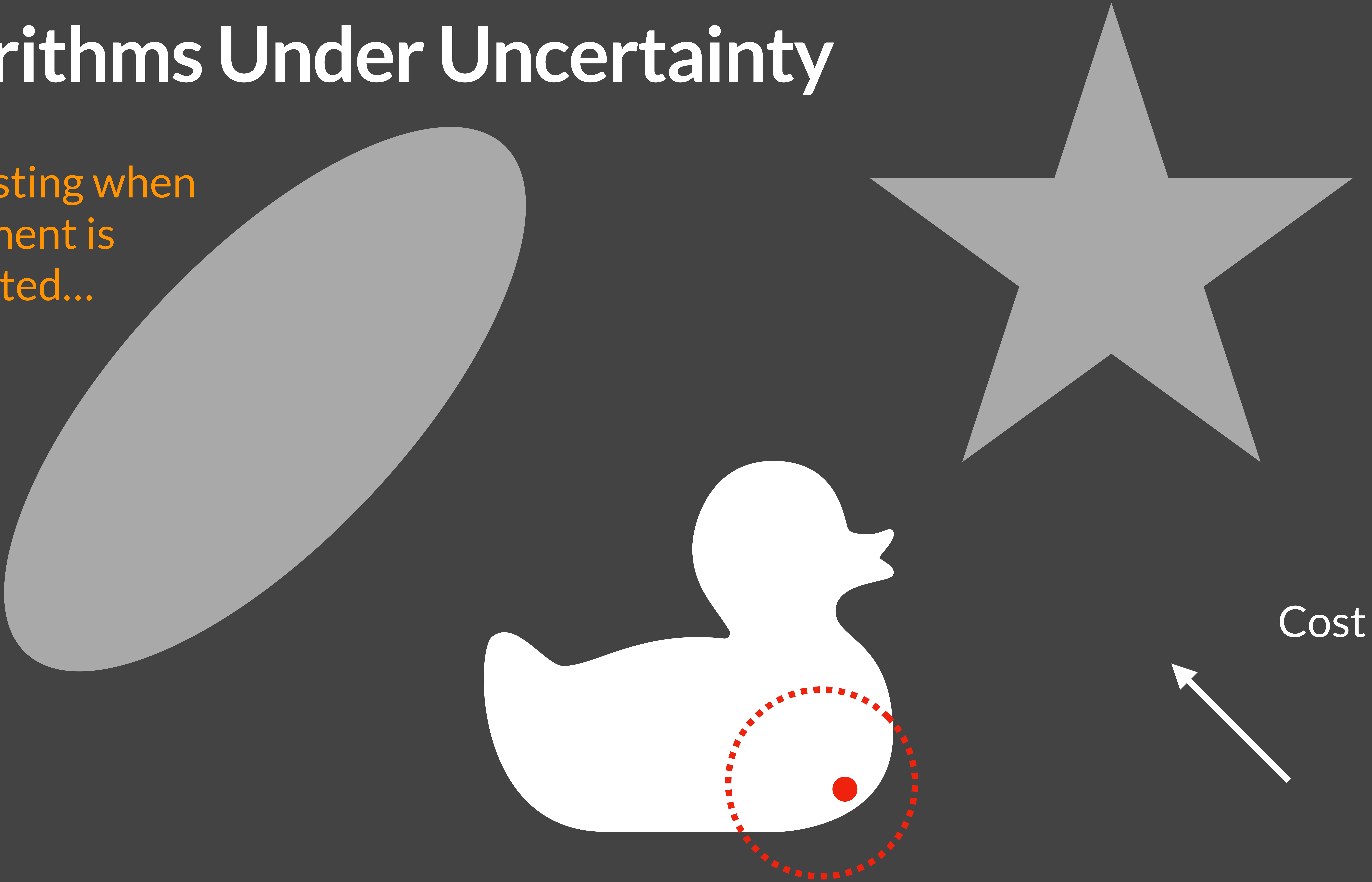

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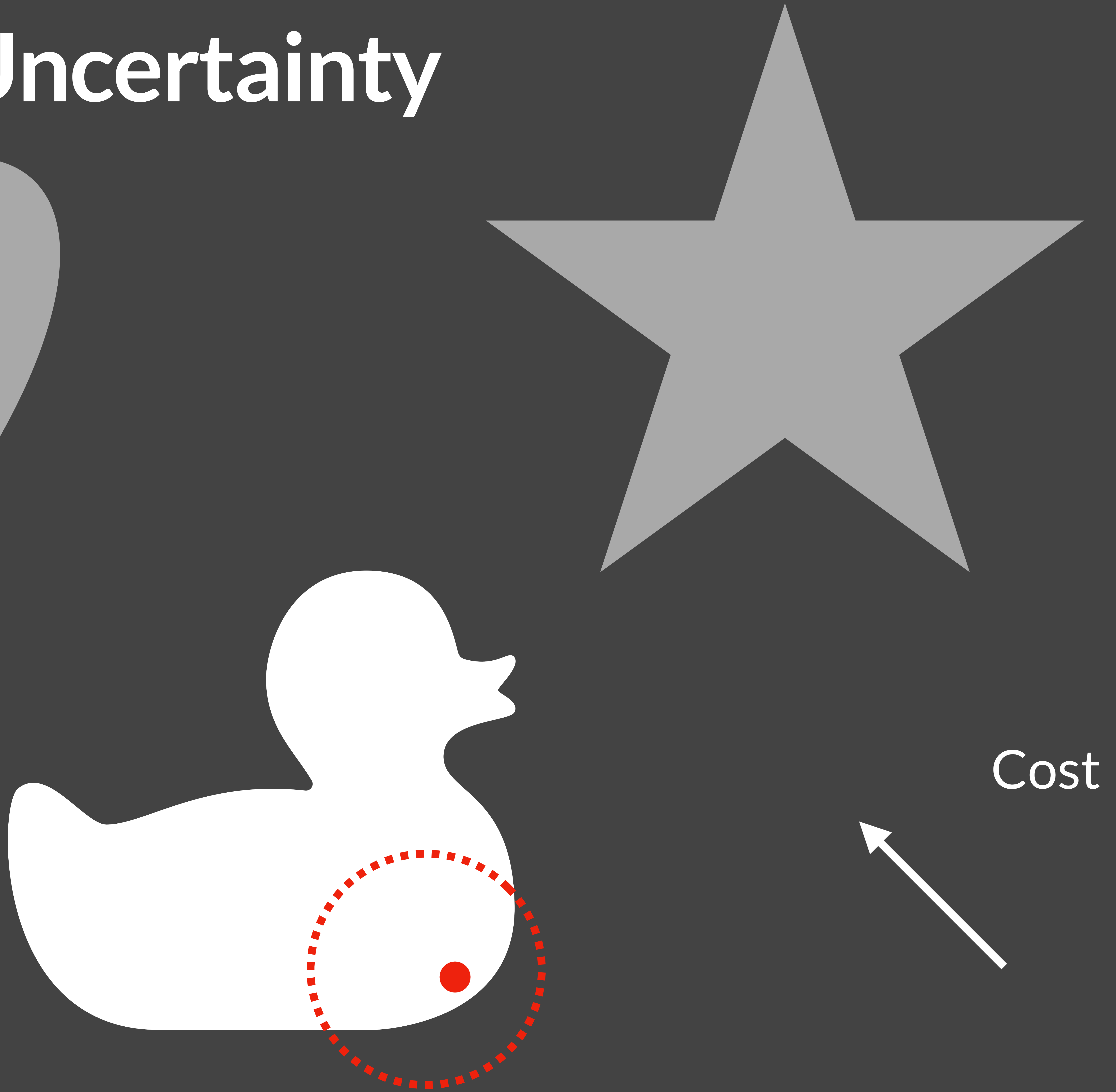
Thesis studies 3 restrictions:

Online — monotone solution

Dynamic — low movement

Streaming — low memory



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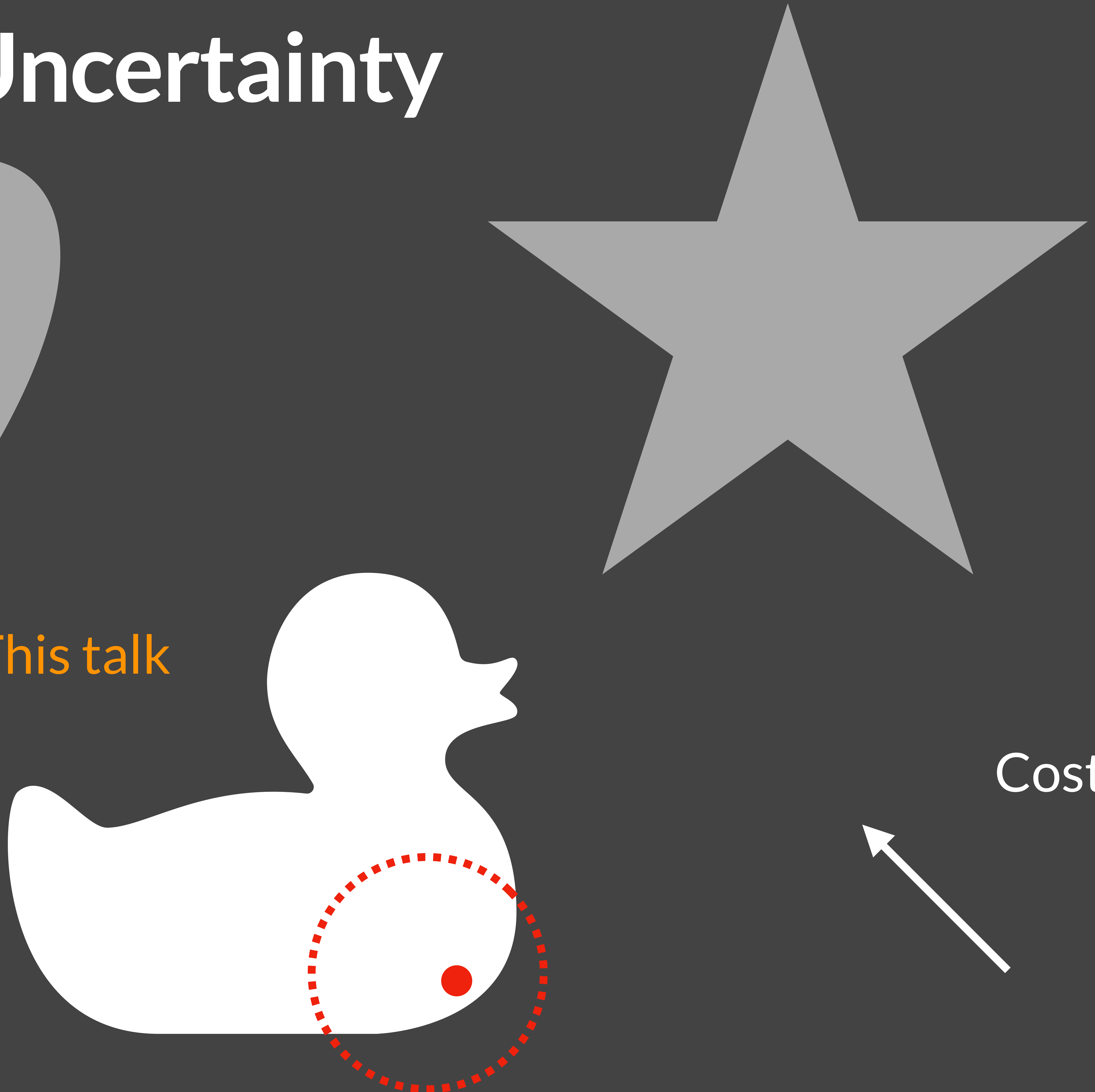

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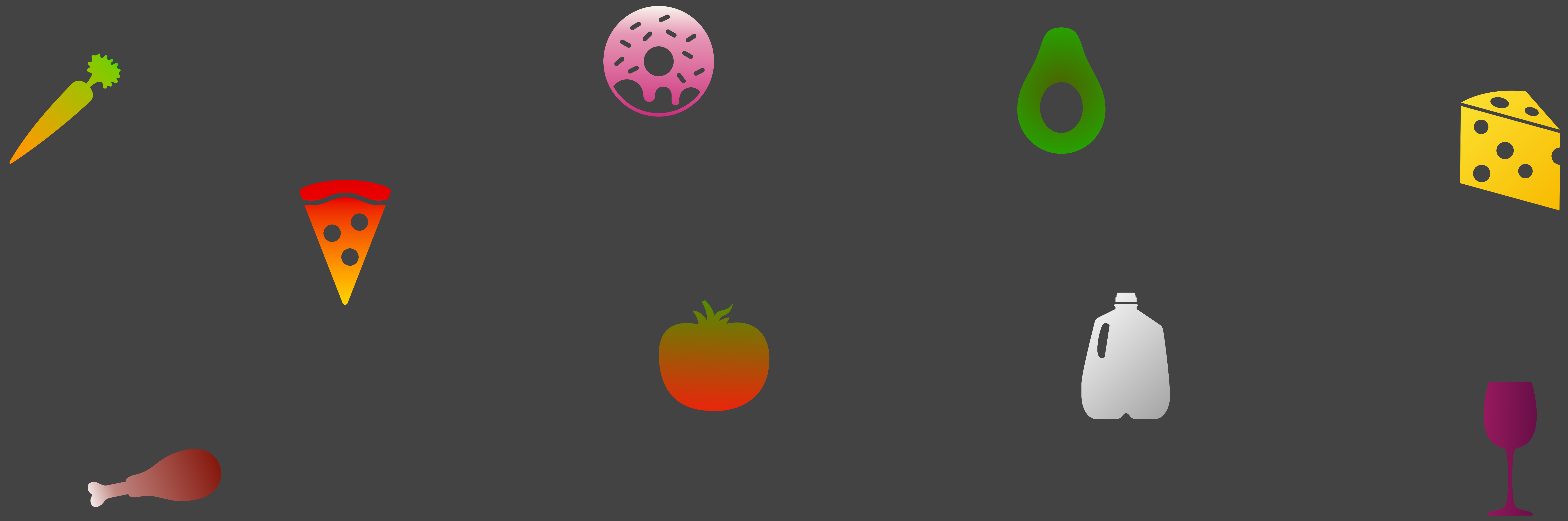

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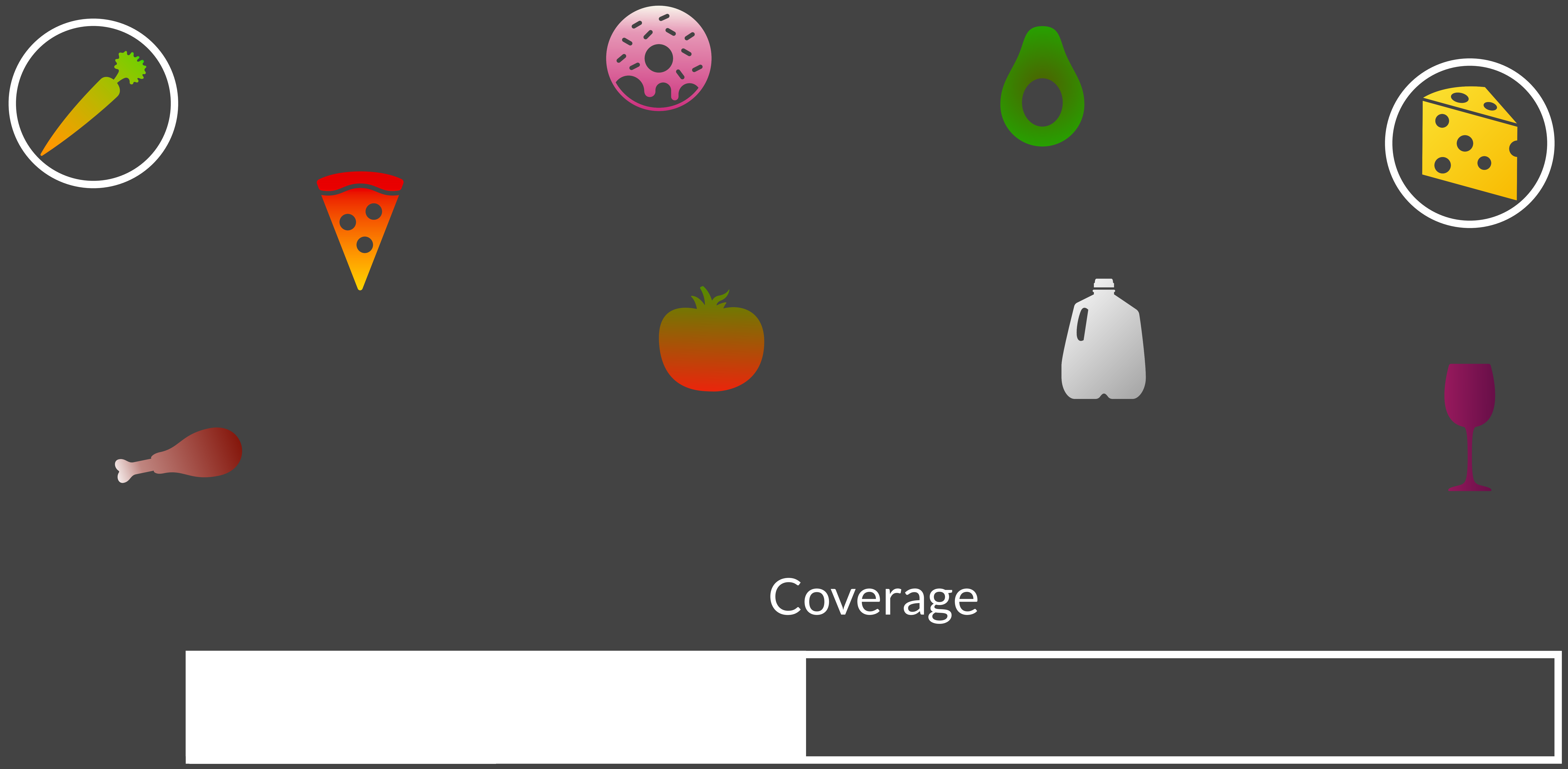

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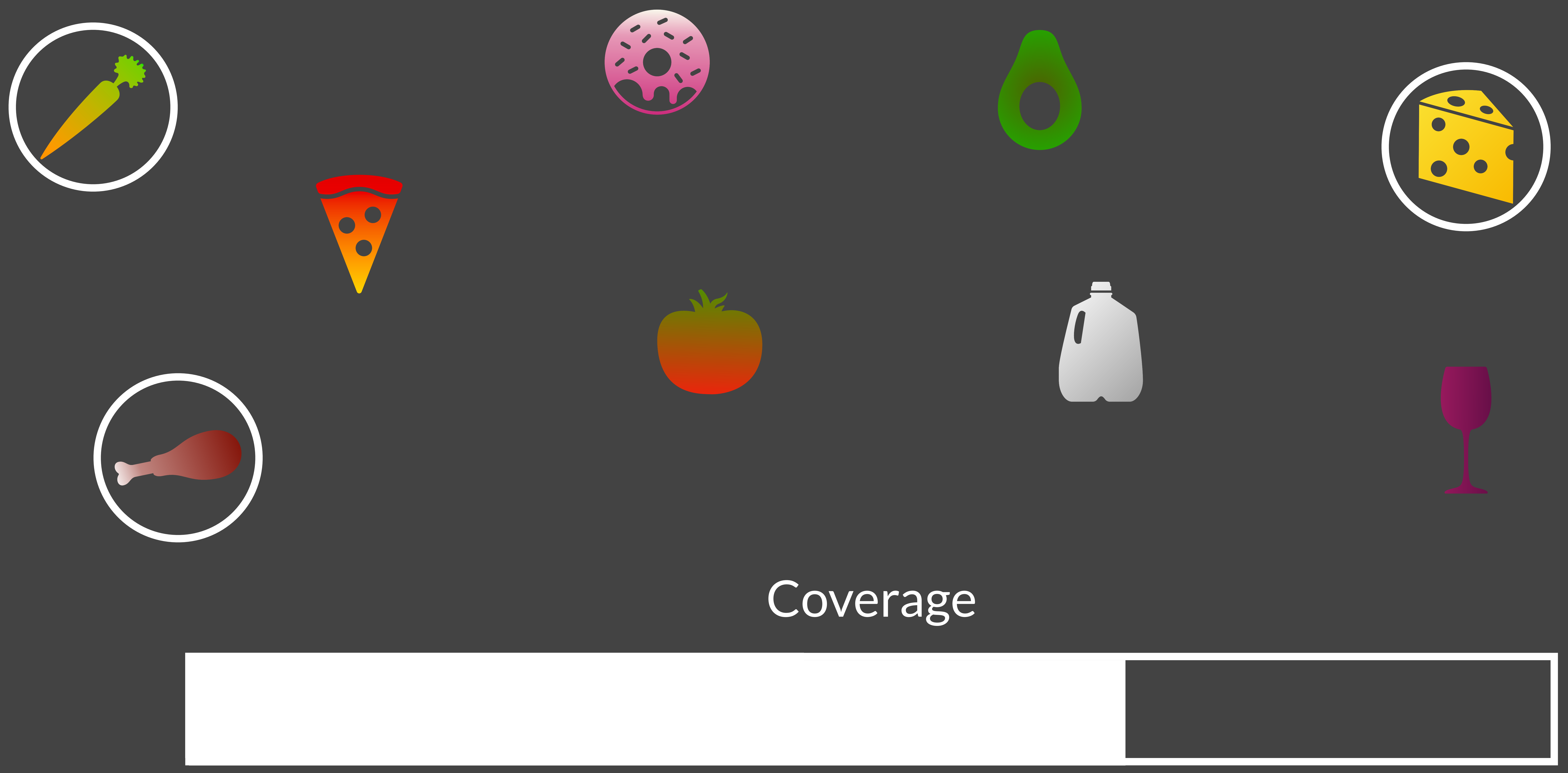

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<u>This talk:</u>

f integer valued,
all costs are 1.

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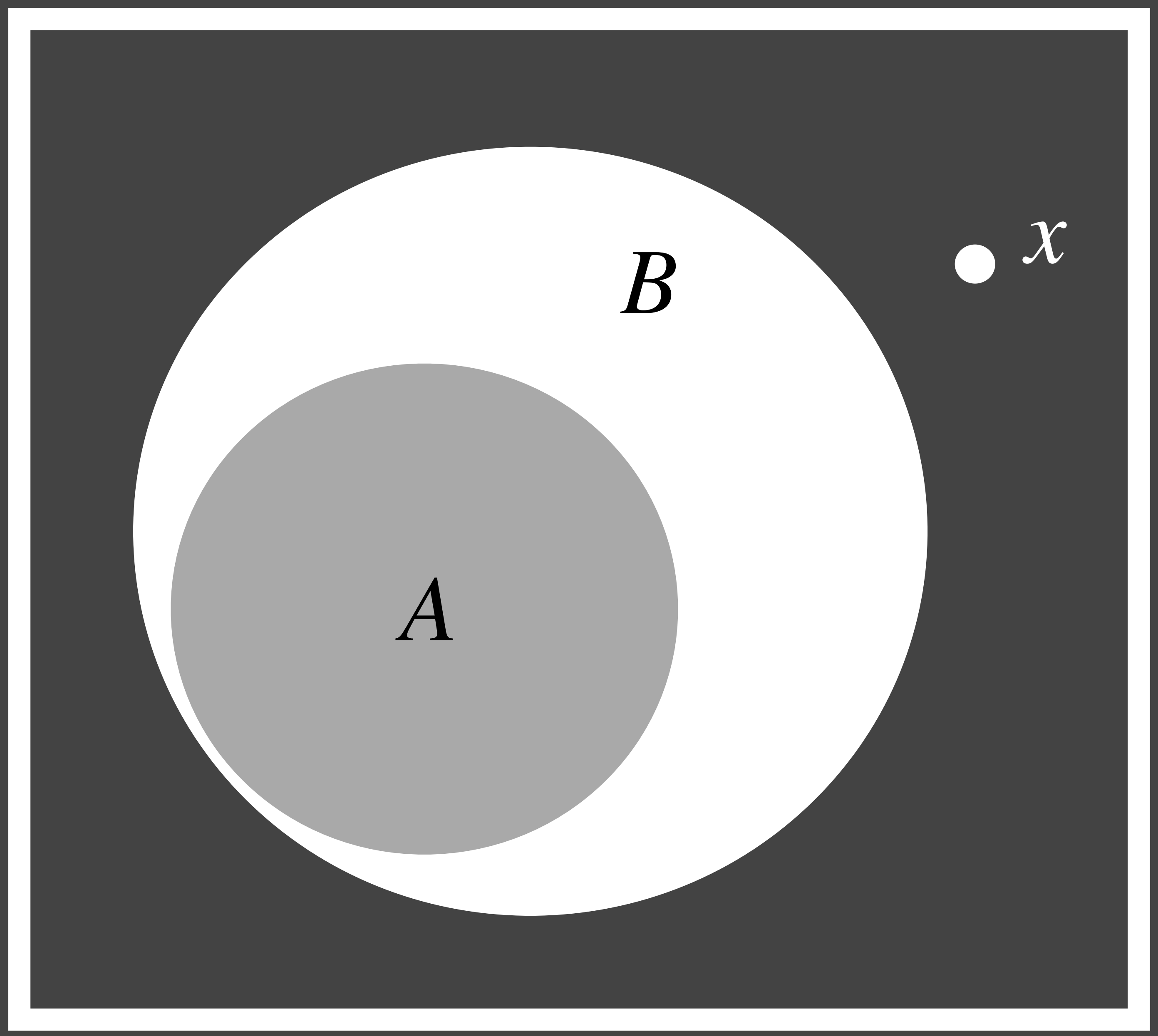

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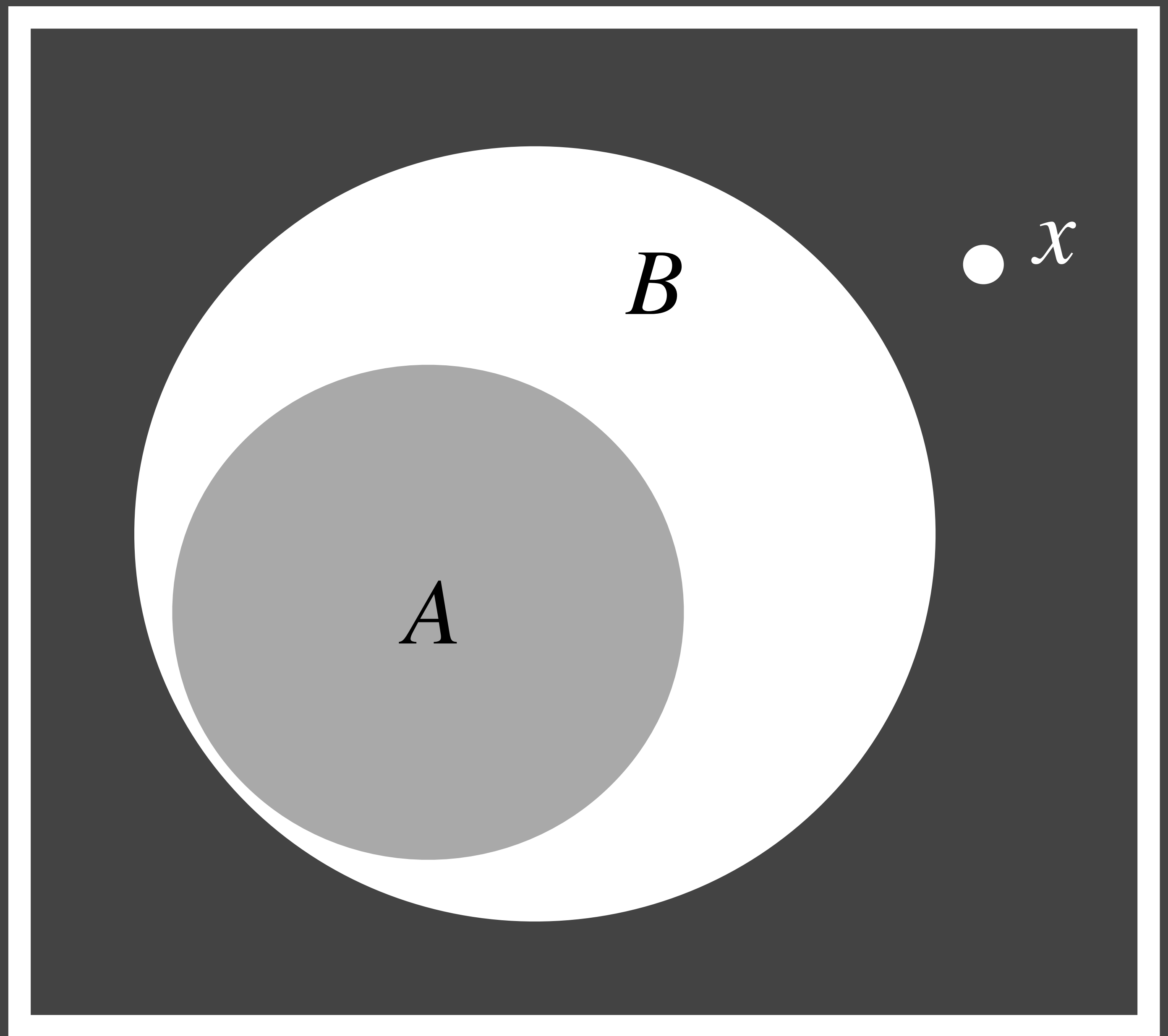

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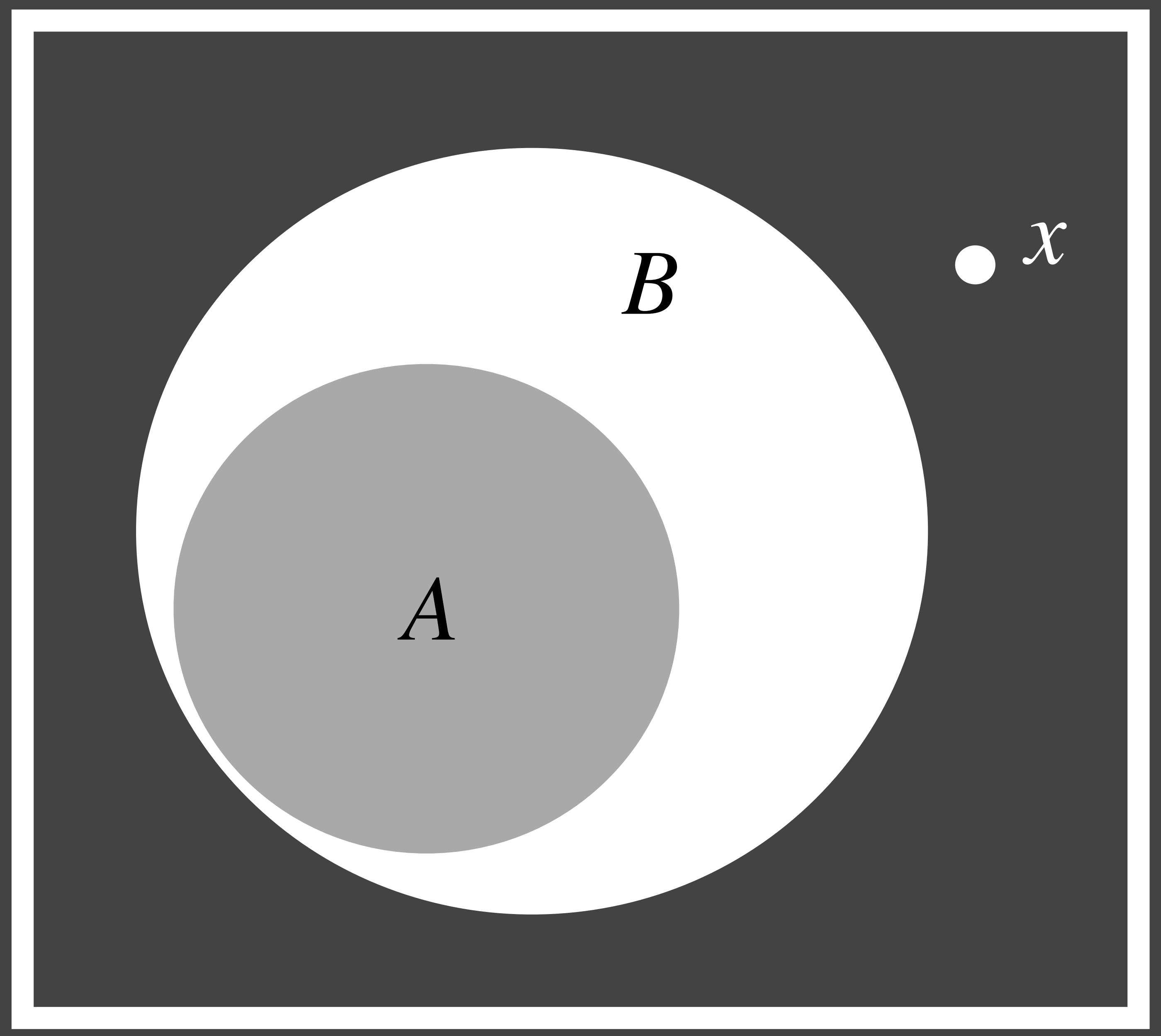

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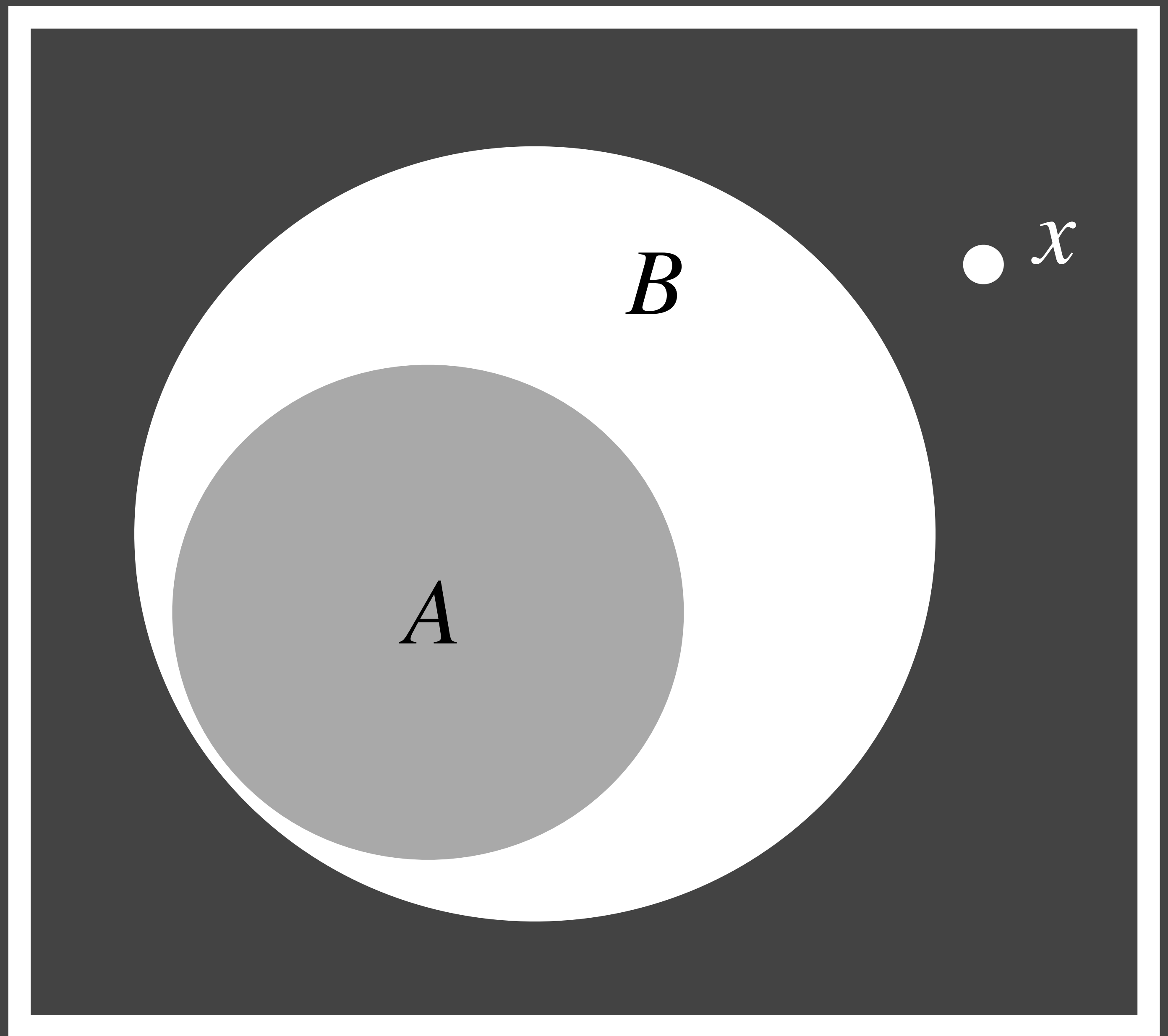

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Porting submod cover to uncertain settings automatically ports all applications!

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Sweet spot between generality and tractability!

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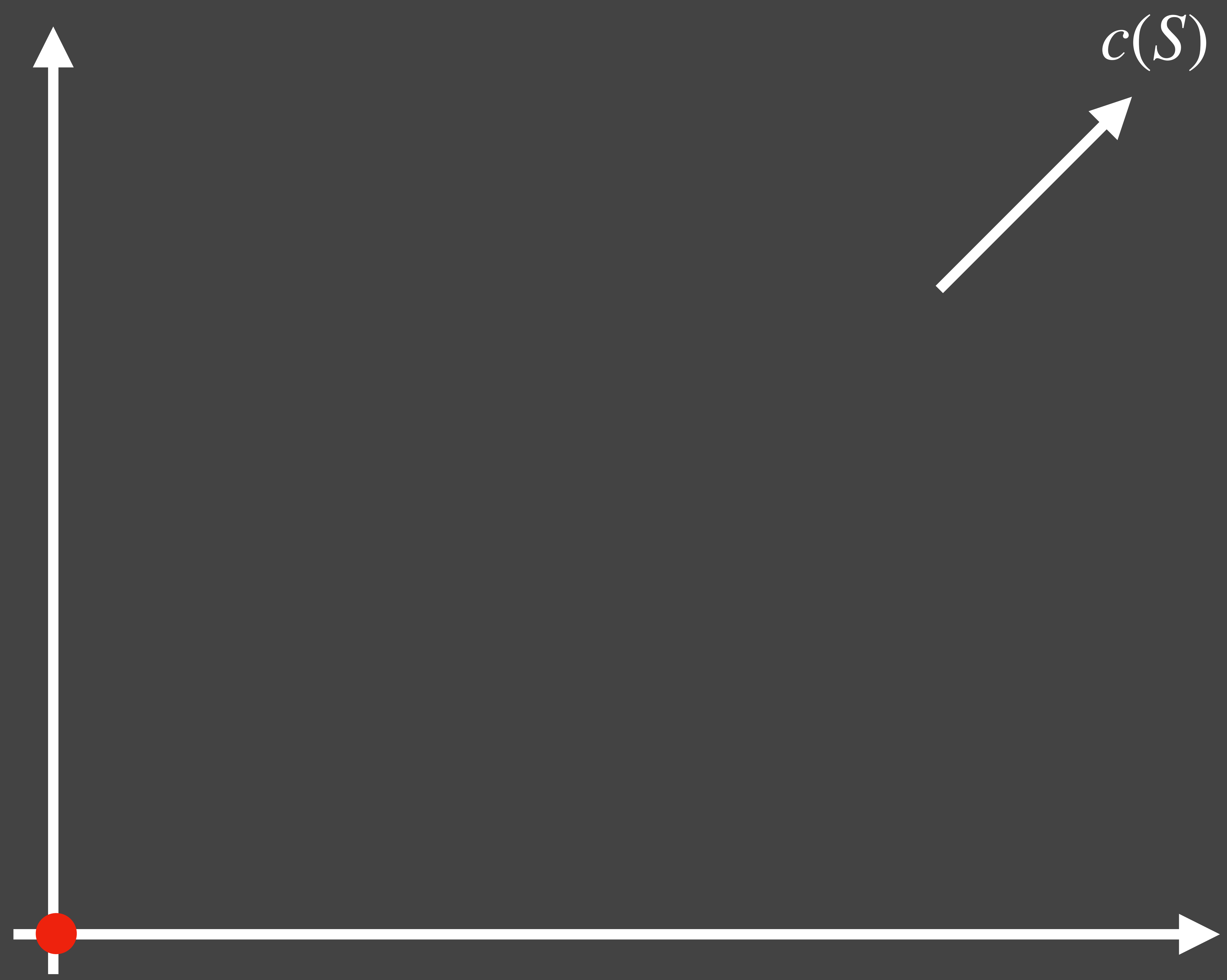

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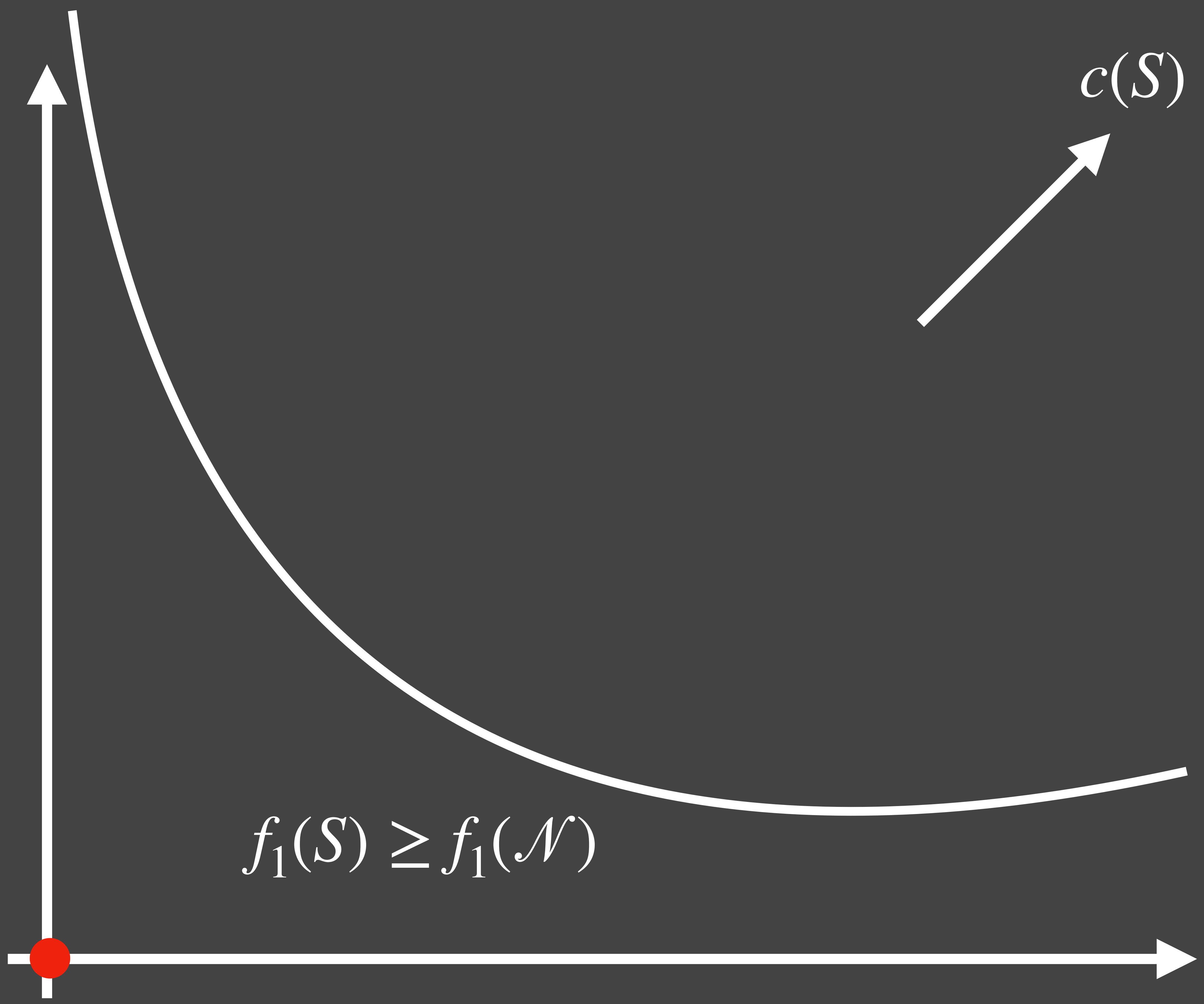

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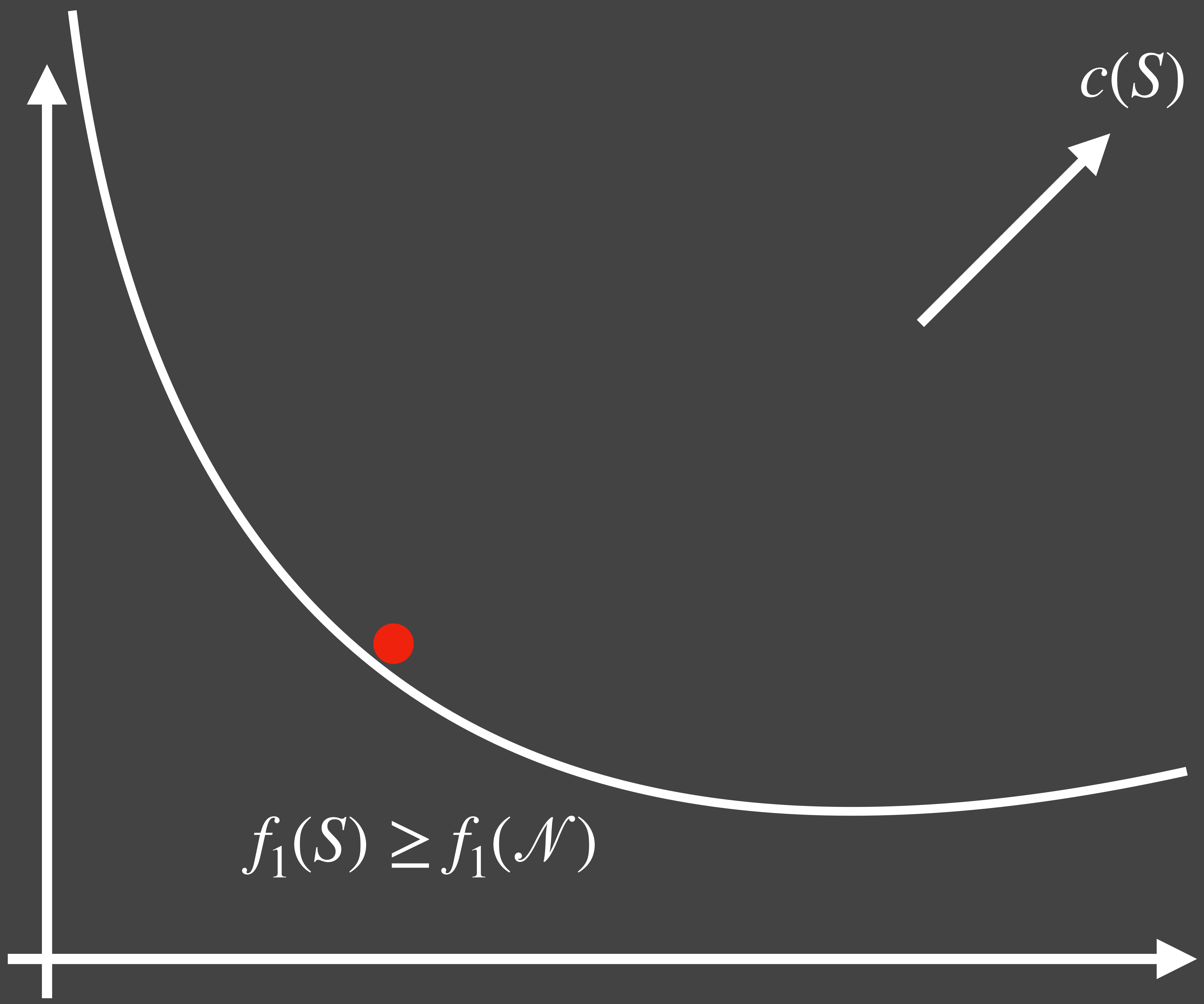

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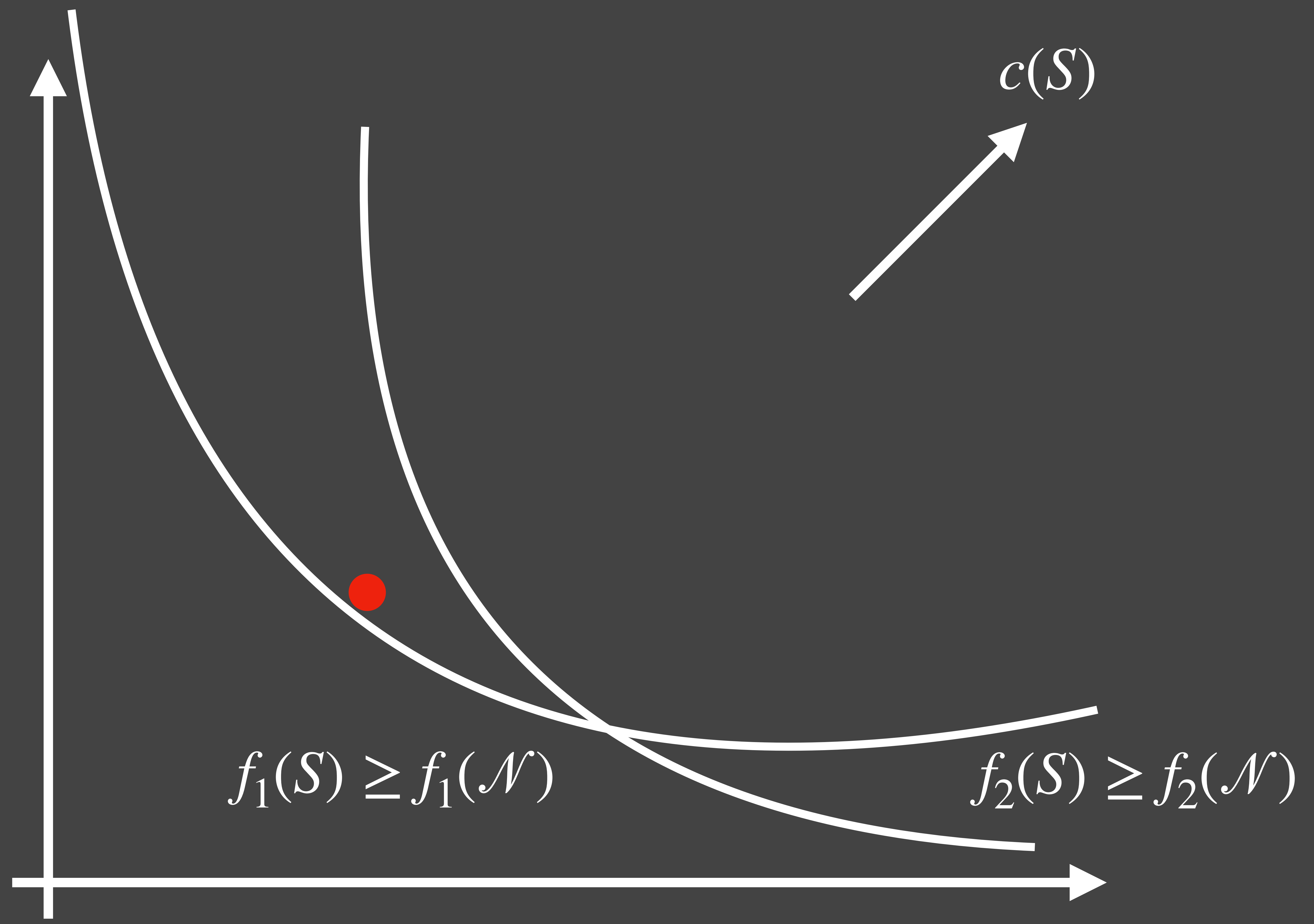

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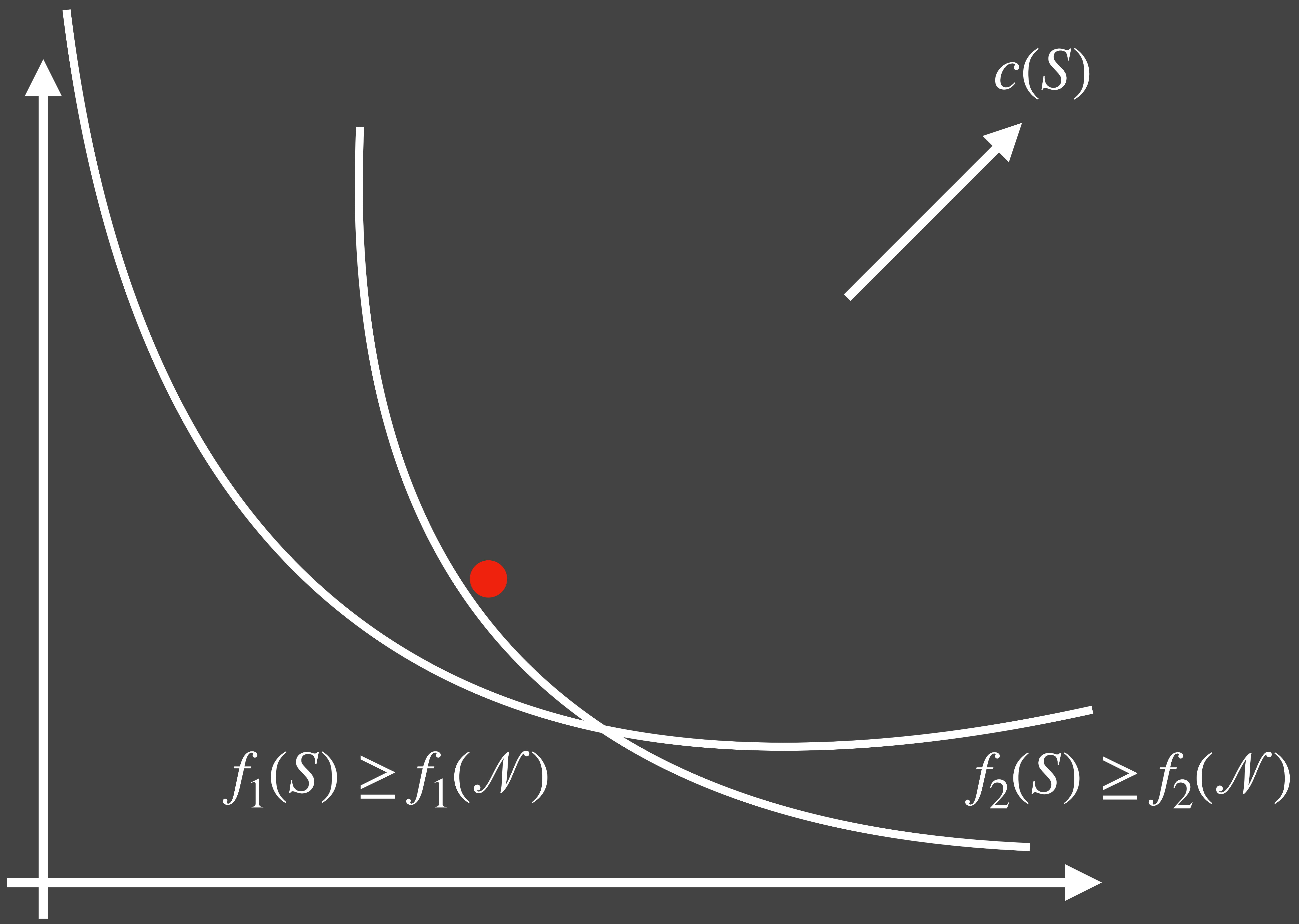

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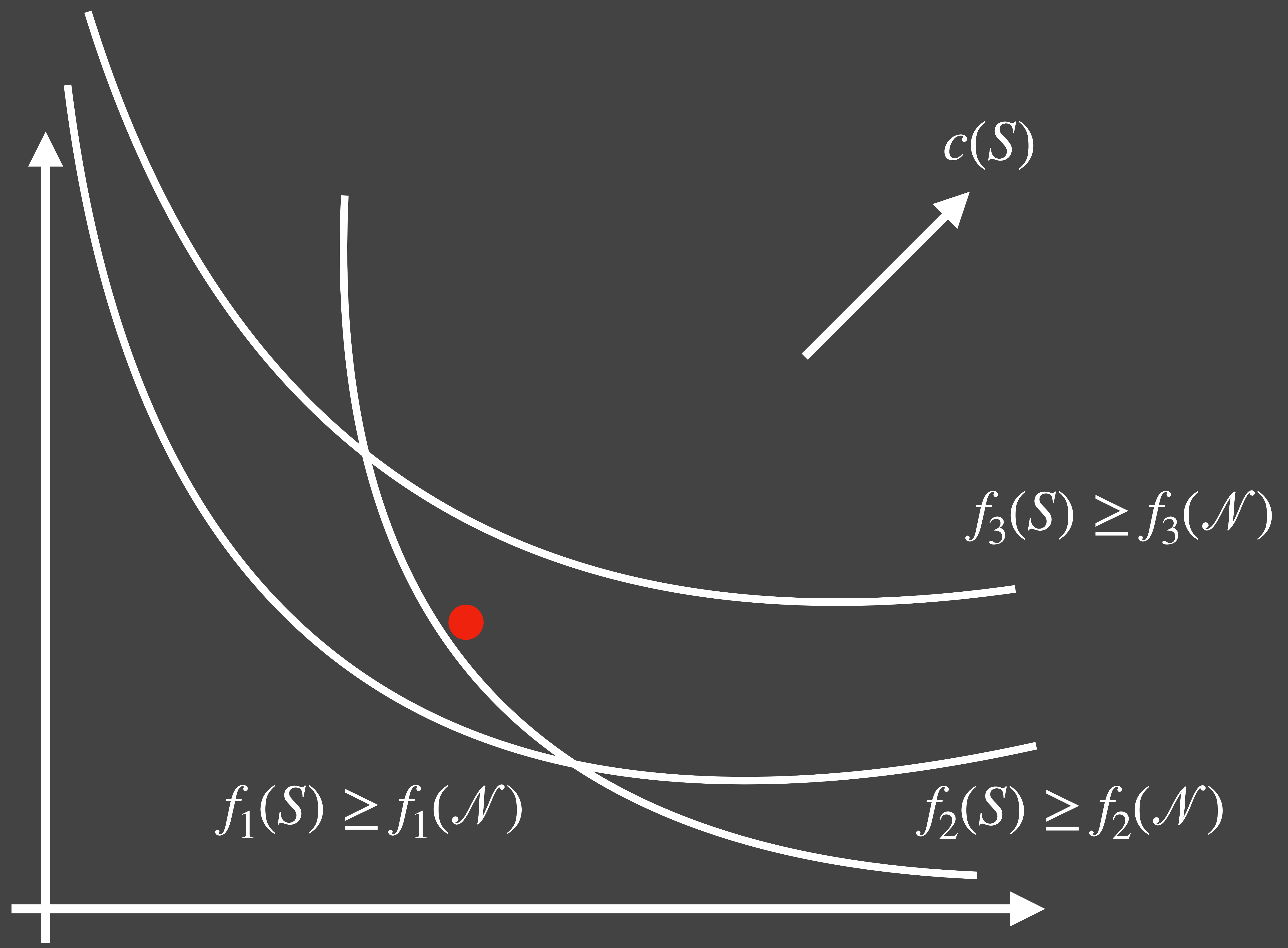

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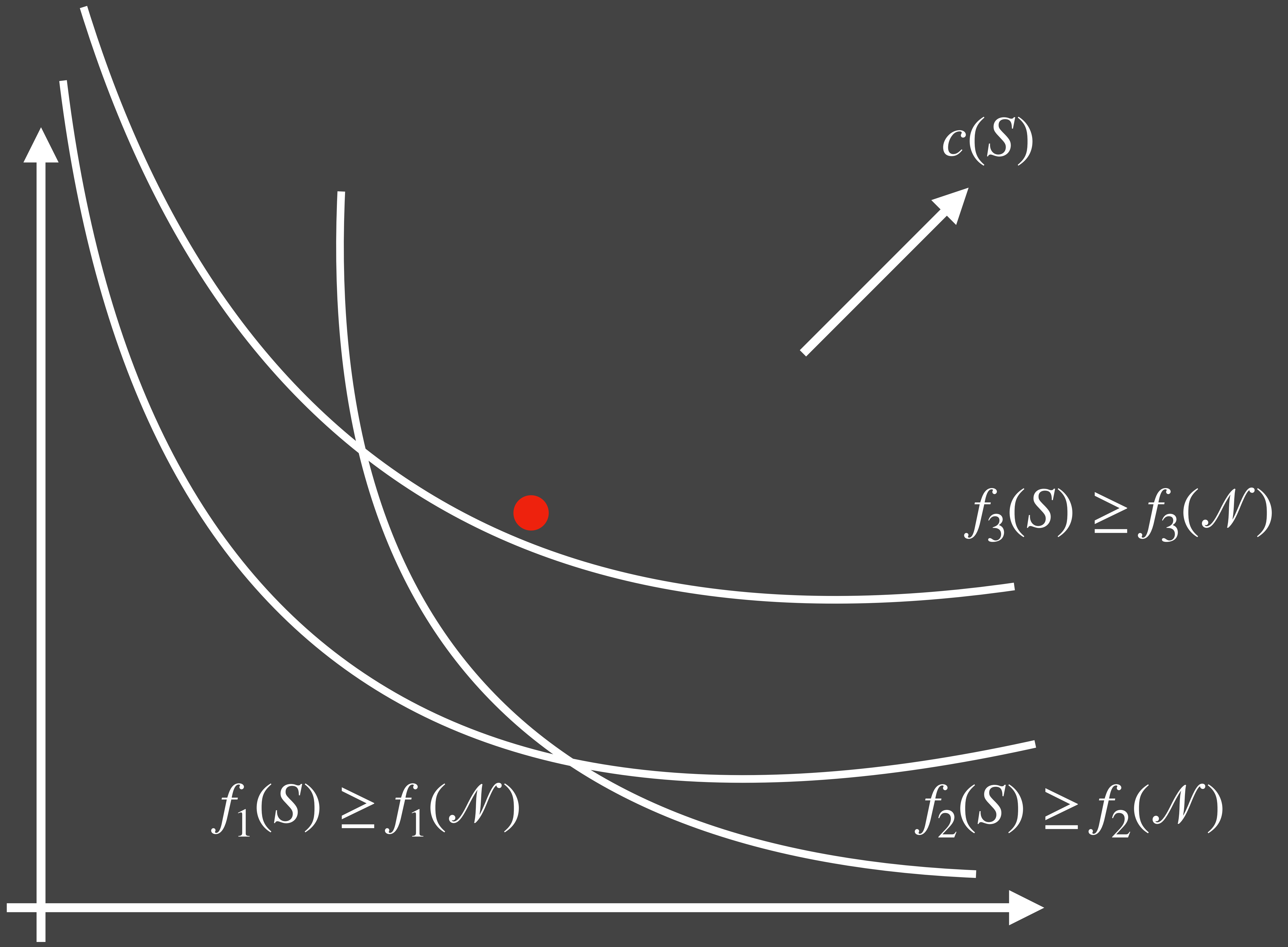

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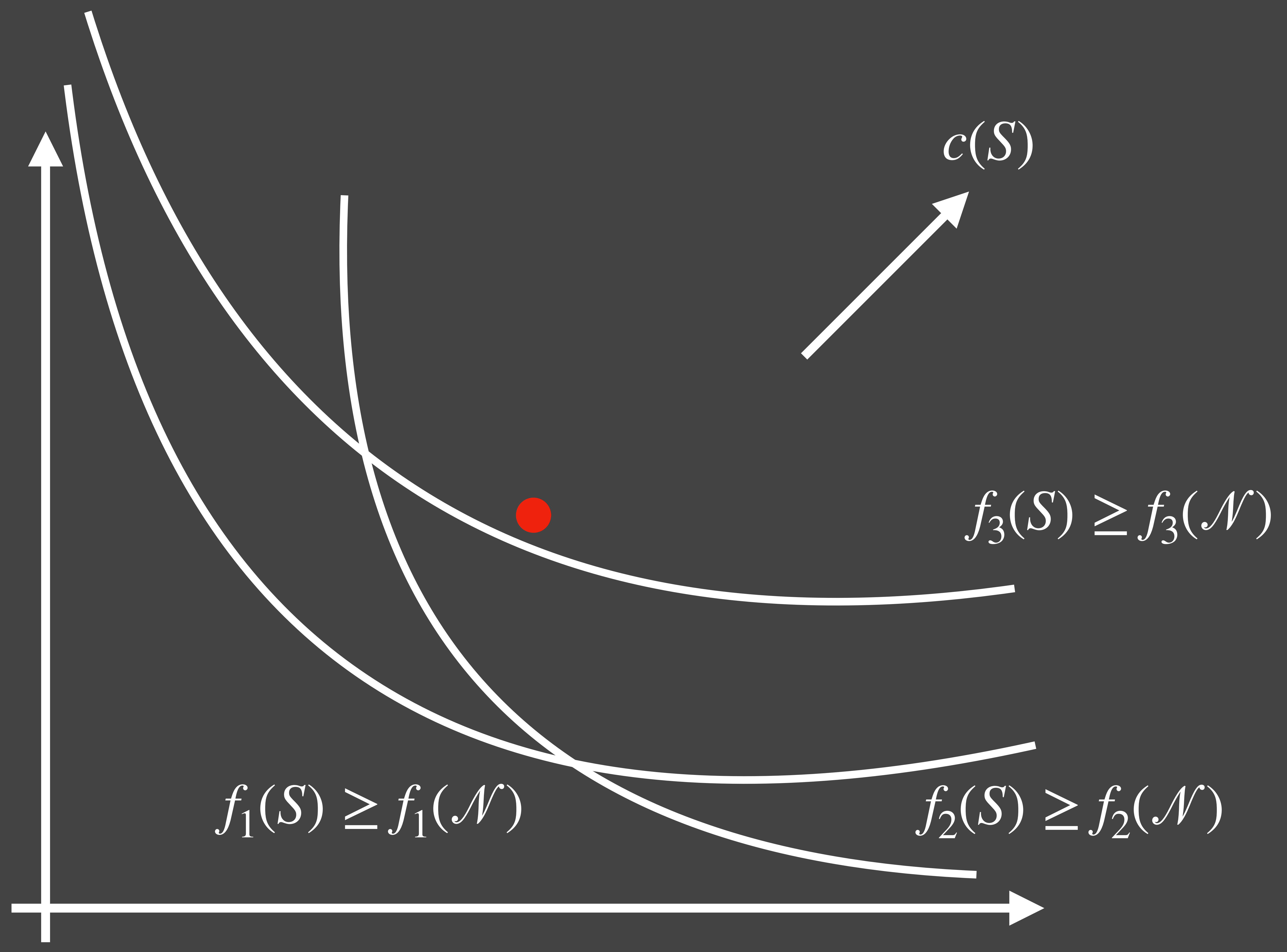

$$F = \sum_i f_i$$

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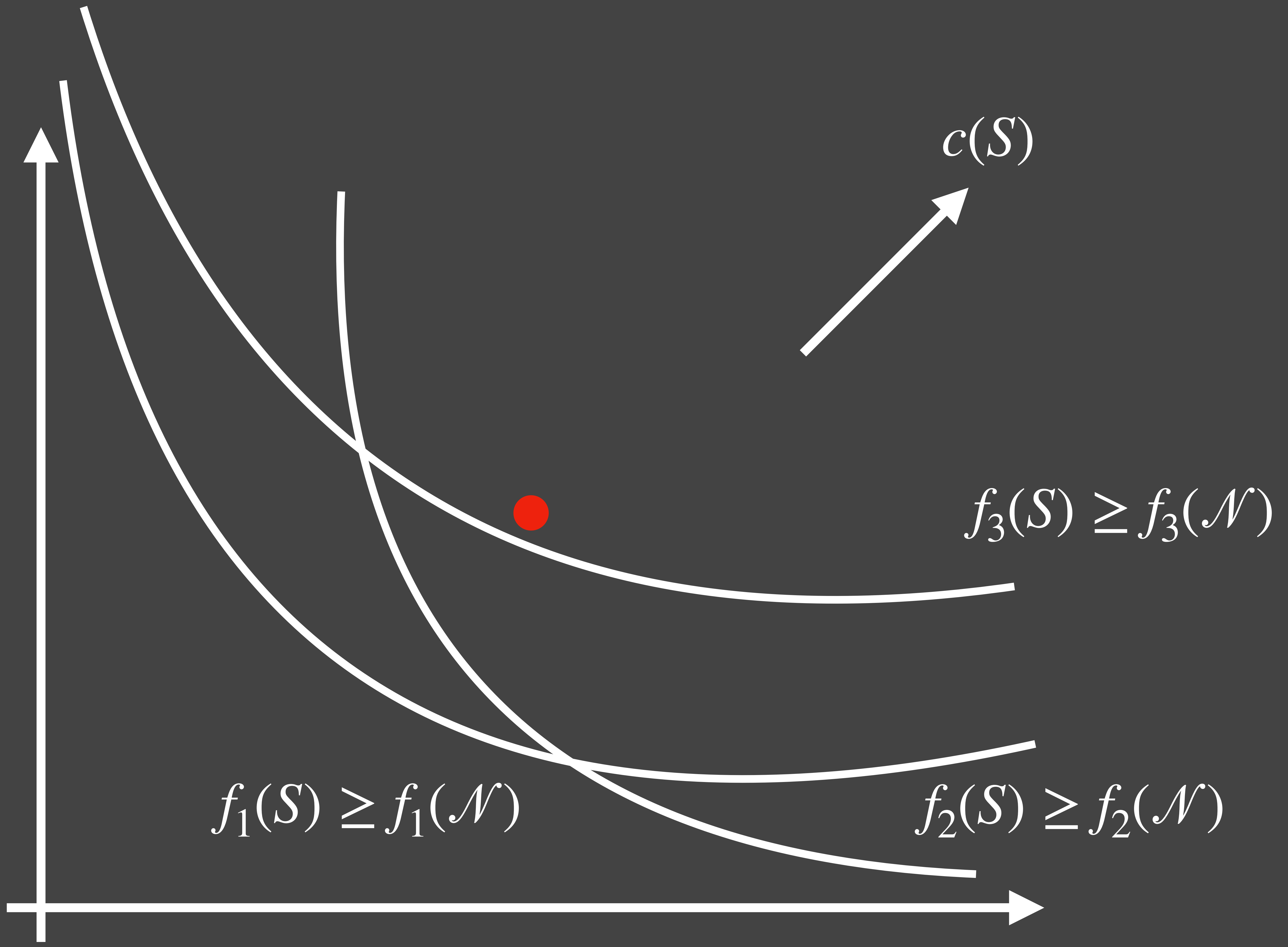

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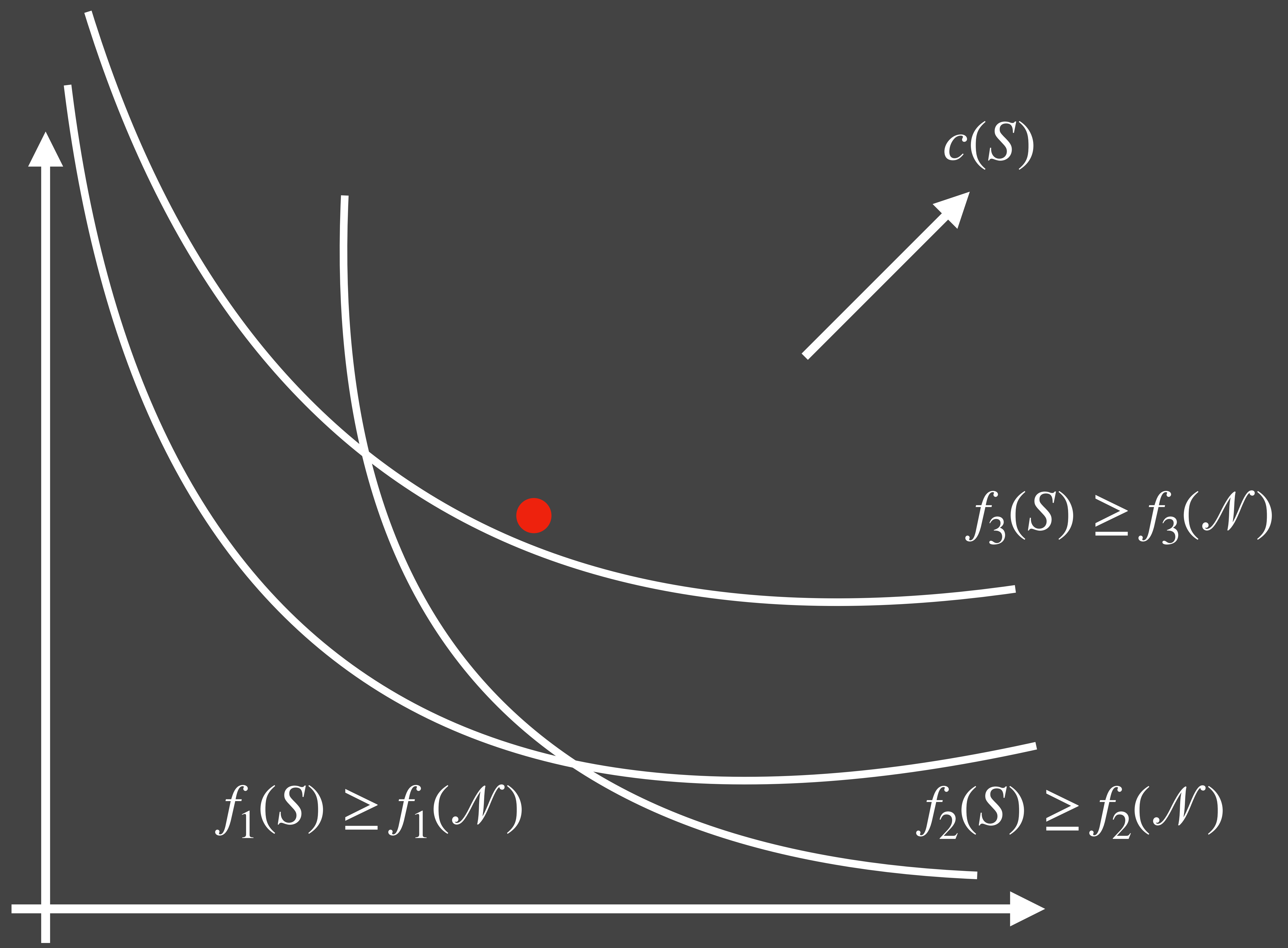

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My PhD Work

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… and Offline

Finding Skewed Subcubes Under a Distribution [Gopalan, L., Wieder ITCS 20]

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Talk Outline

➔ Intro

Part I — Online/Dynamic Submodular Cover

Part II — Application: Block-Aware Caching

Part III — Random Order Online Set Cover

Conclusion

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Part I — Online/Dynamic Submodular Cover

with Anupam Gupta

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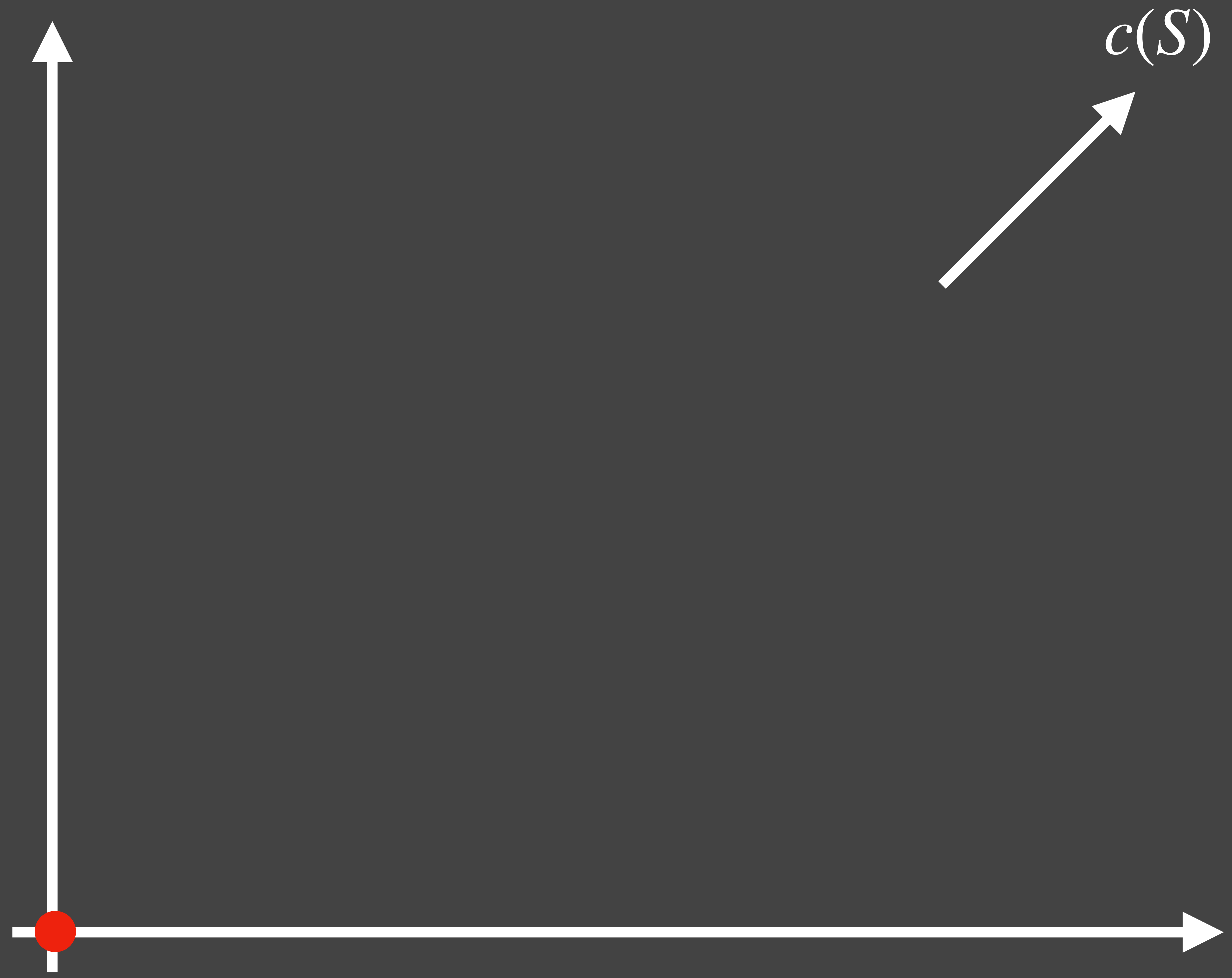

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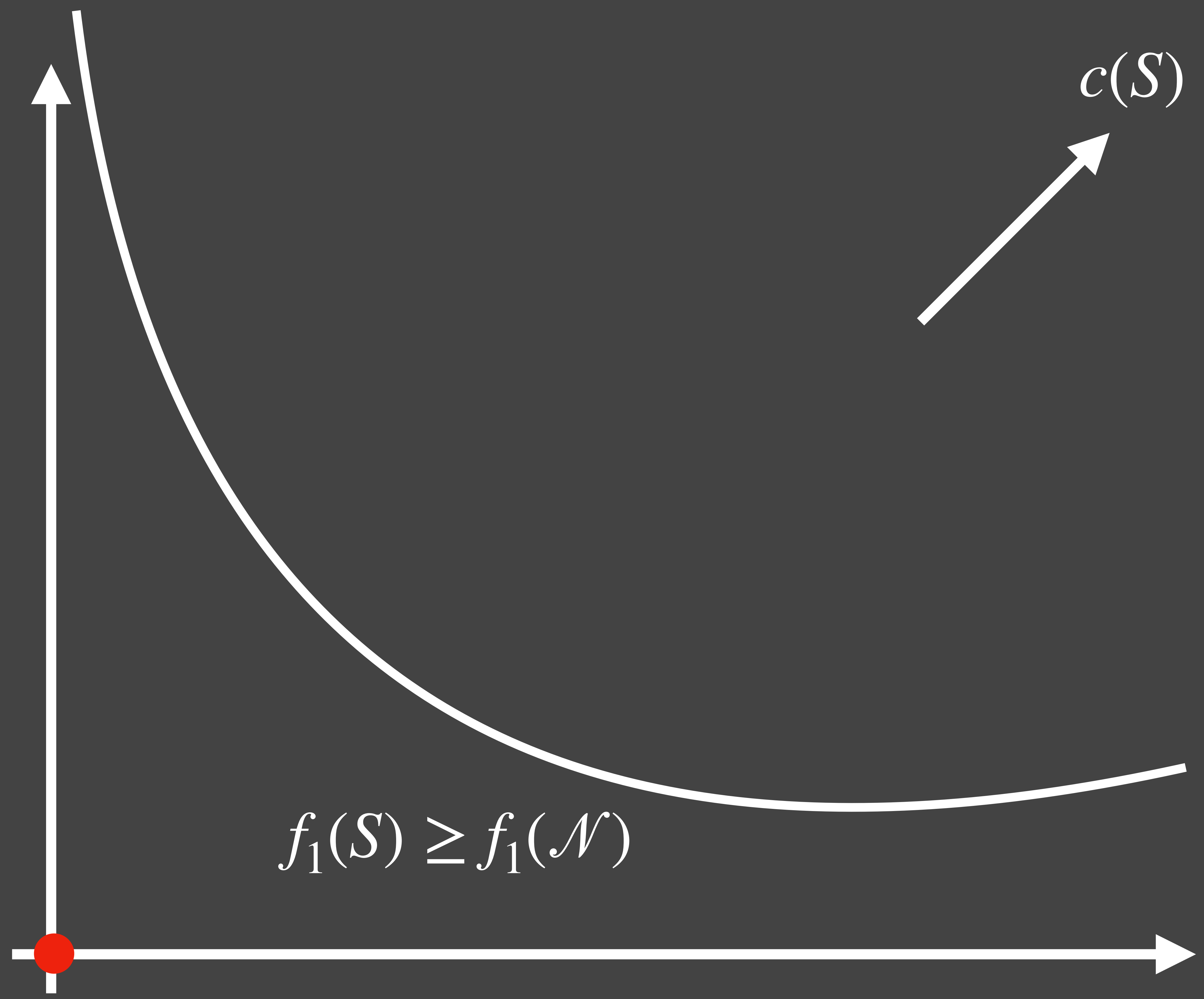

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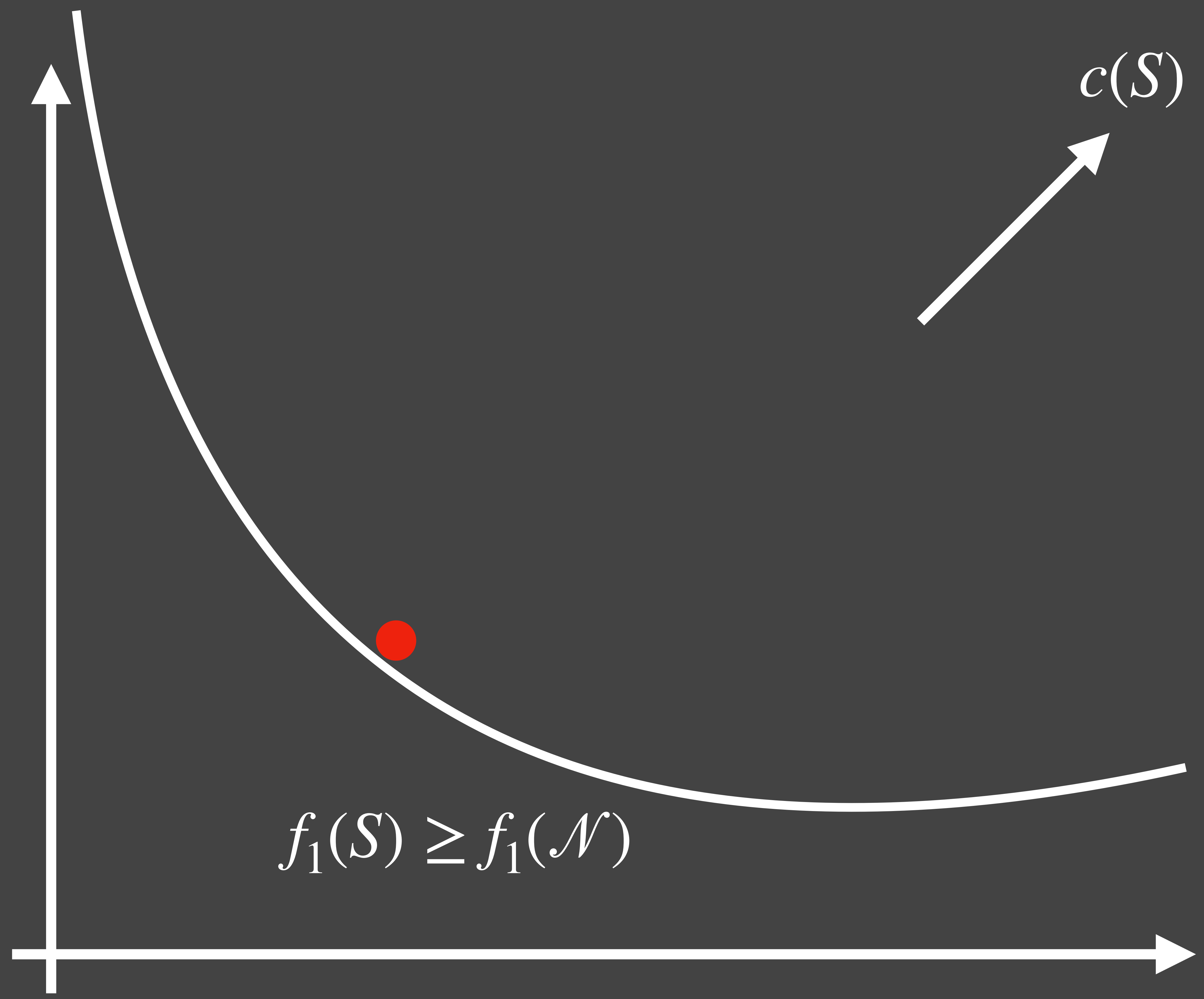

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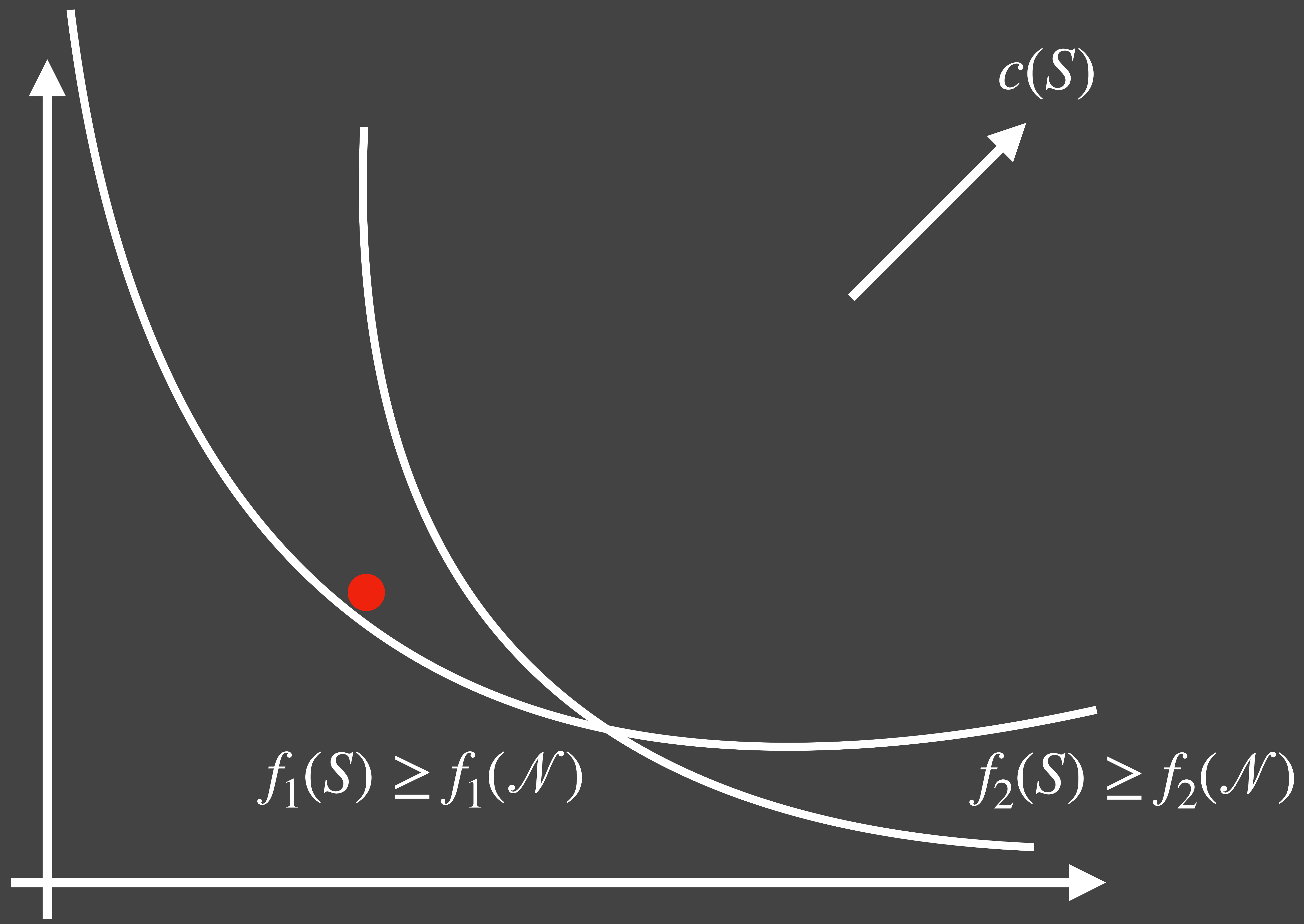

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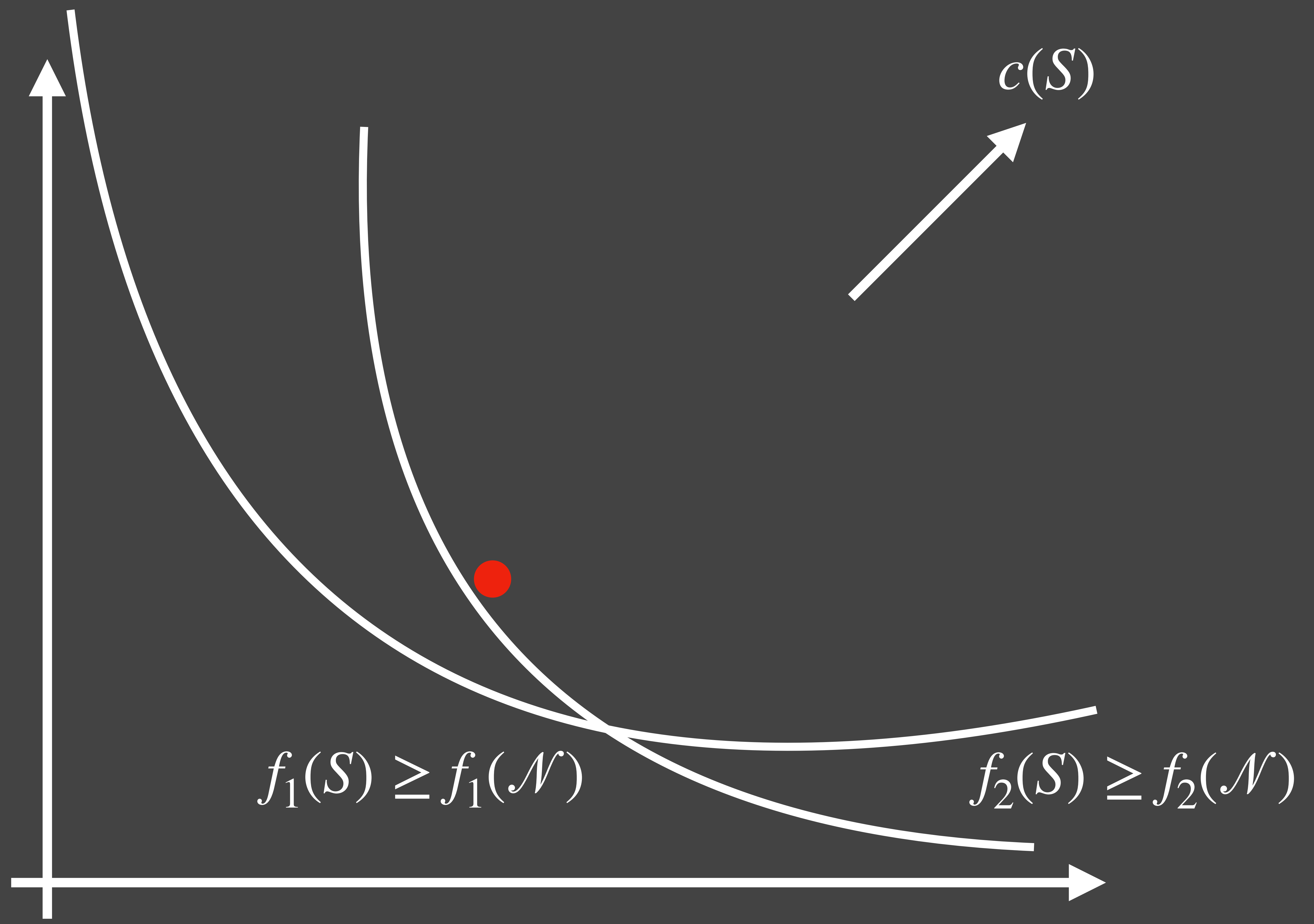

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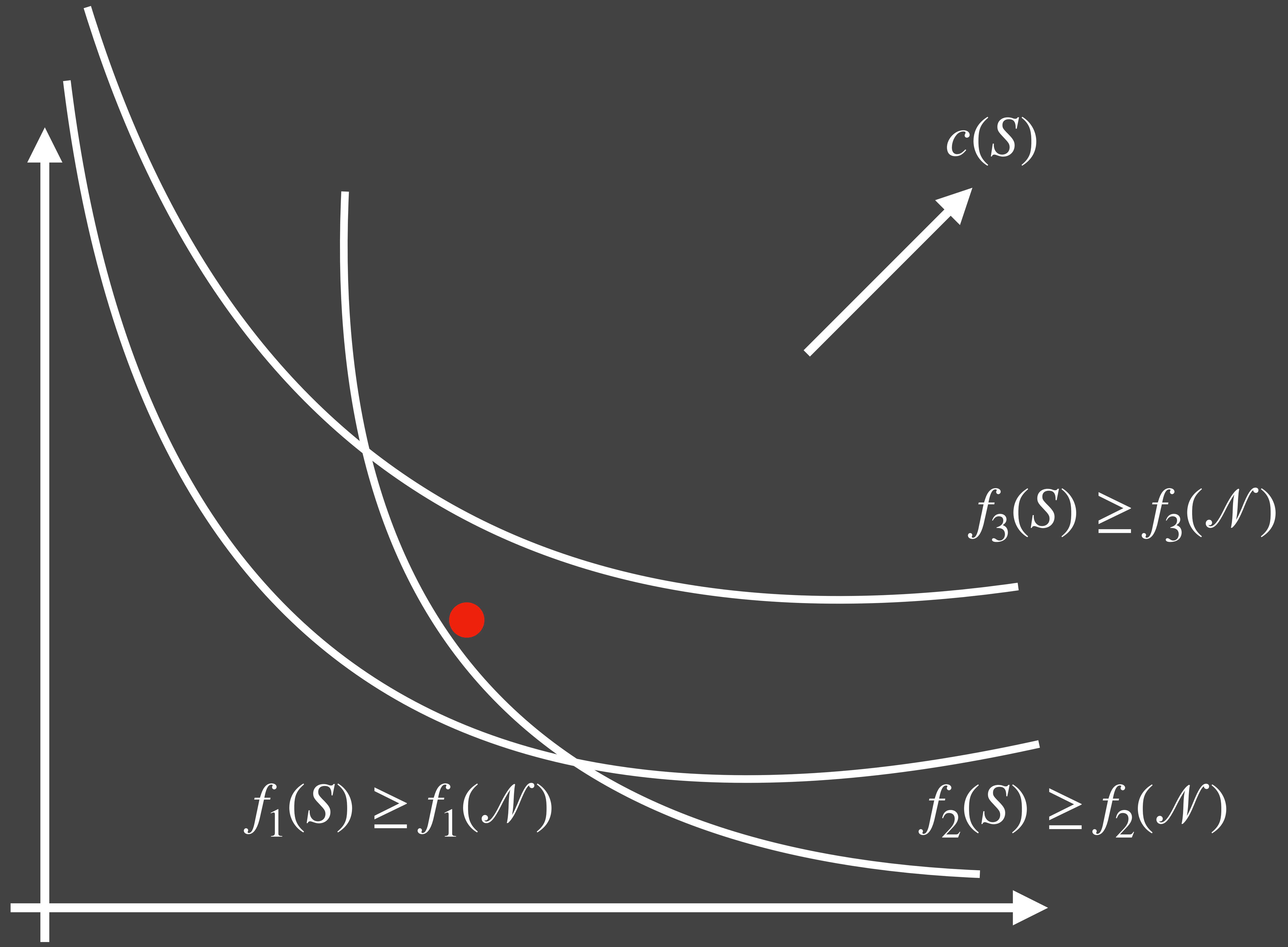

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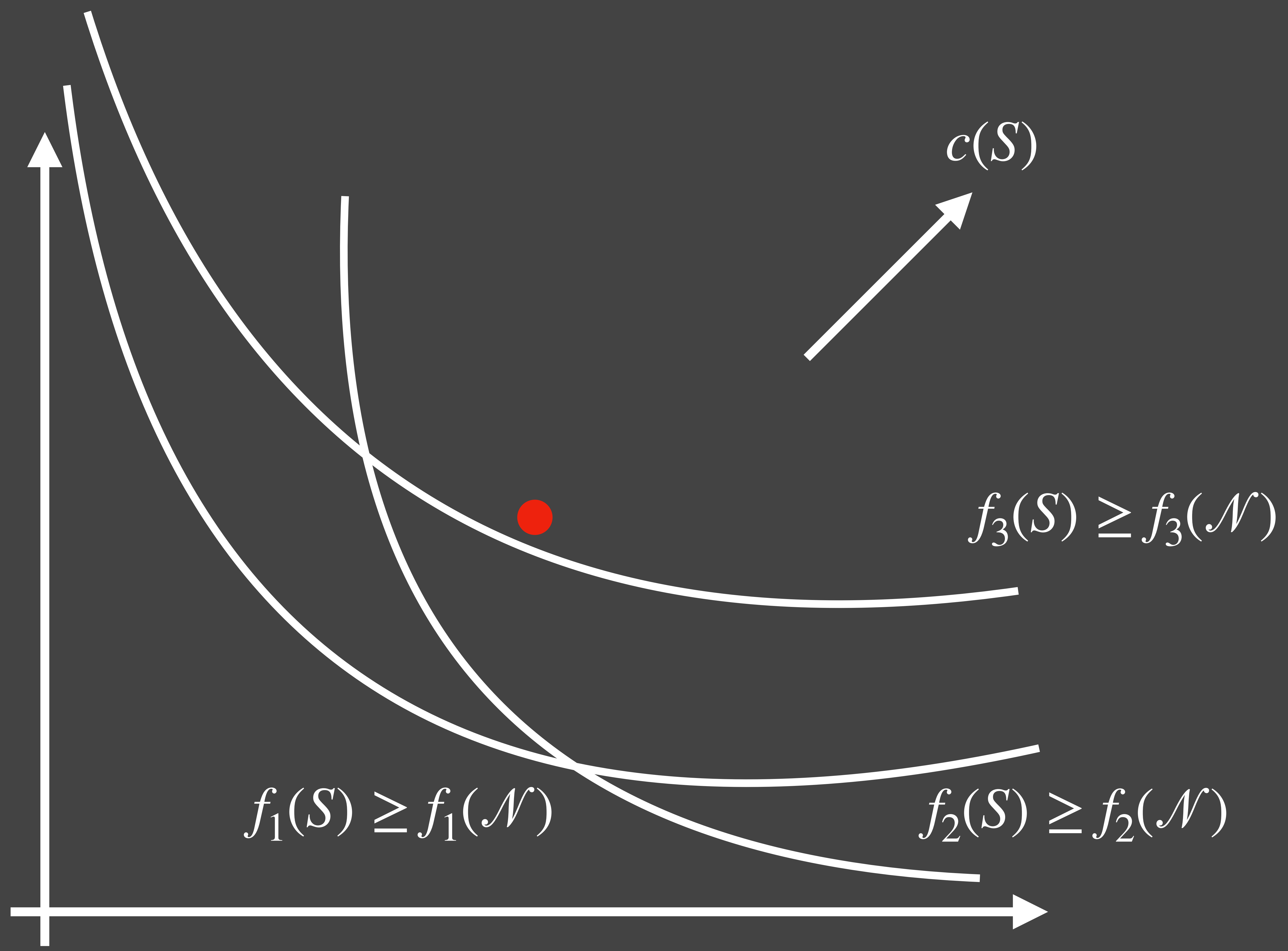

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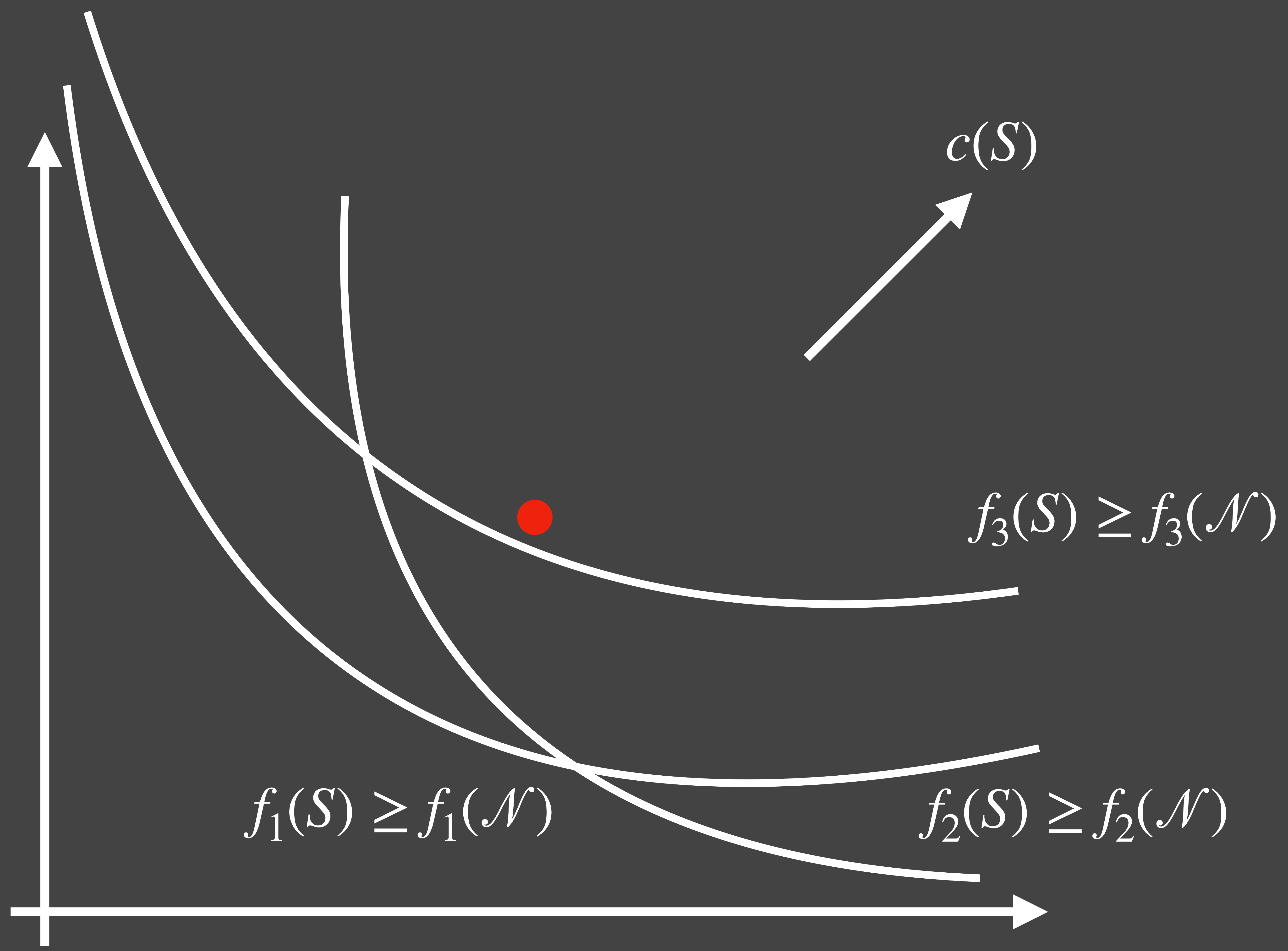

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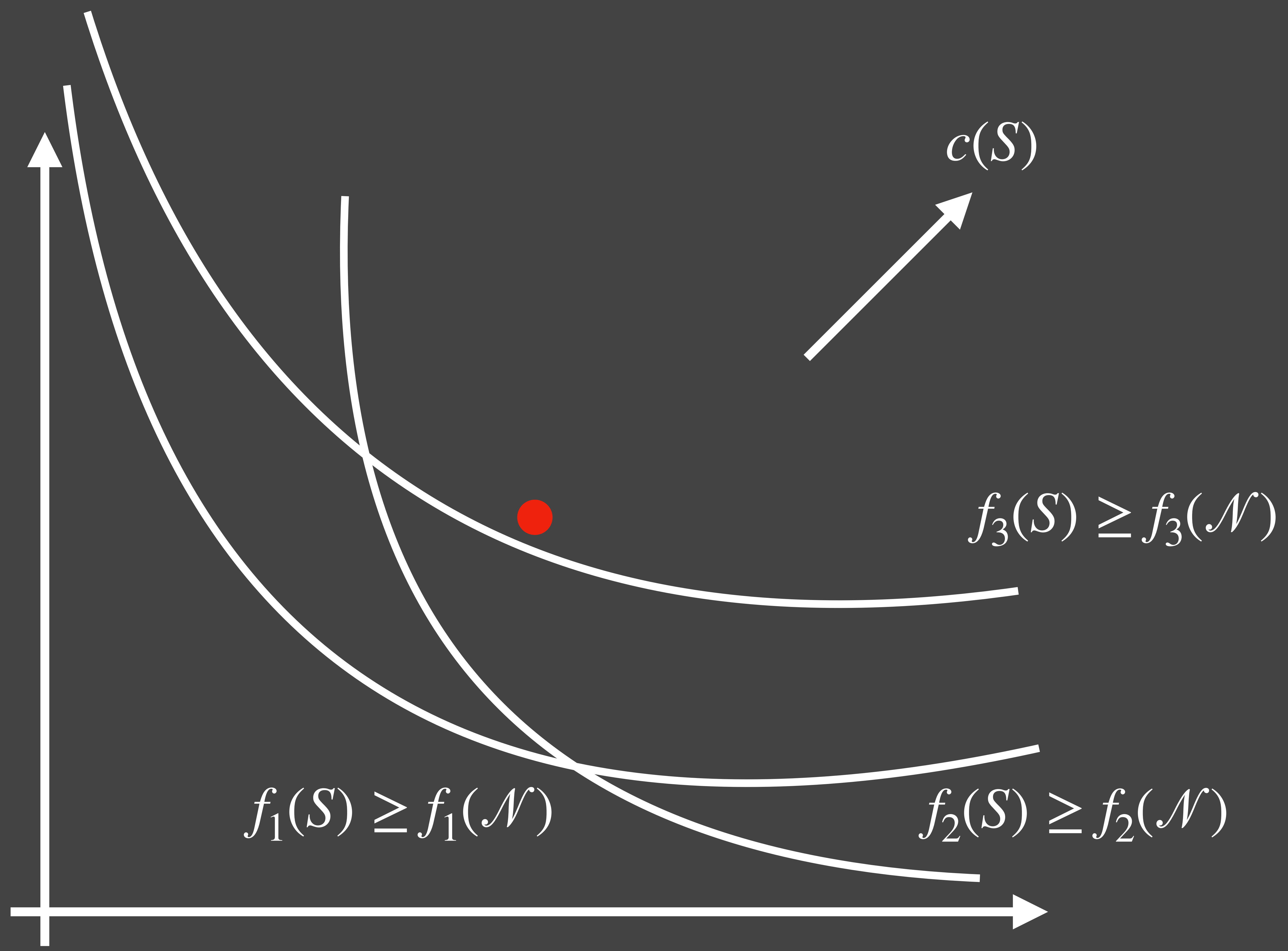

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S can only grow over time...

Special Case: Online Set Cover

[Alon Awerbuch Azar Buchbinder Naor 03]

s_1

s_2

s_3

s_4

s_5

s_6

Special Case: Online Set Cover

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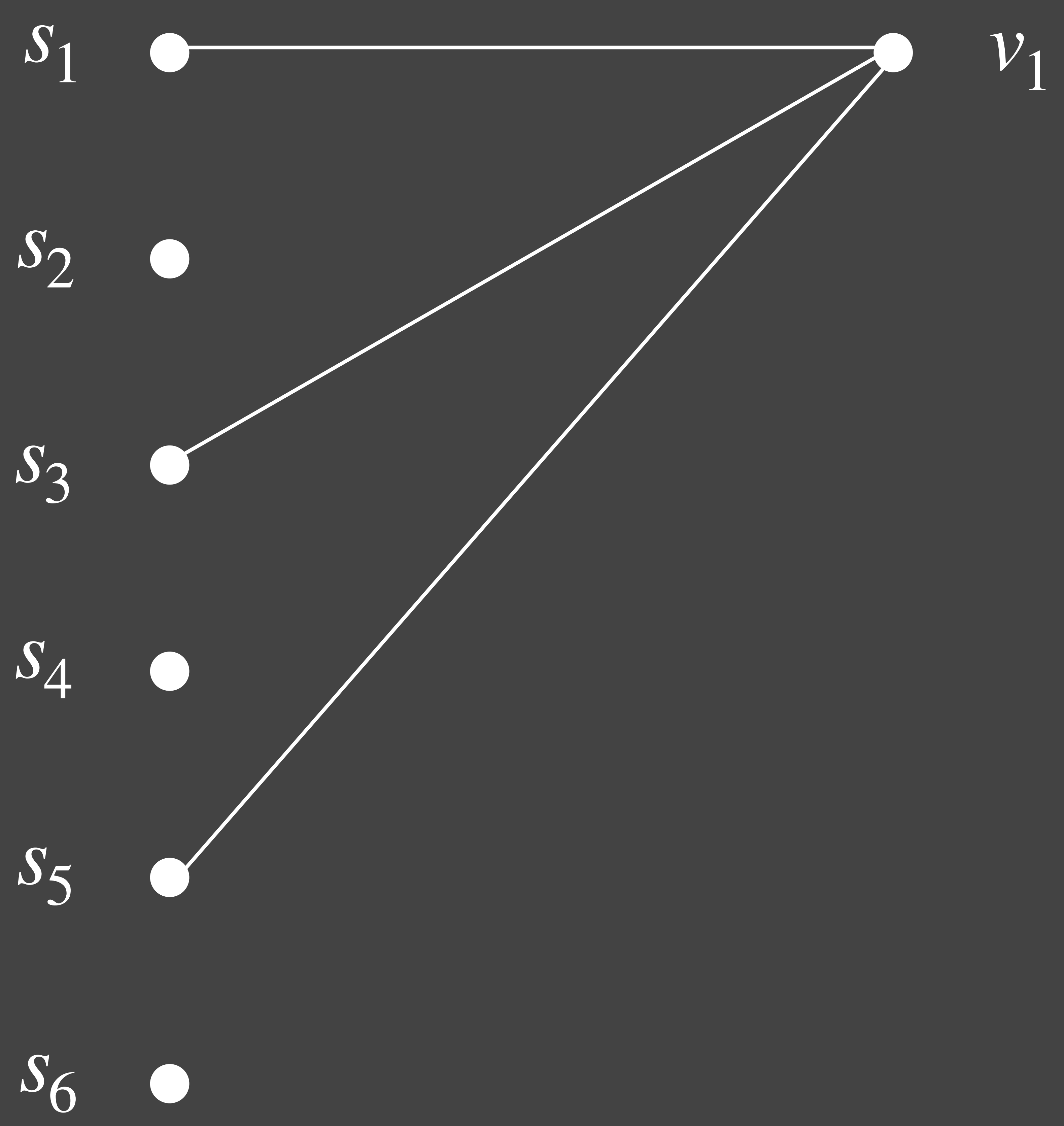

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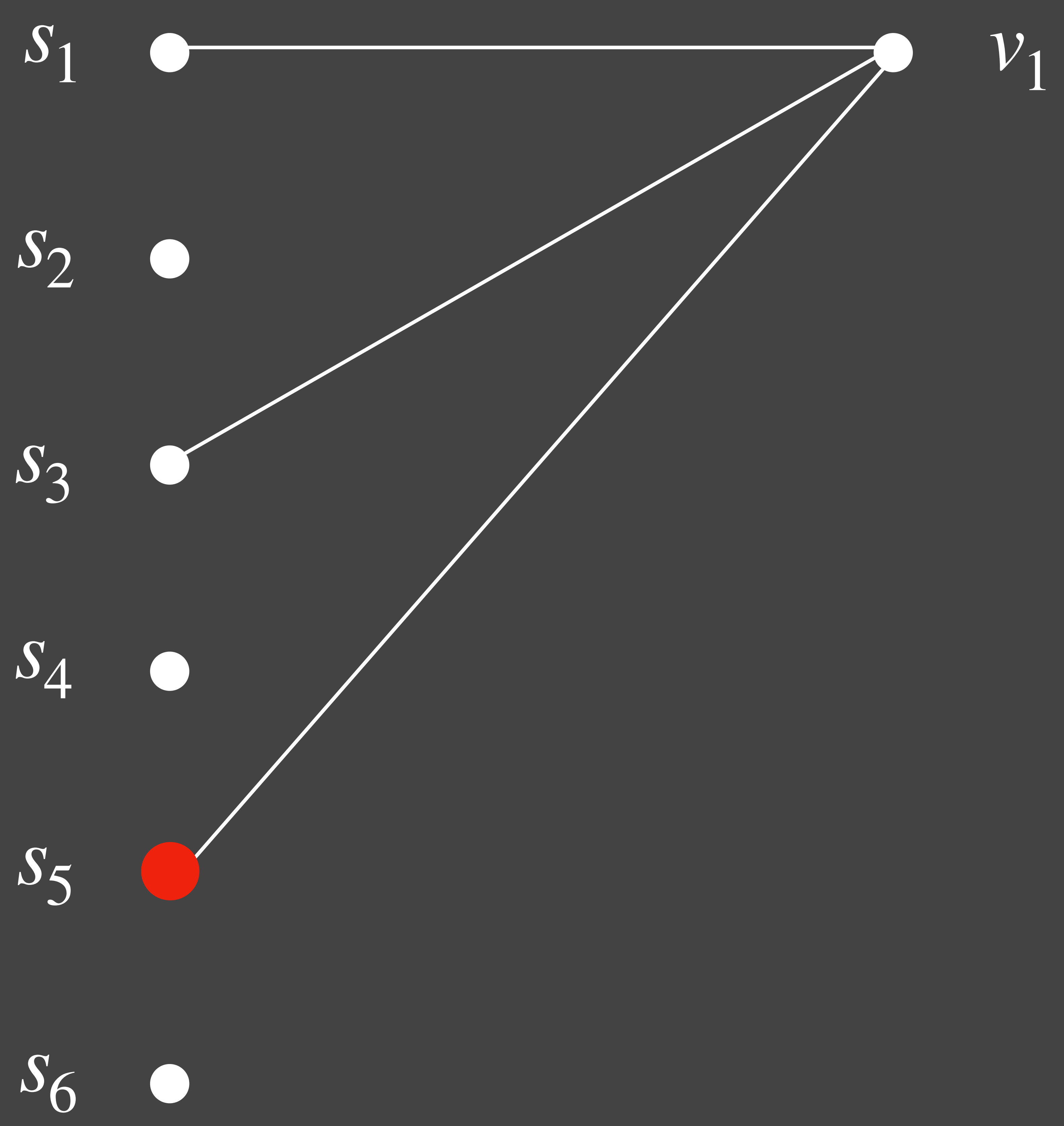

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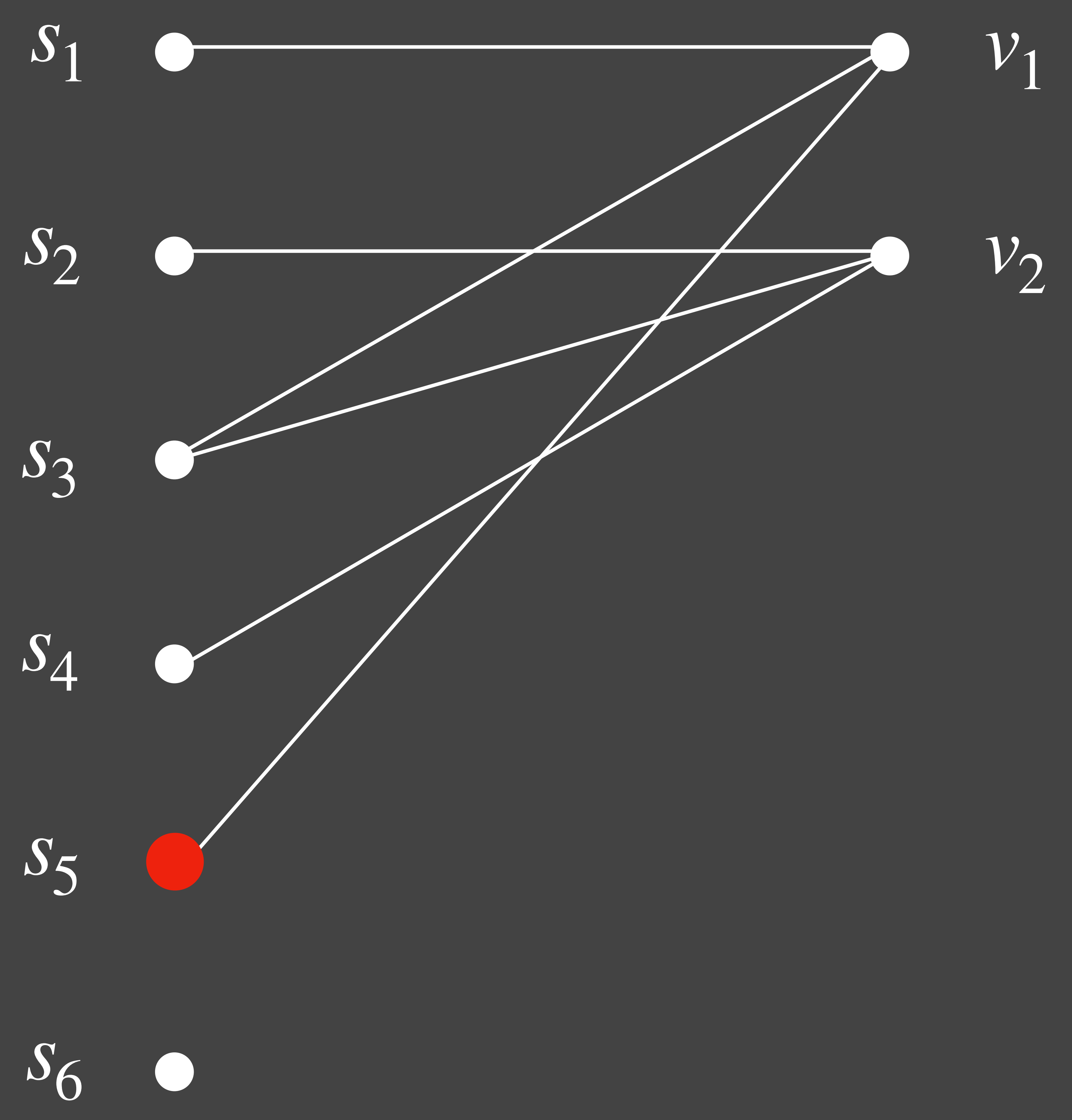

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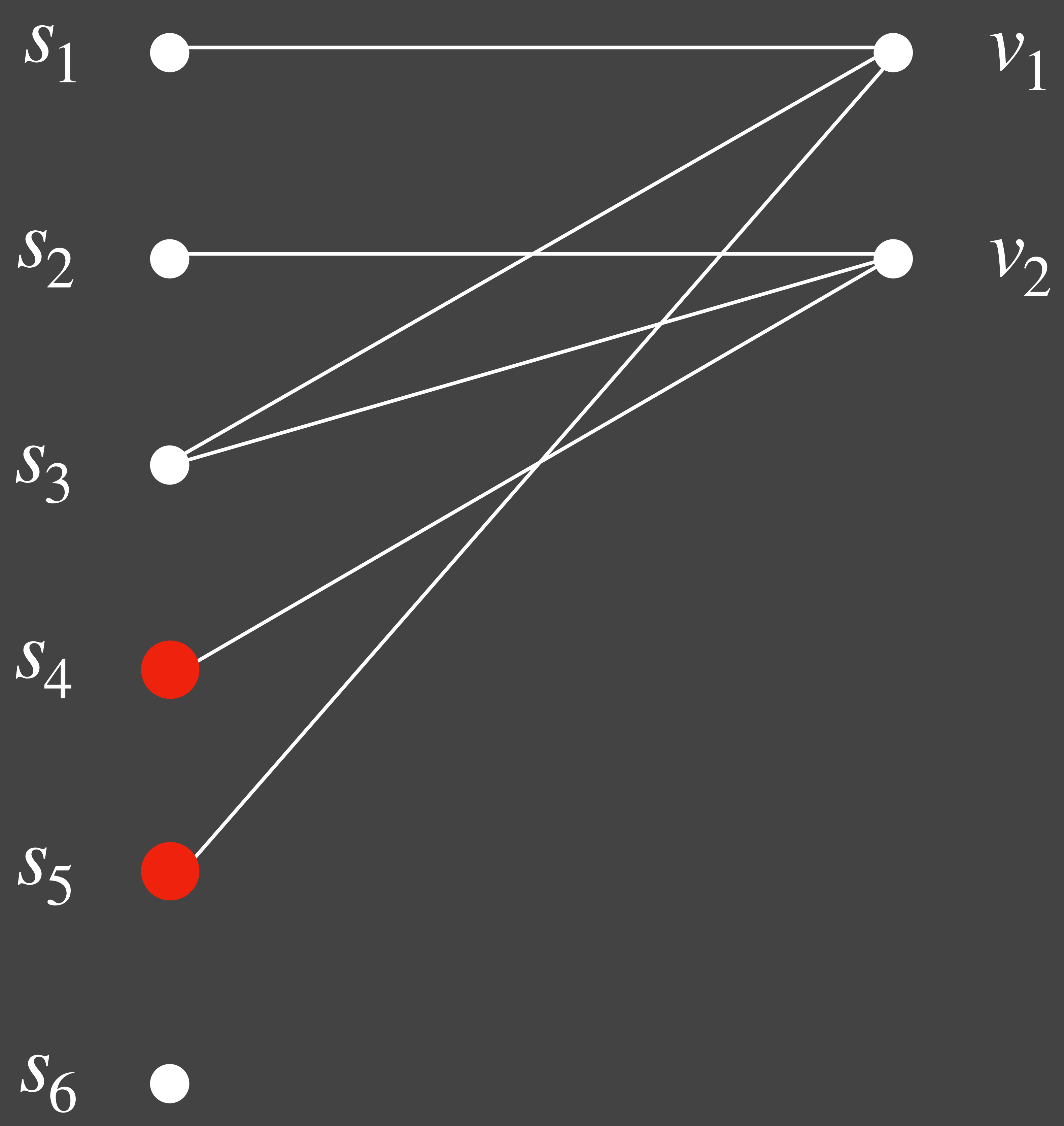

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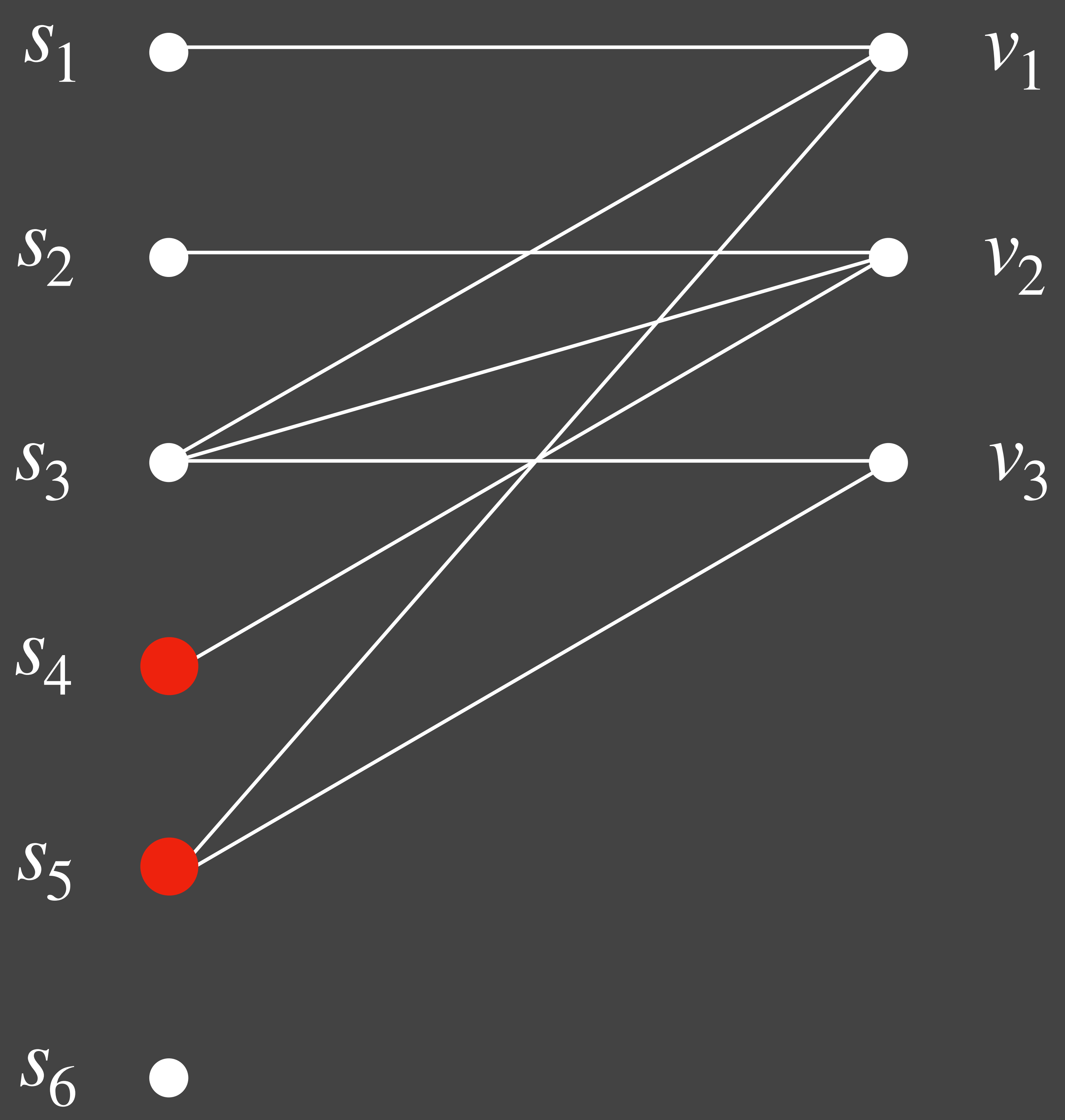

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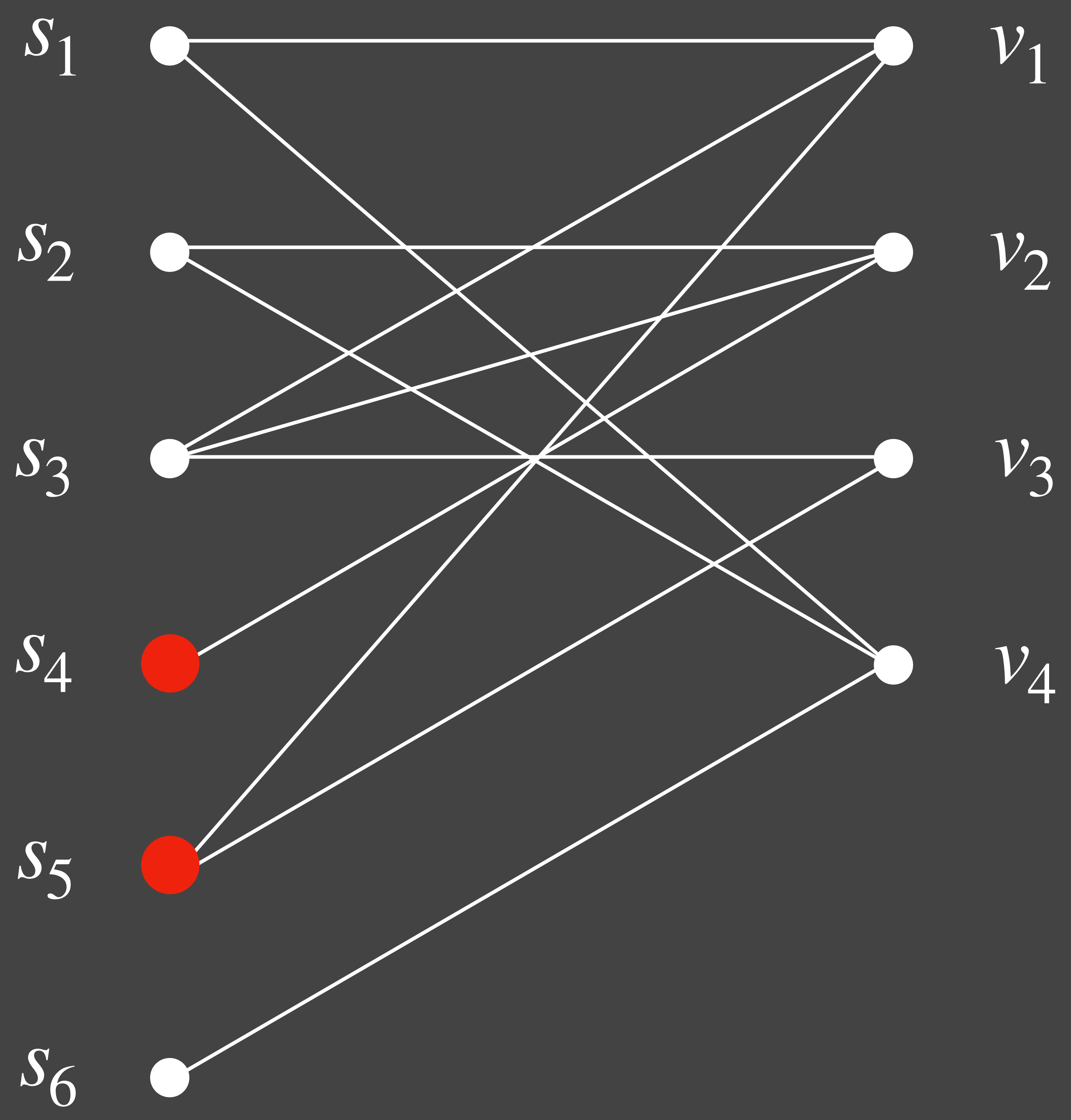

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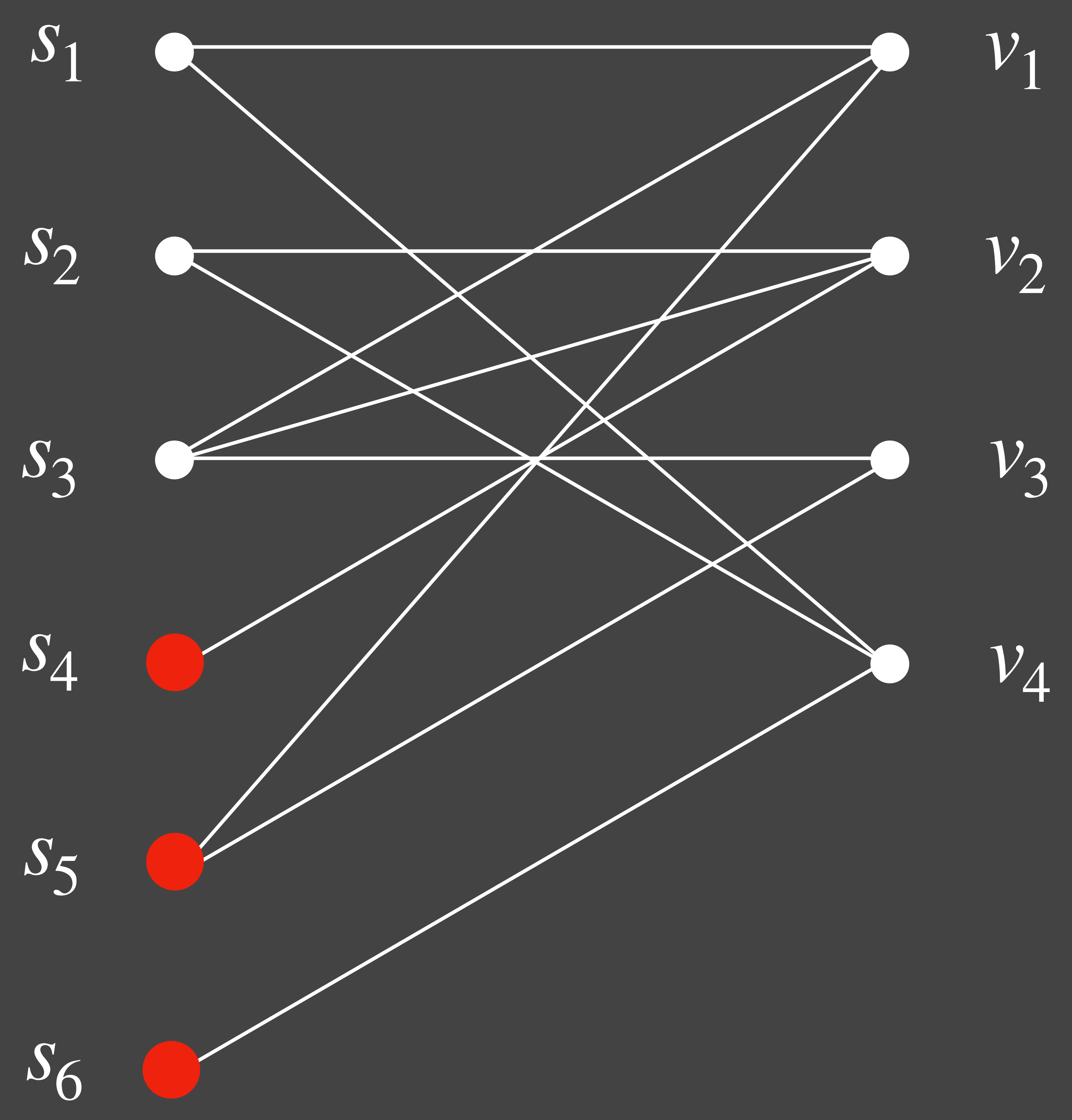

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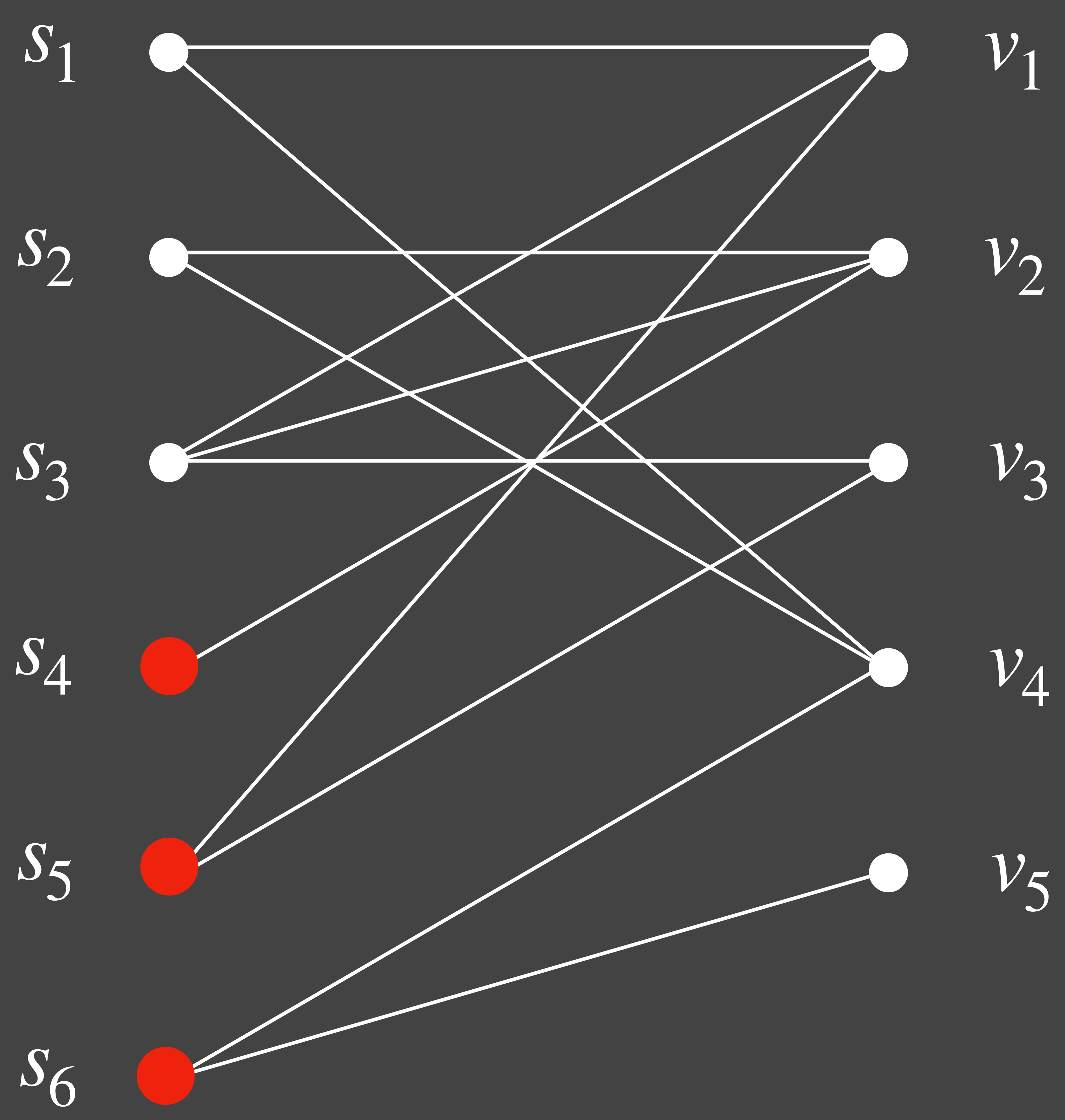

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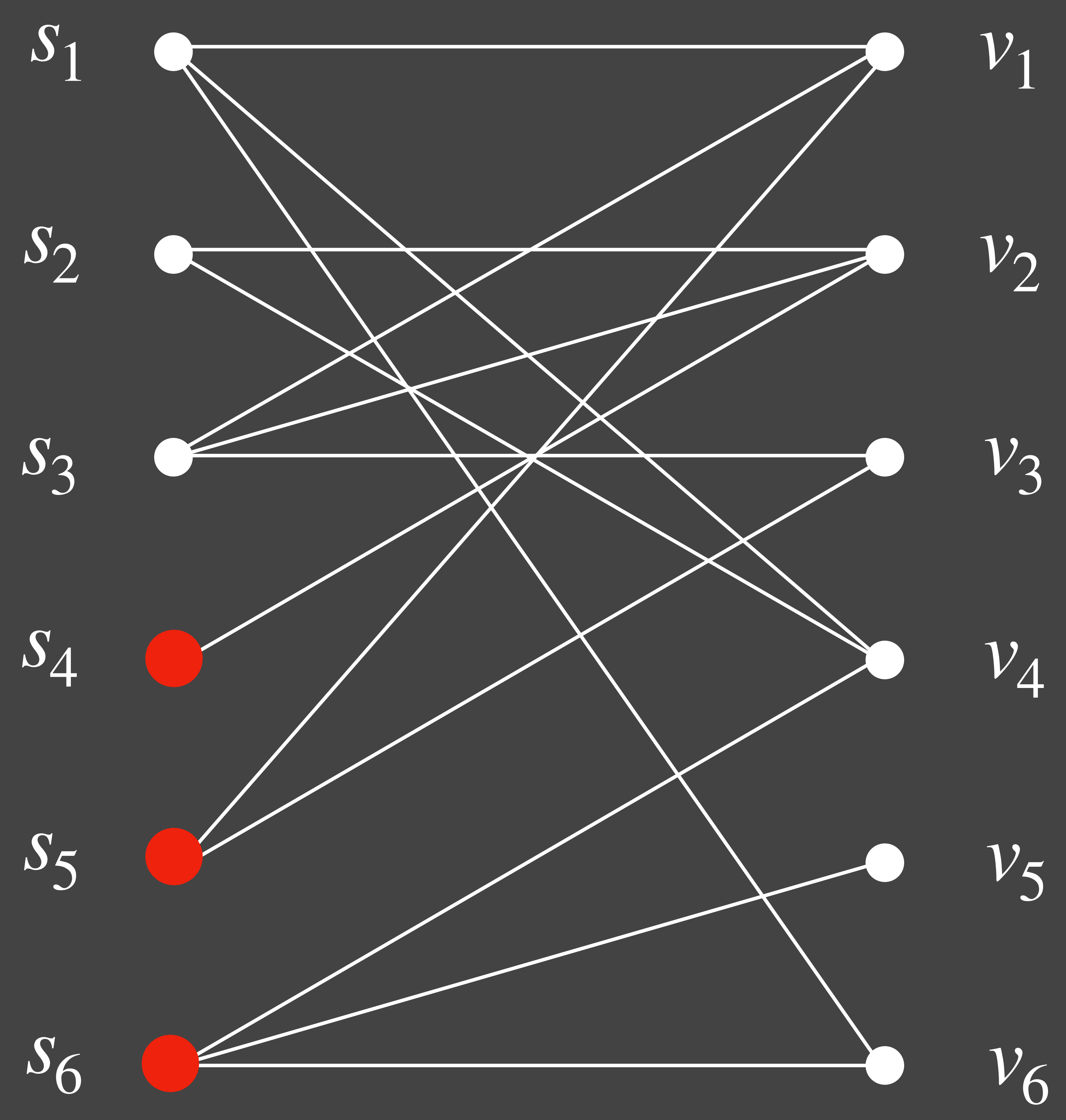

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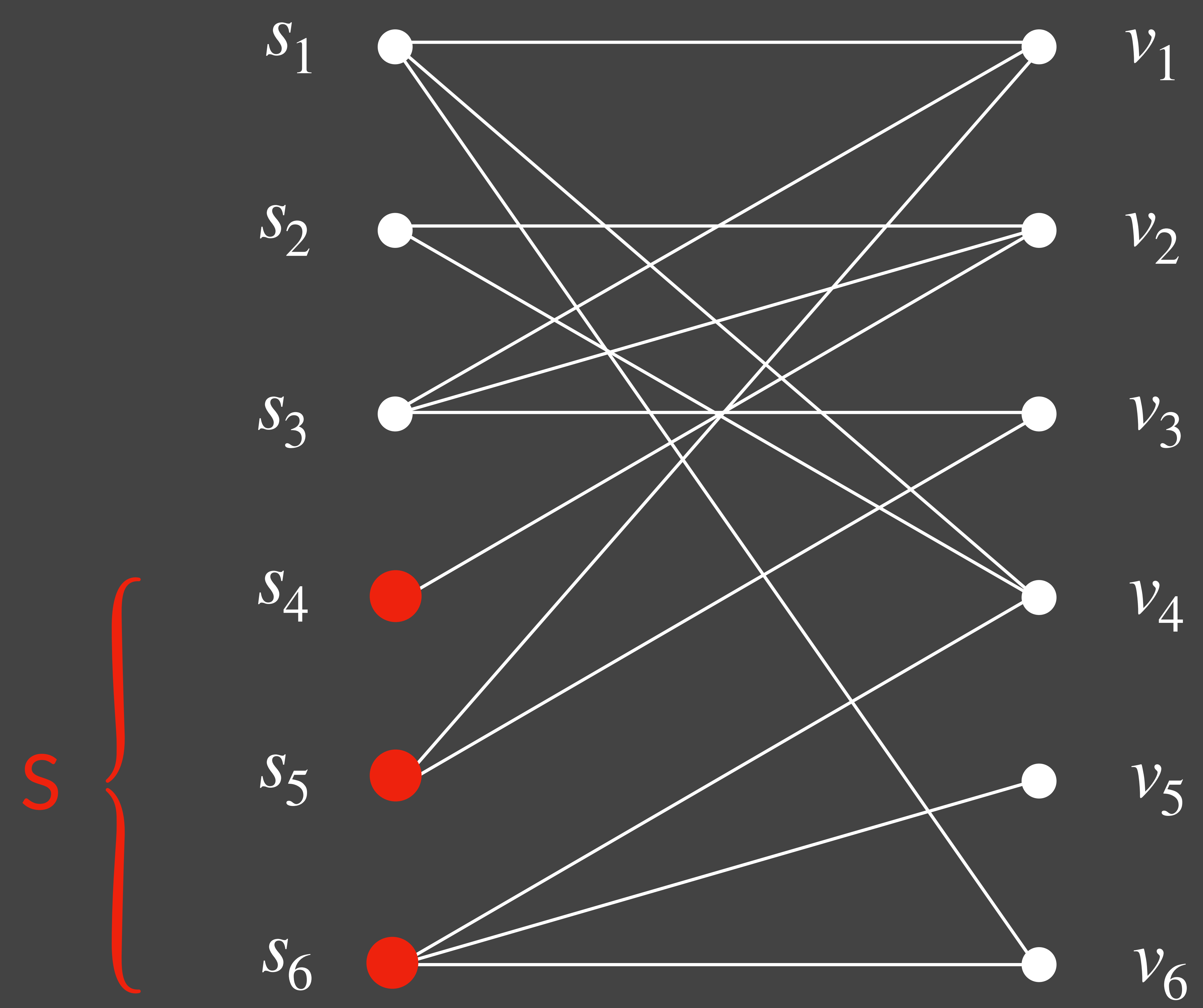

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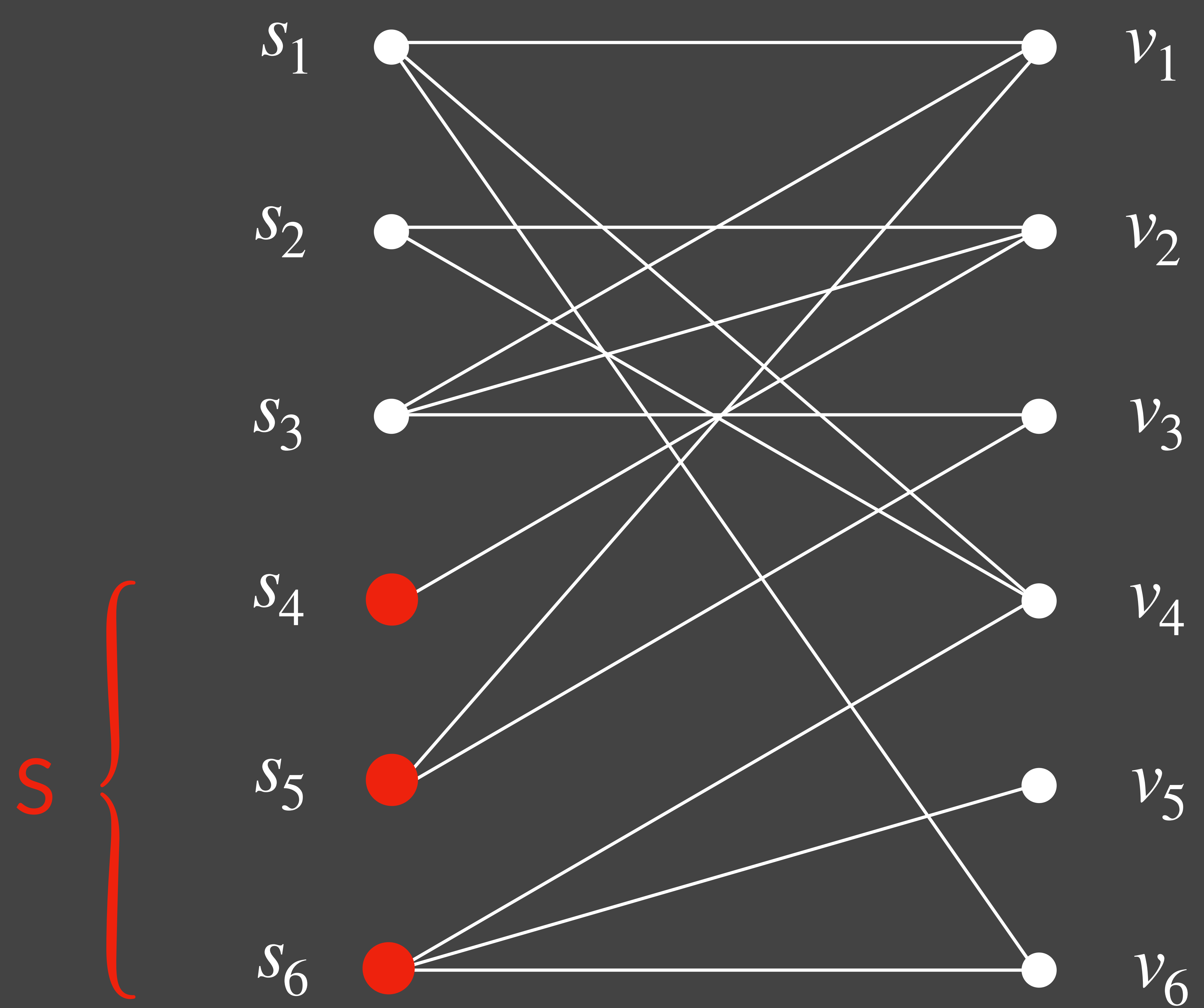

$$f_i(S) = \begin{cases} 1 & \text{if } v_i \text{ covered by } S \\ 0 & \text{otherwise} \end{cases}$$

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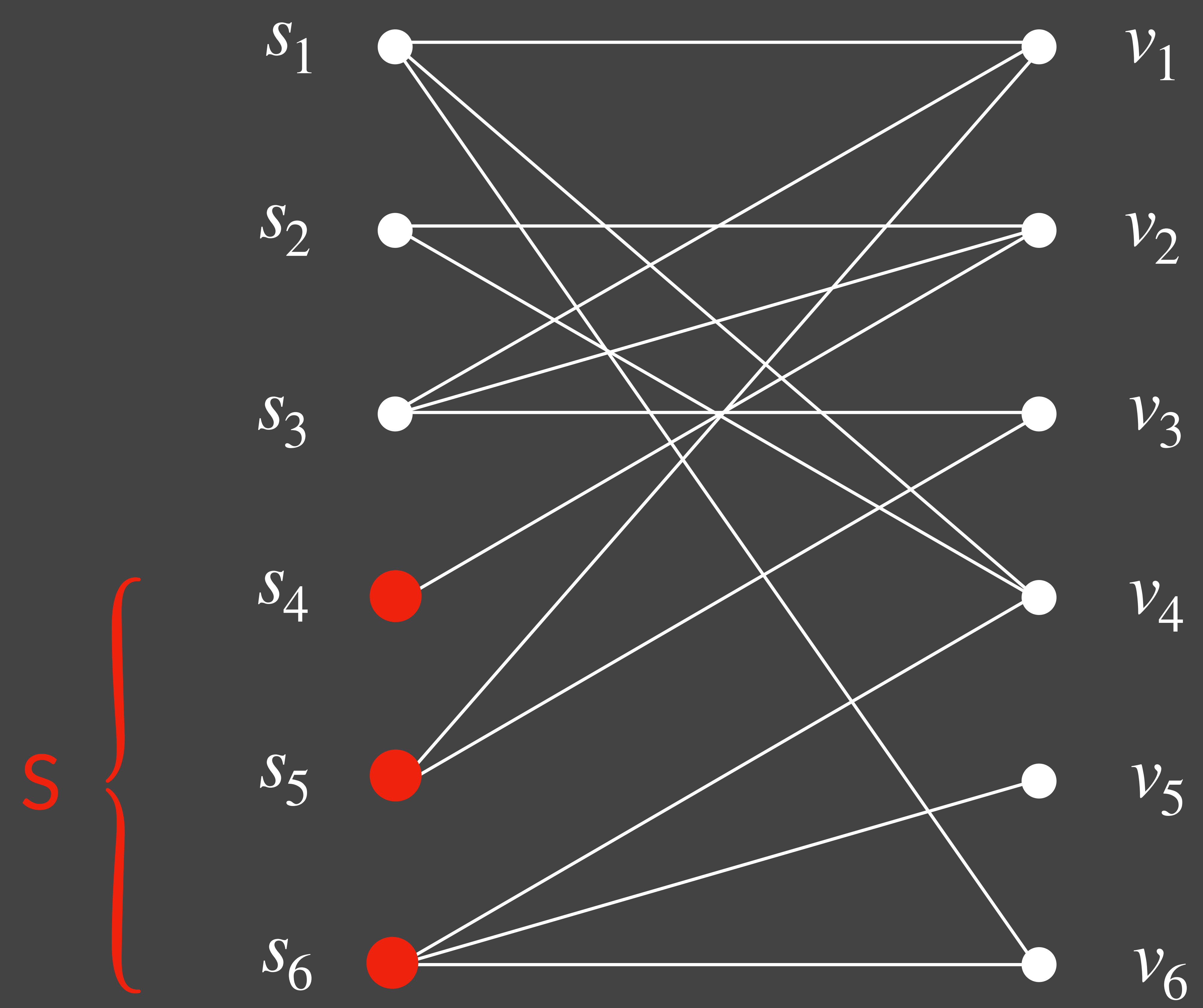

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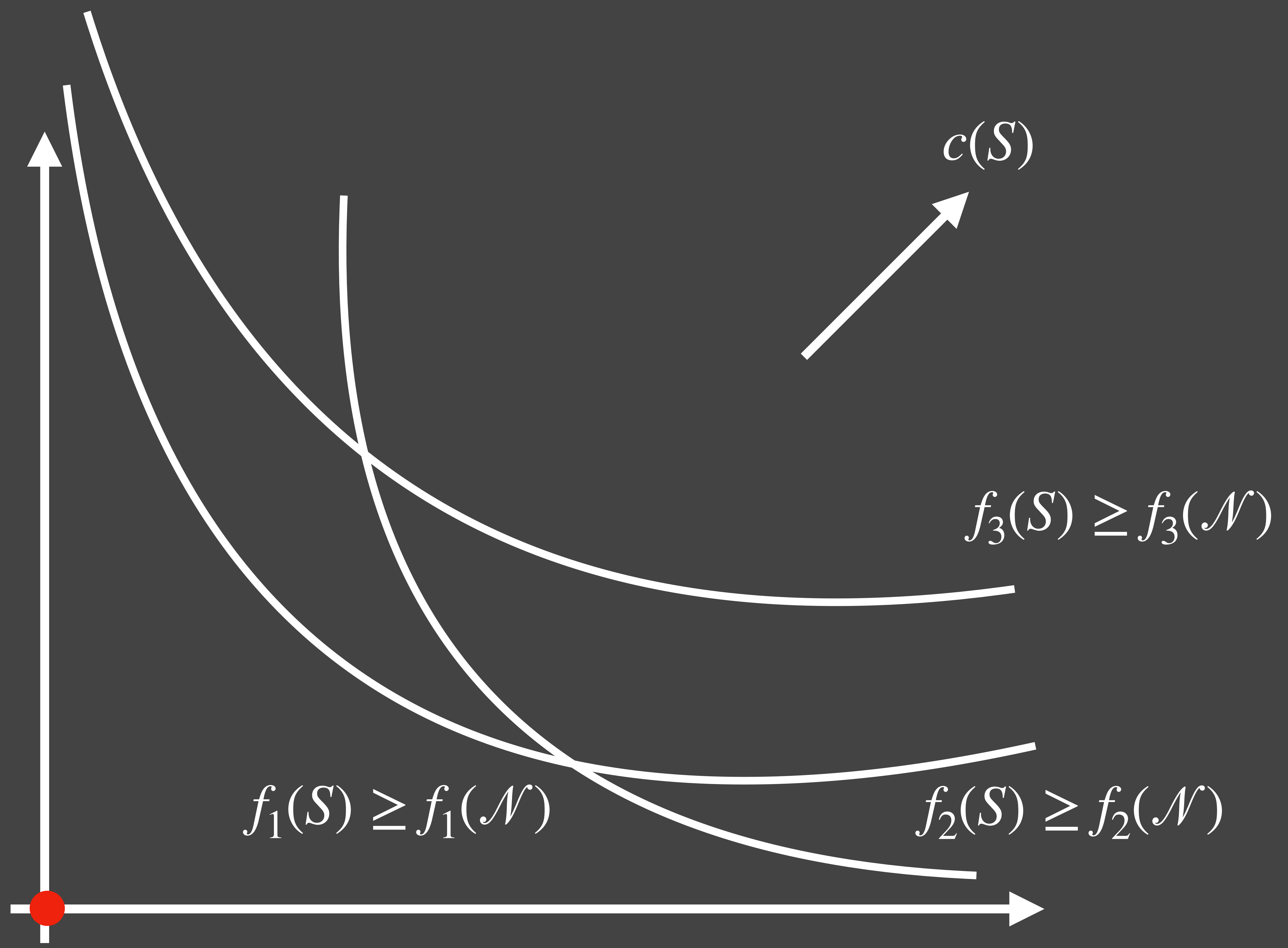
Online Submodular Cover Results



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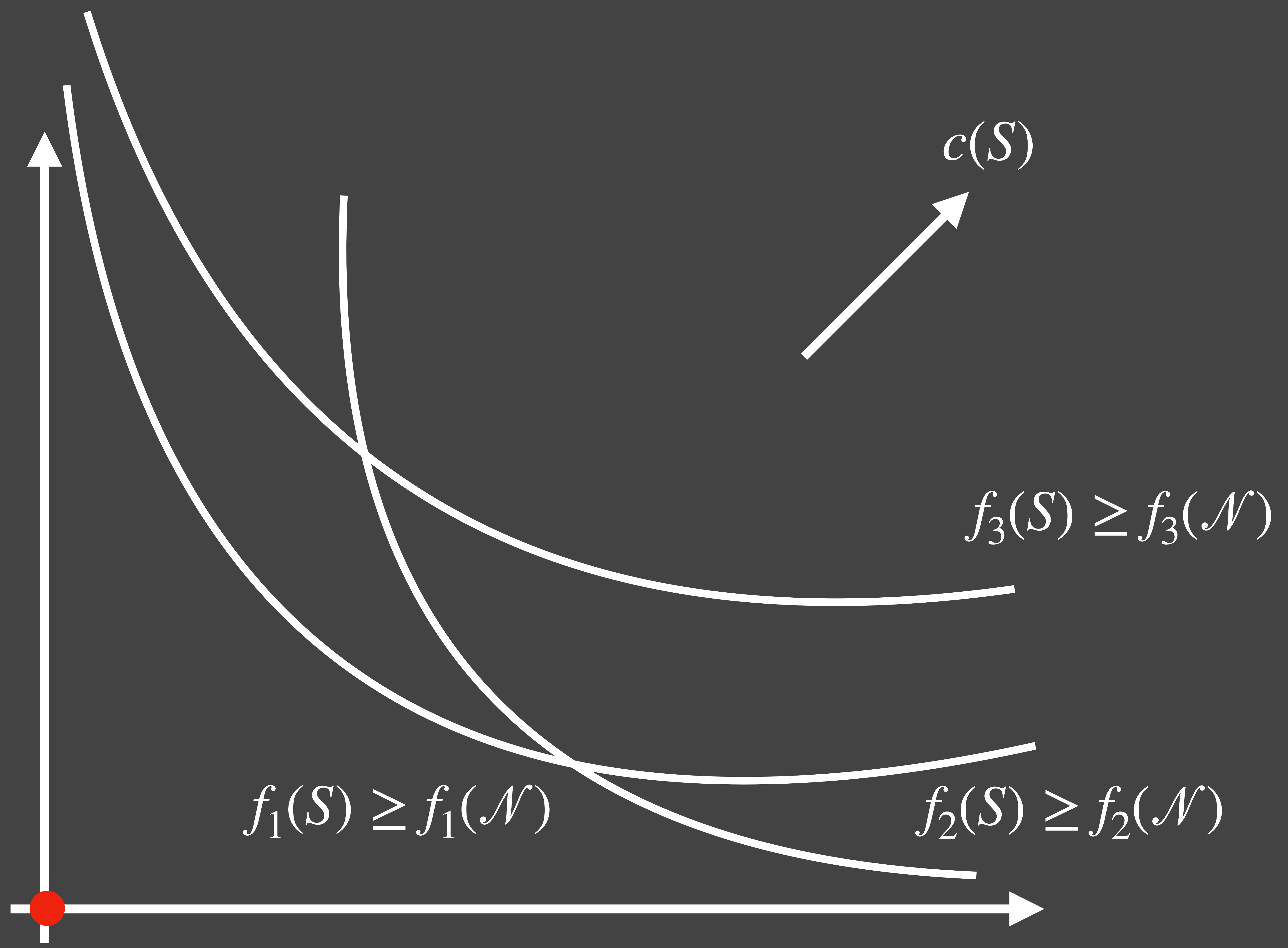
Online Submodular Cover Results



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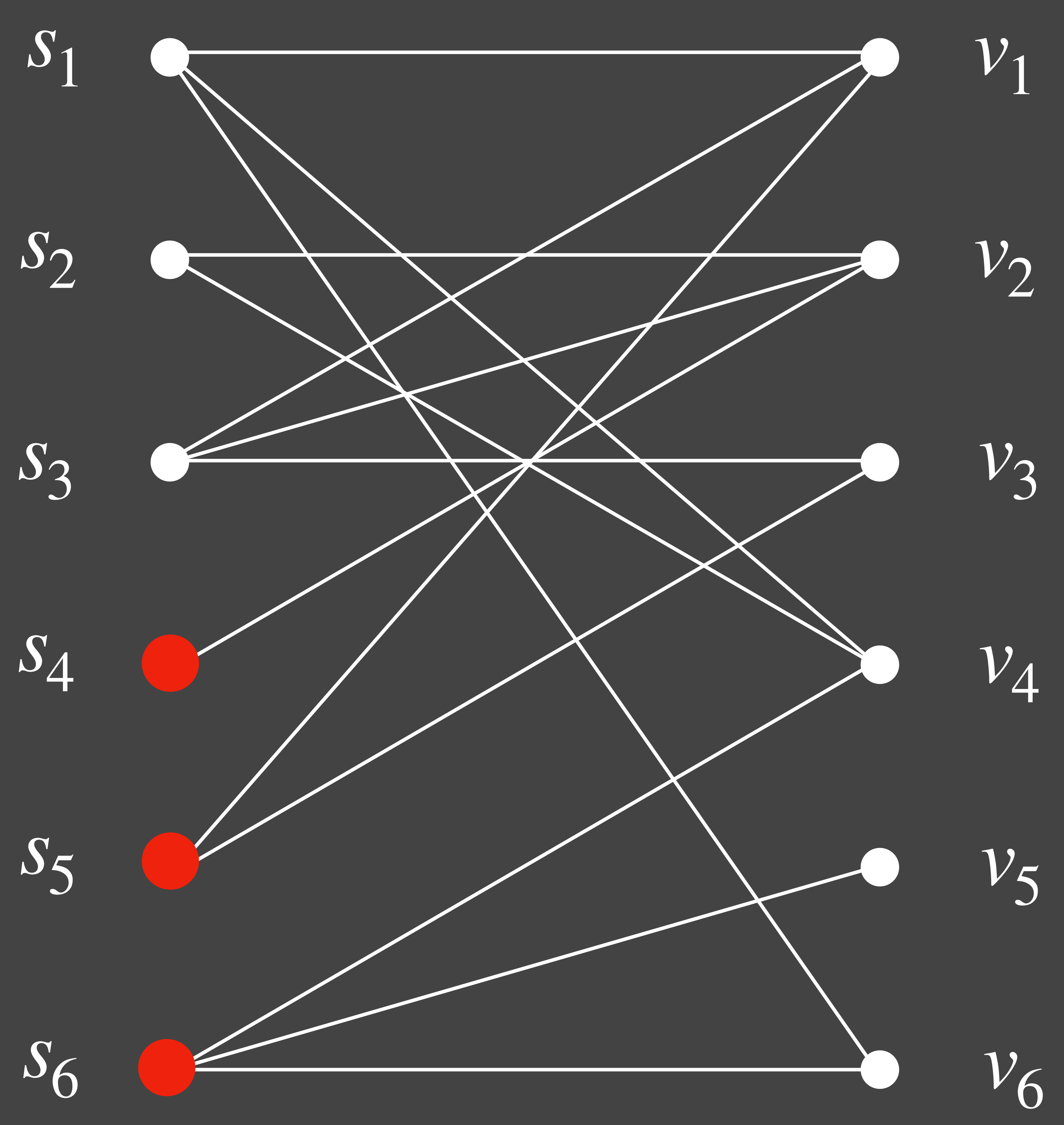
Theorem [Gupta L. SODA20]:

*There is a **randomized poly time** algo for **Online Submod Cover** with expected competitive ratio:*

$$O(\log m \cdot \log F(\mathcal{N})).$$

Special Case: Online Set Cover



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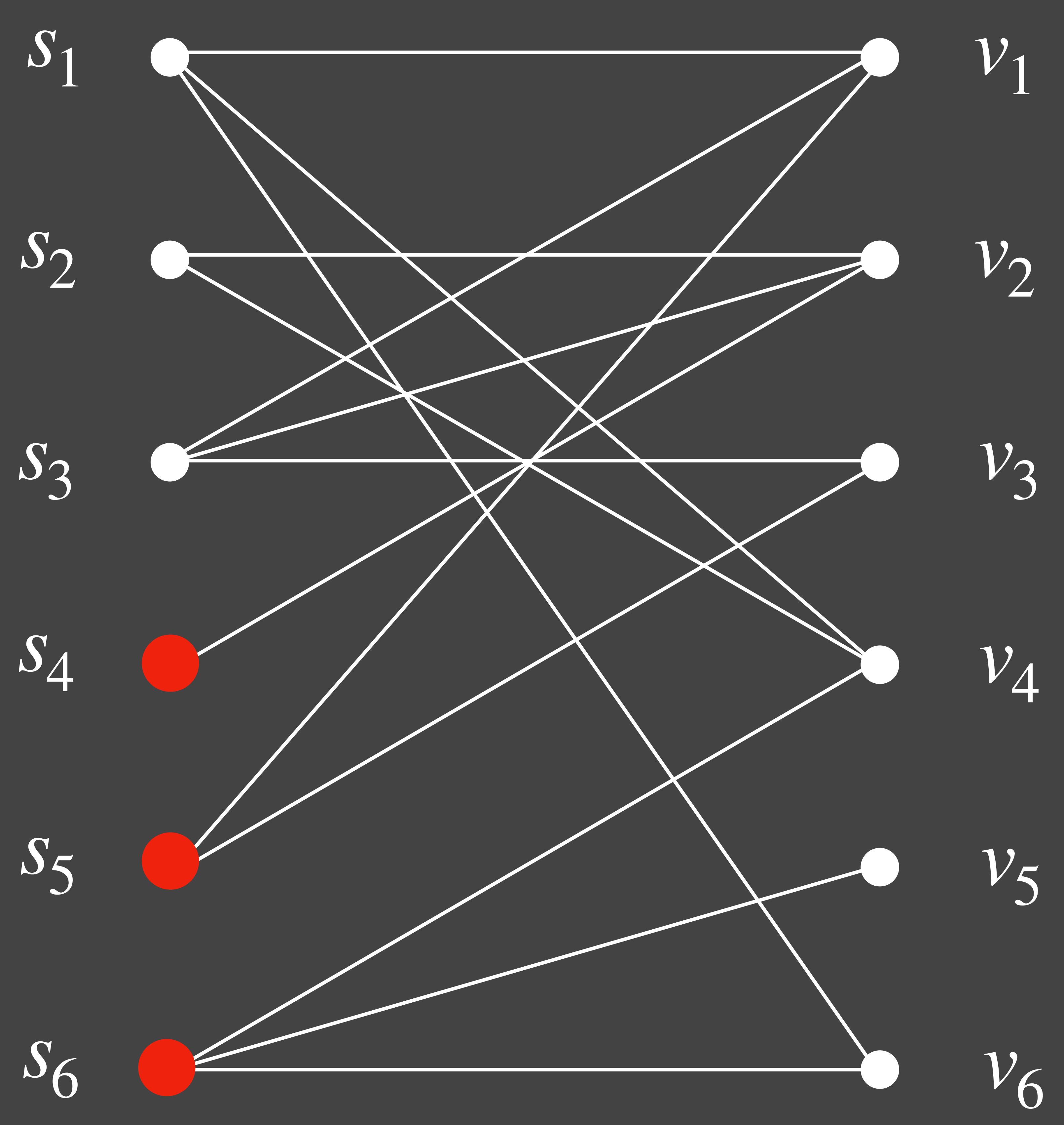

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Theorem (Online):

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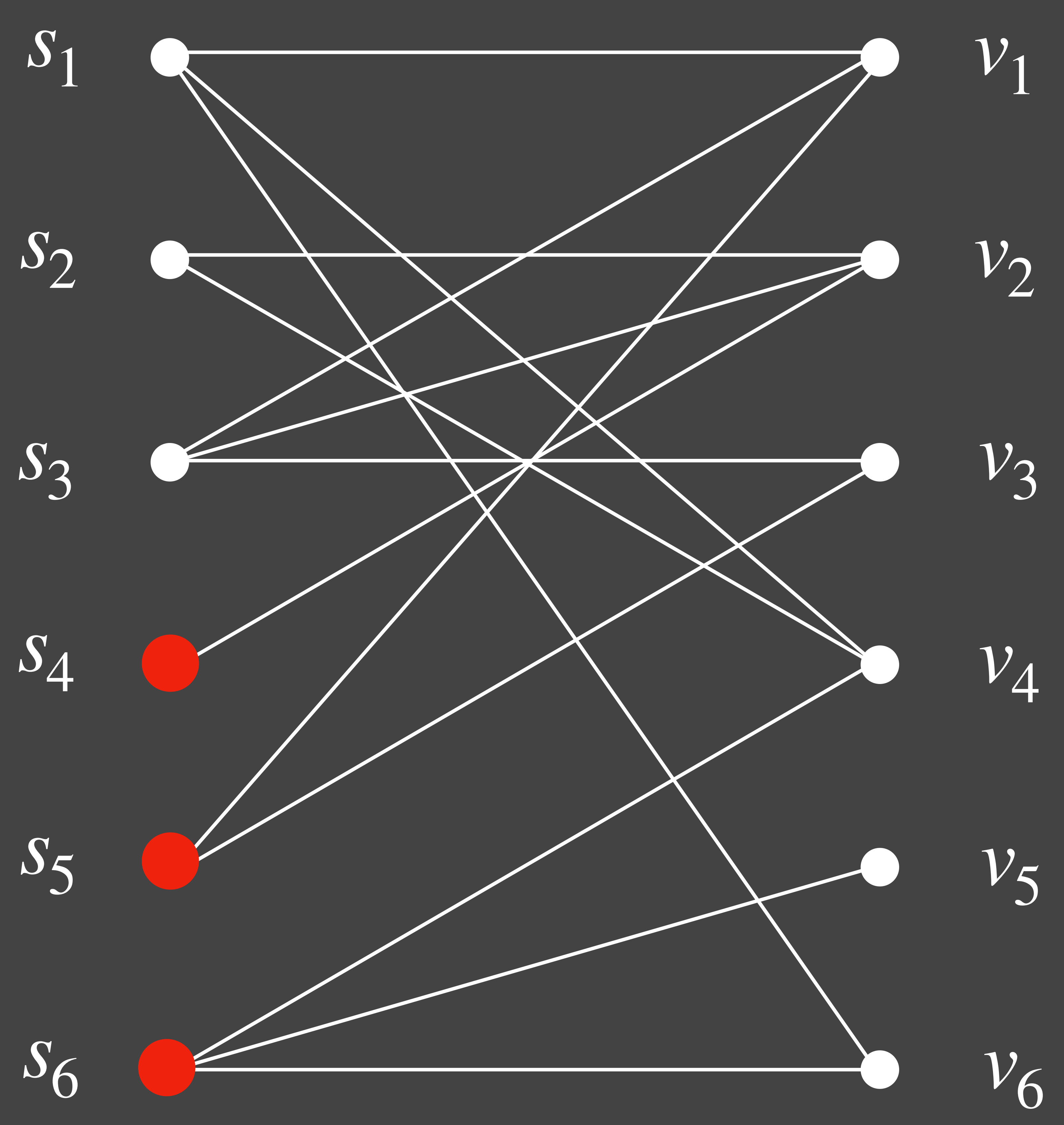

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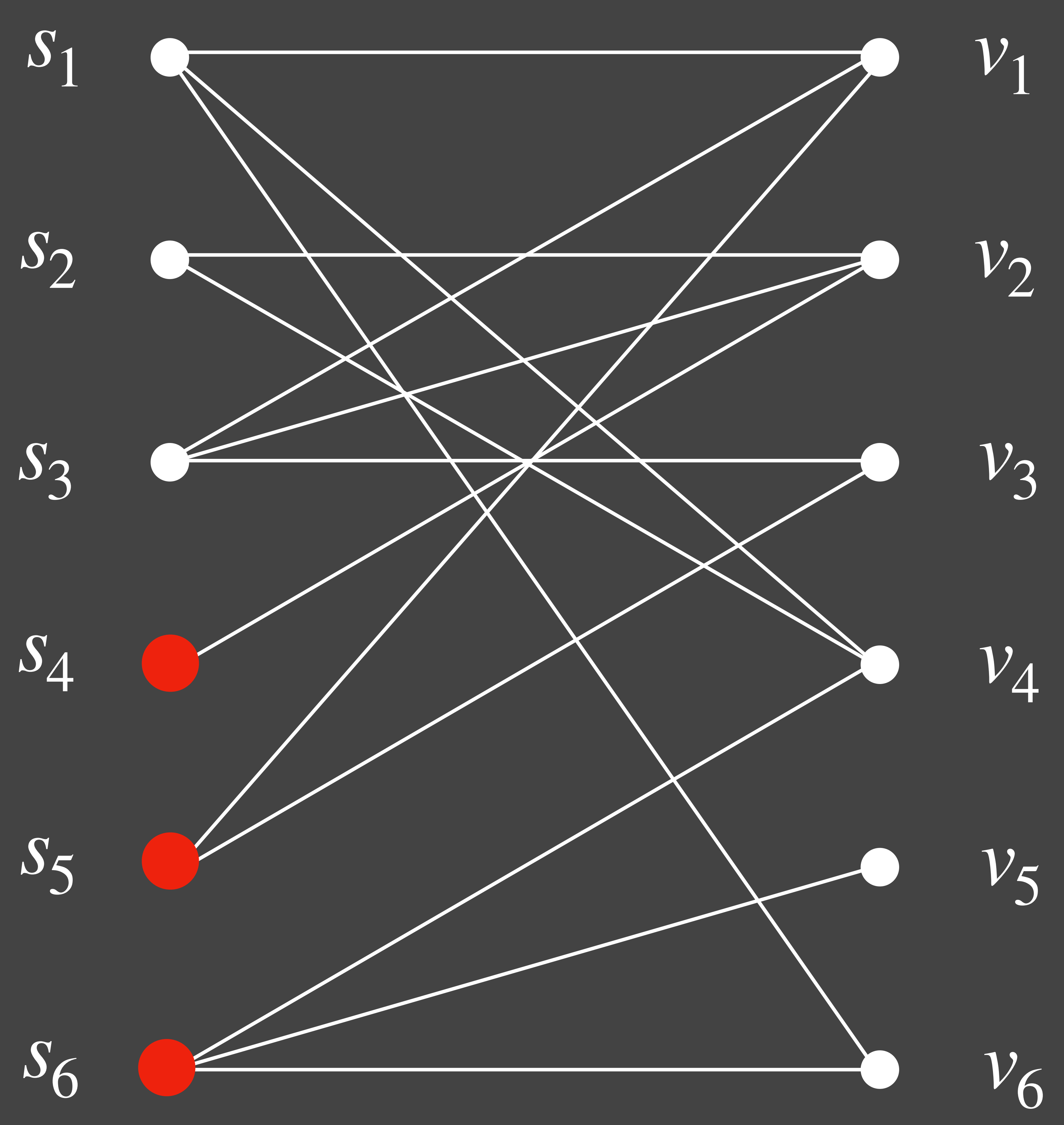

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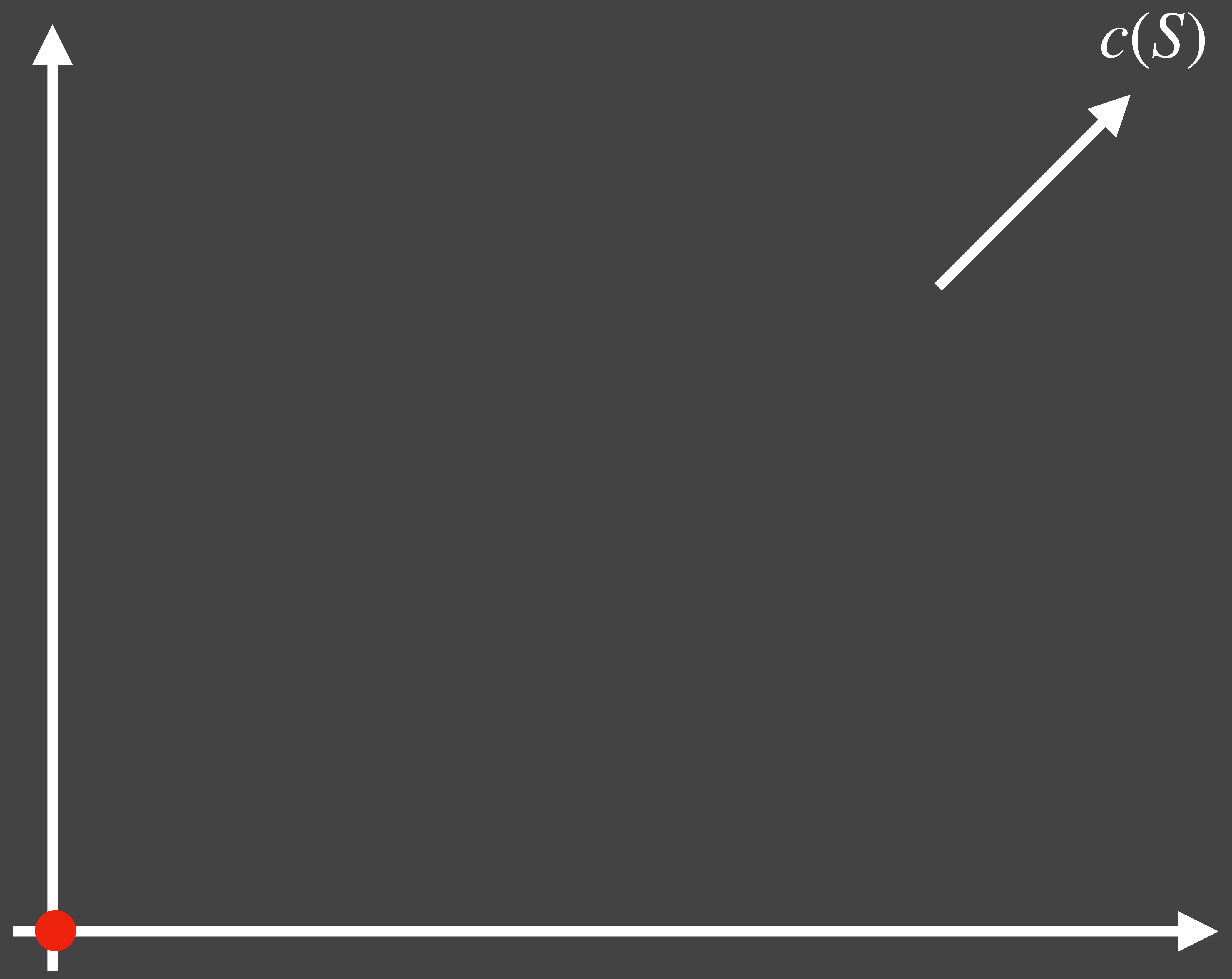
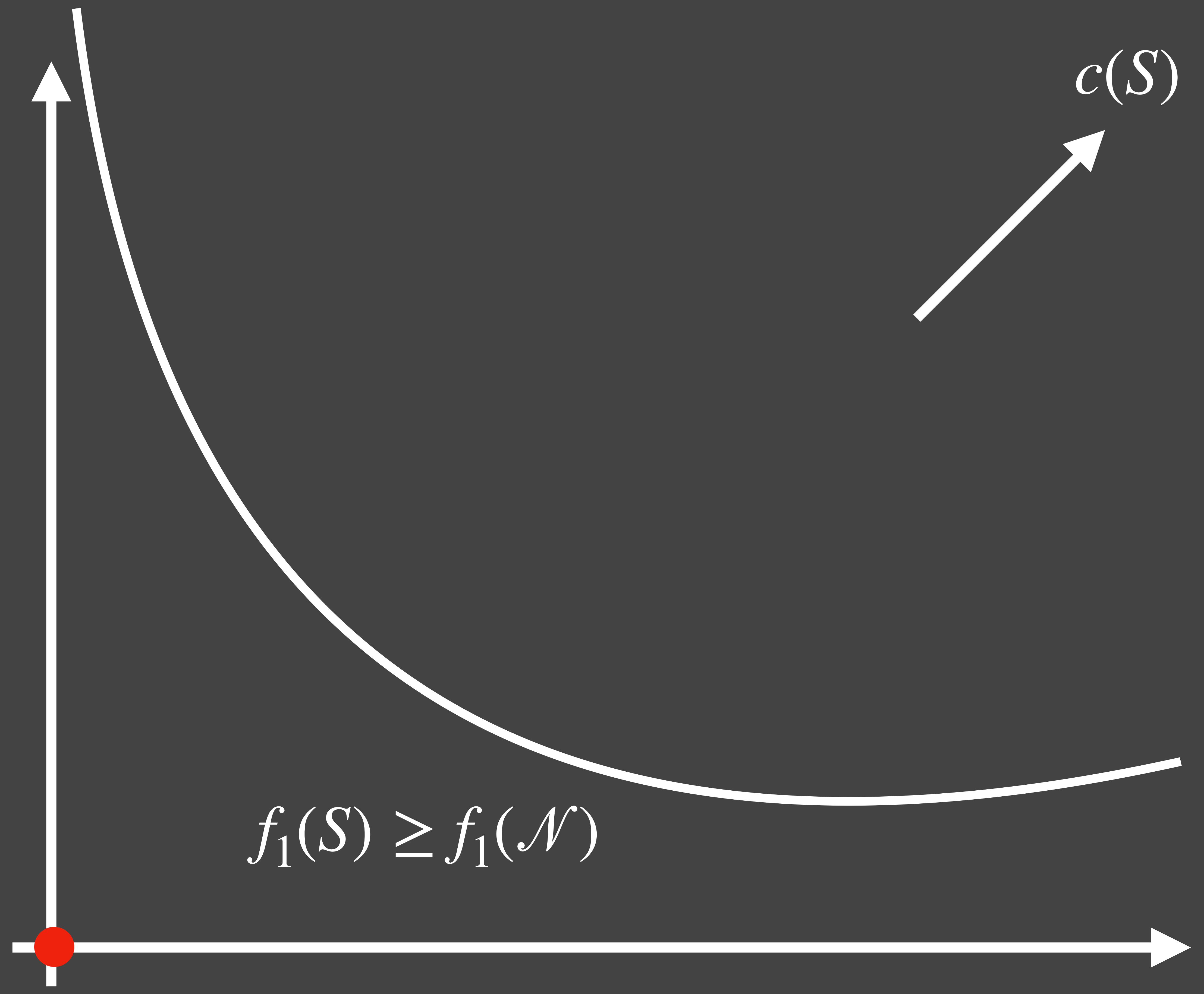
Generalizes [Alon+ 03]

Fully-Dynamic Submodular Cover



$$F = \sum_i f_i$$

Fully-Dynamic Submodular Cover



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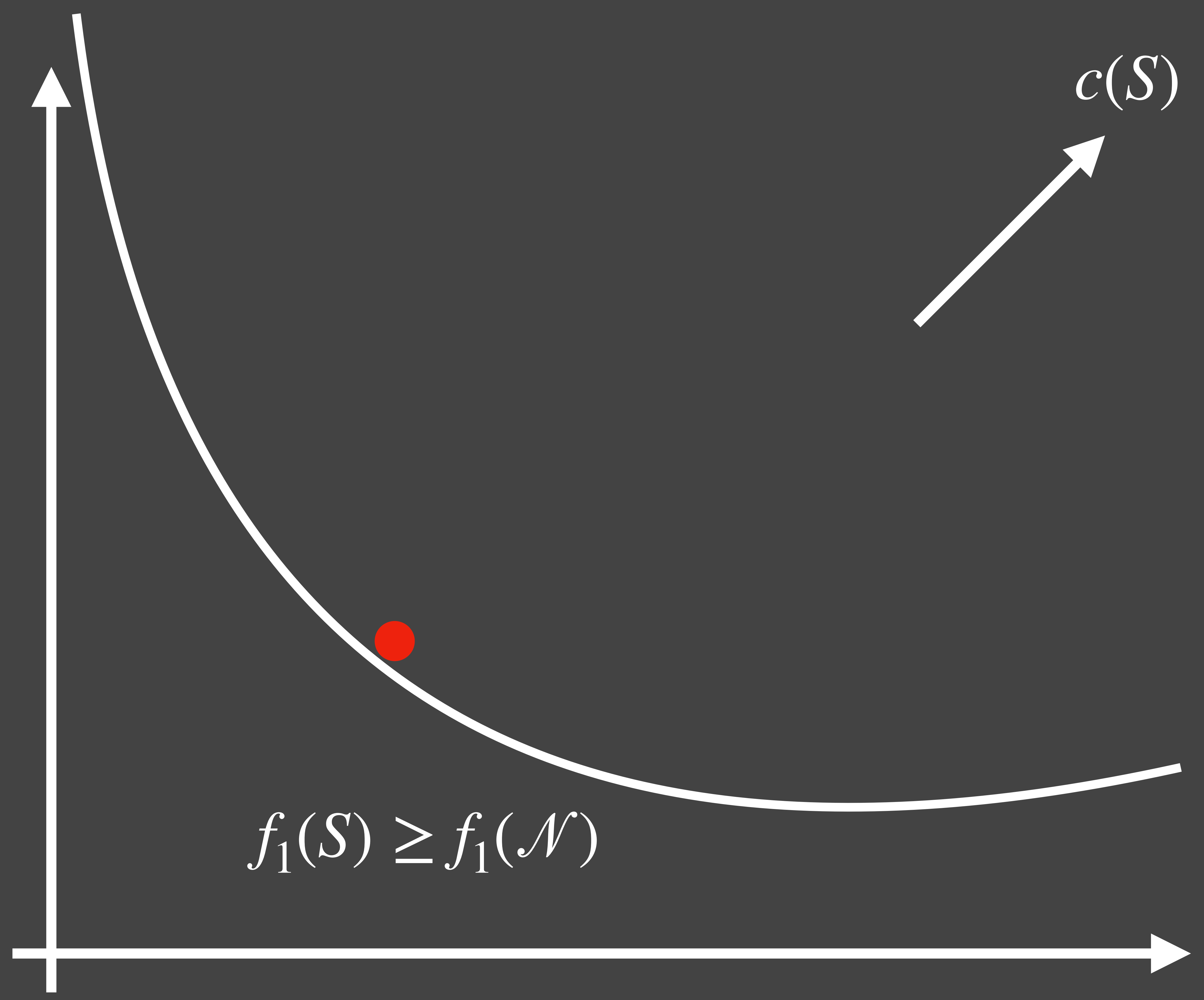

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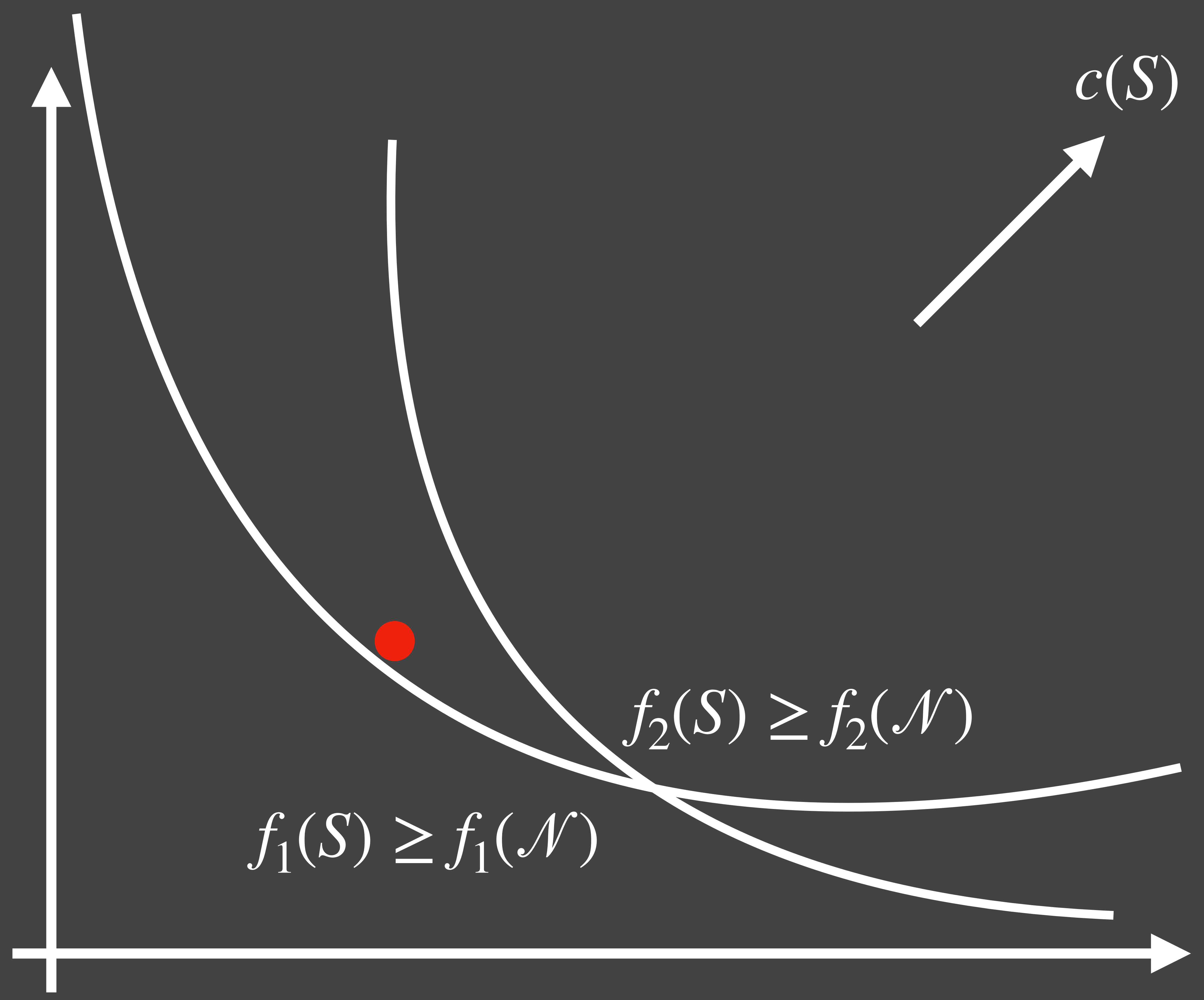

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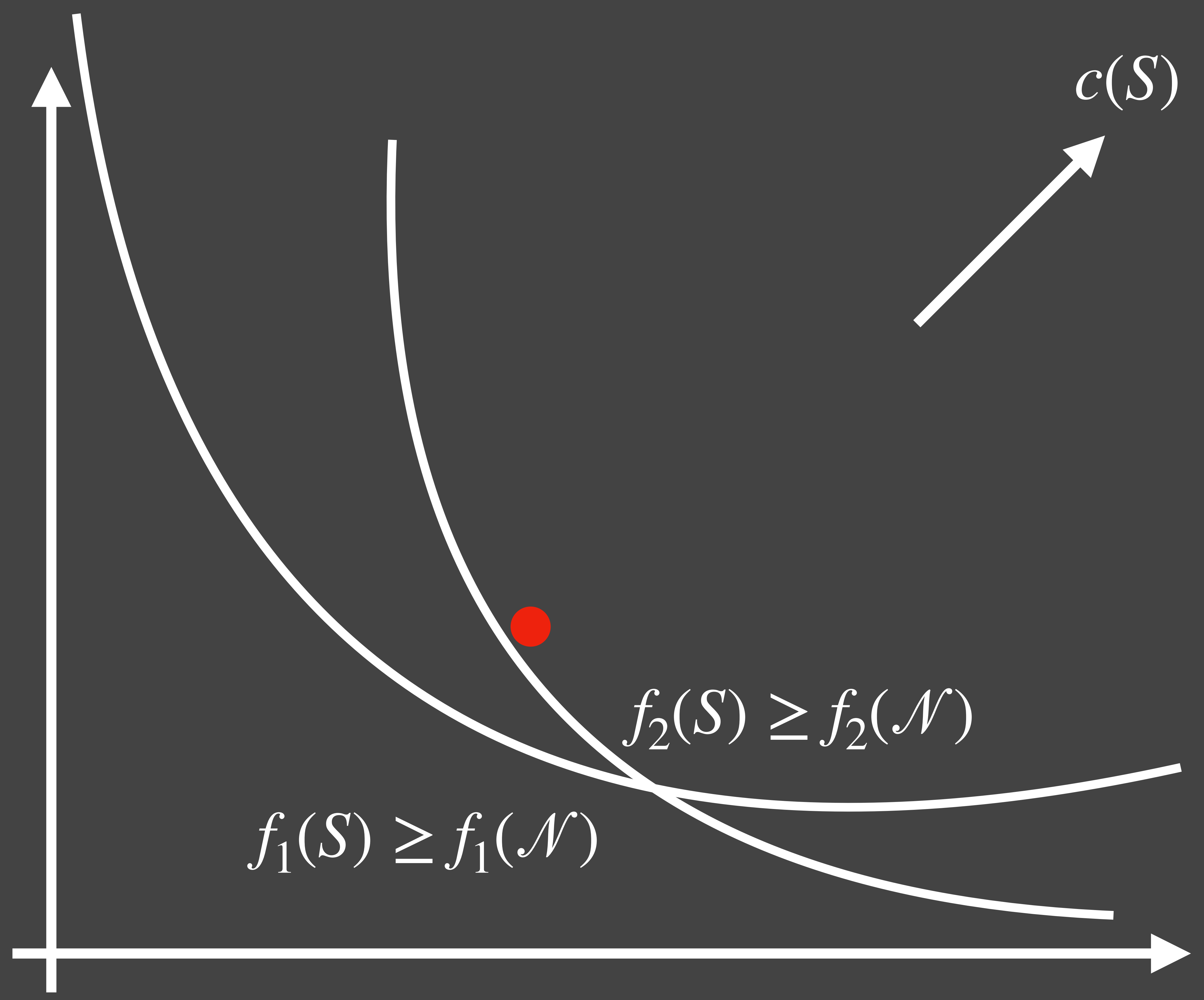

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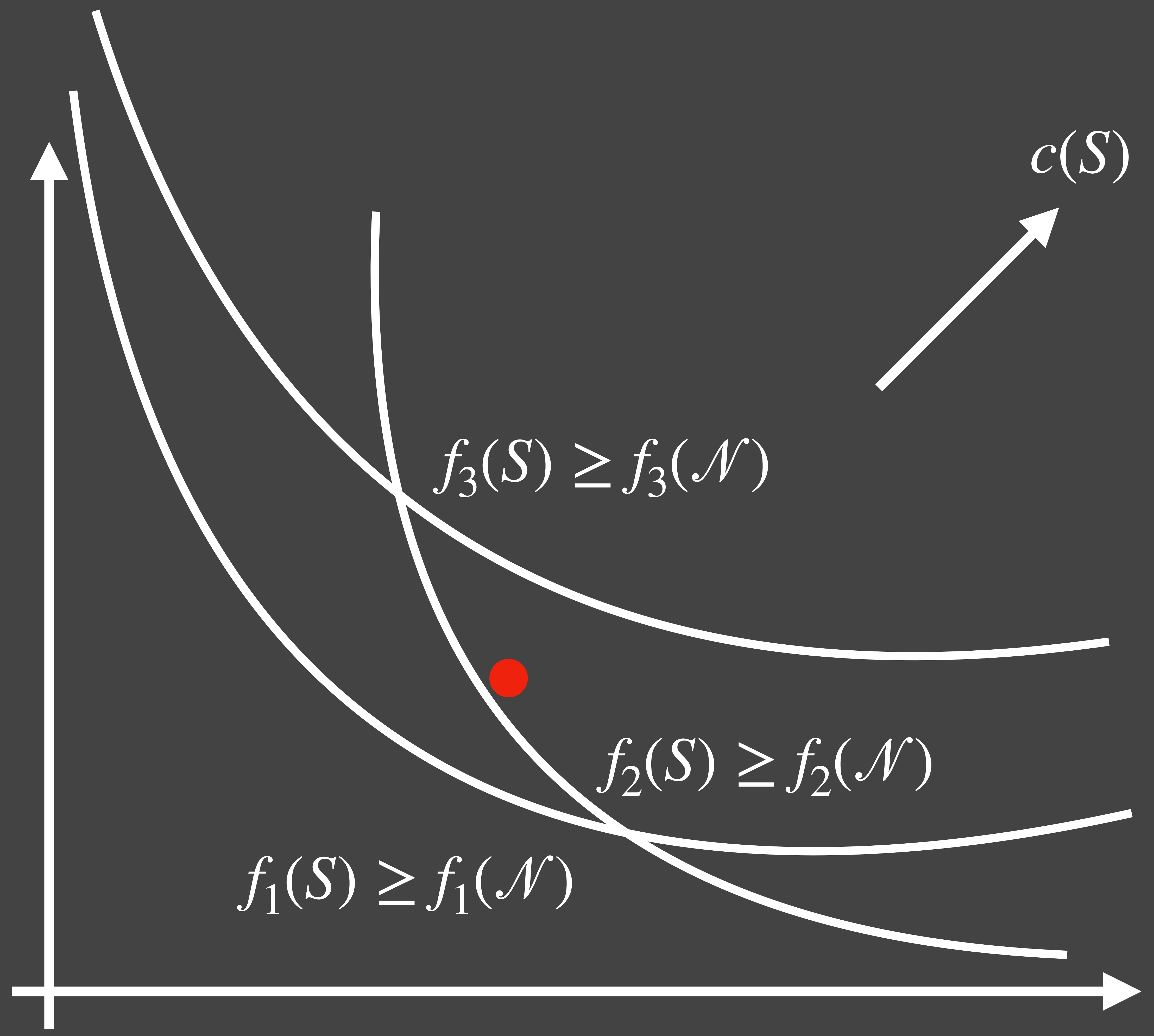

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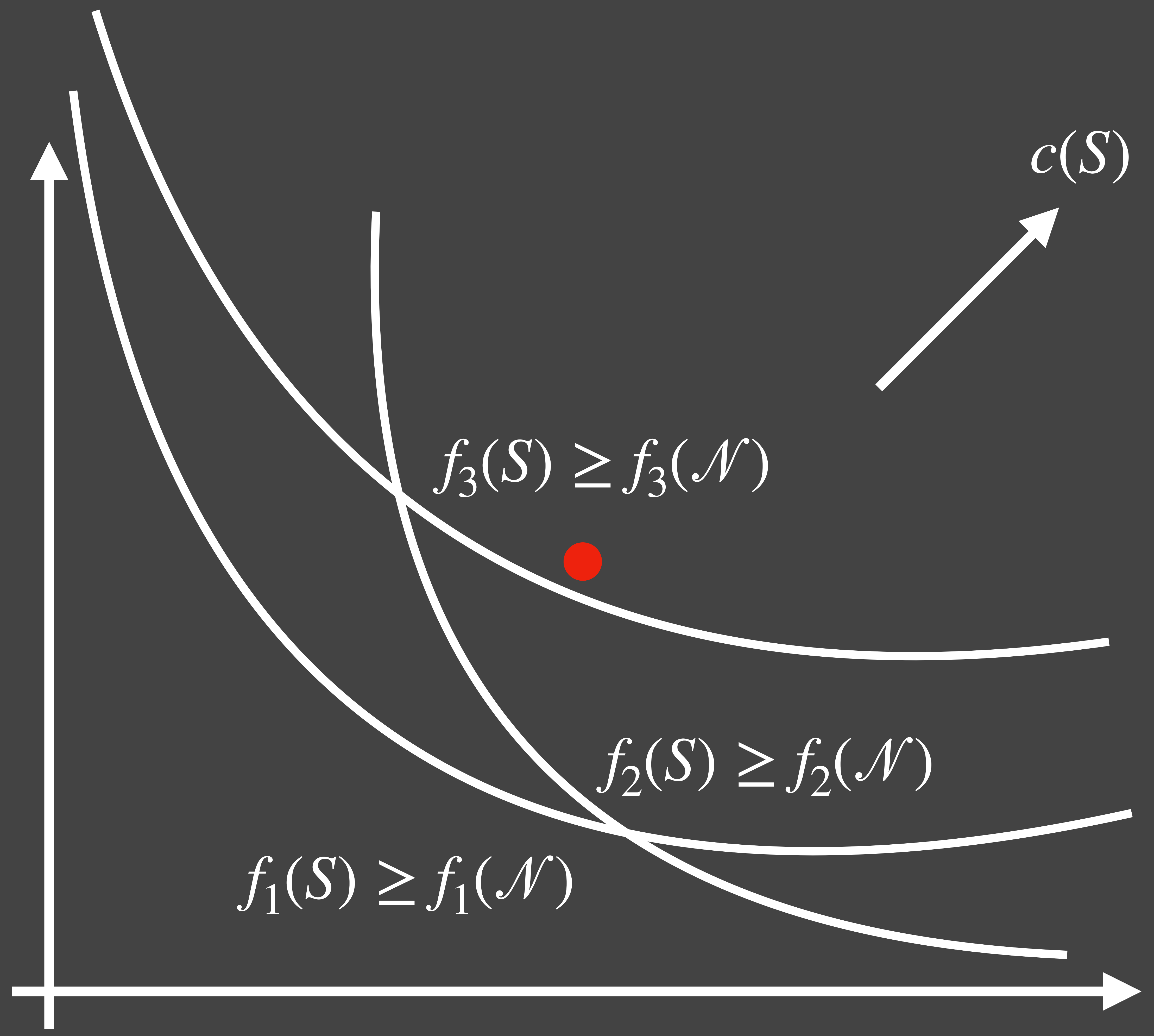

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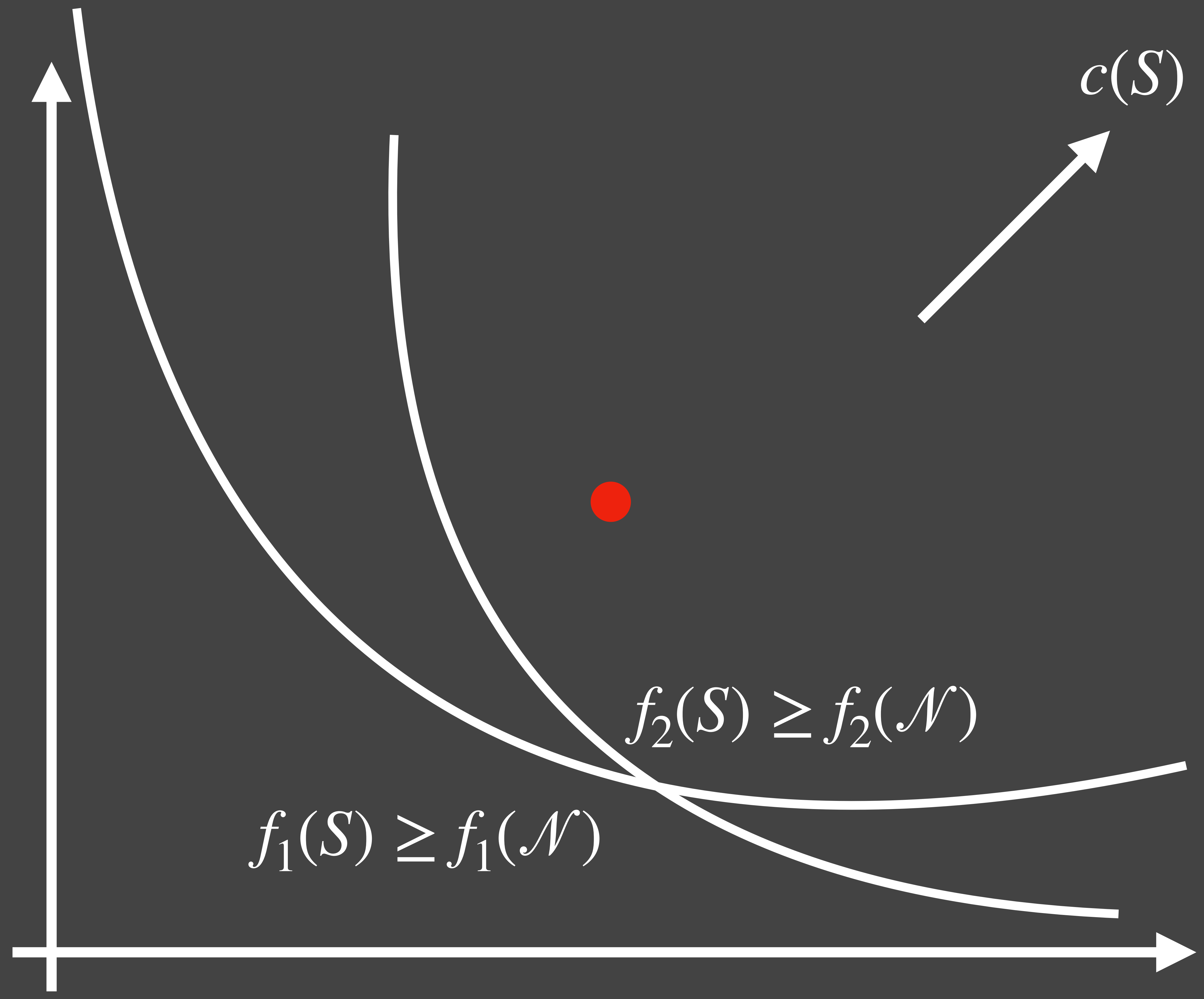

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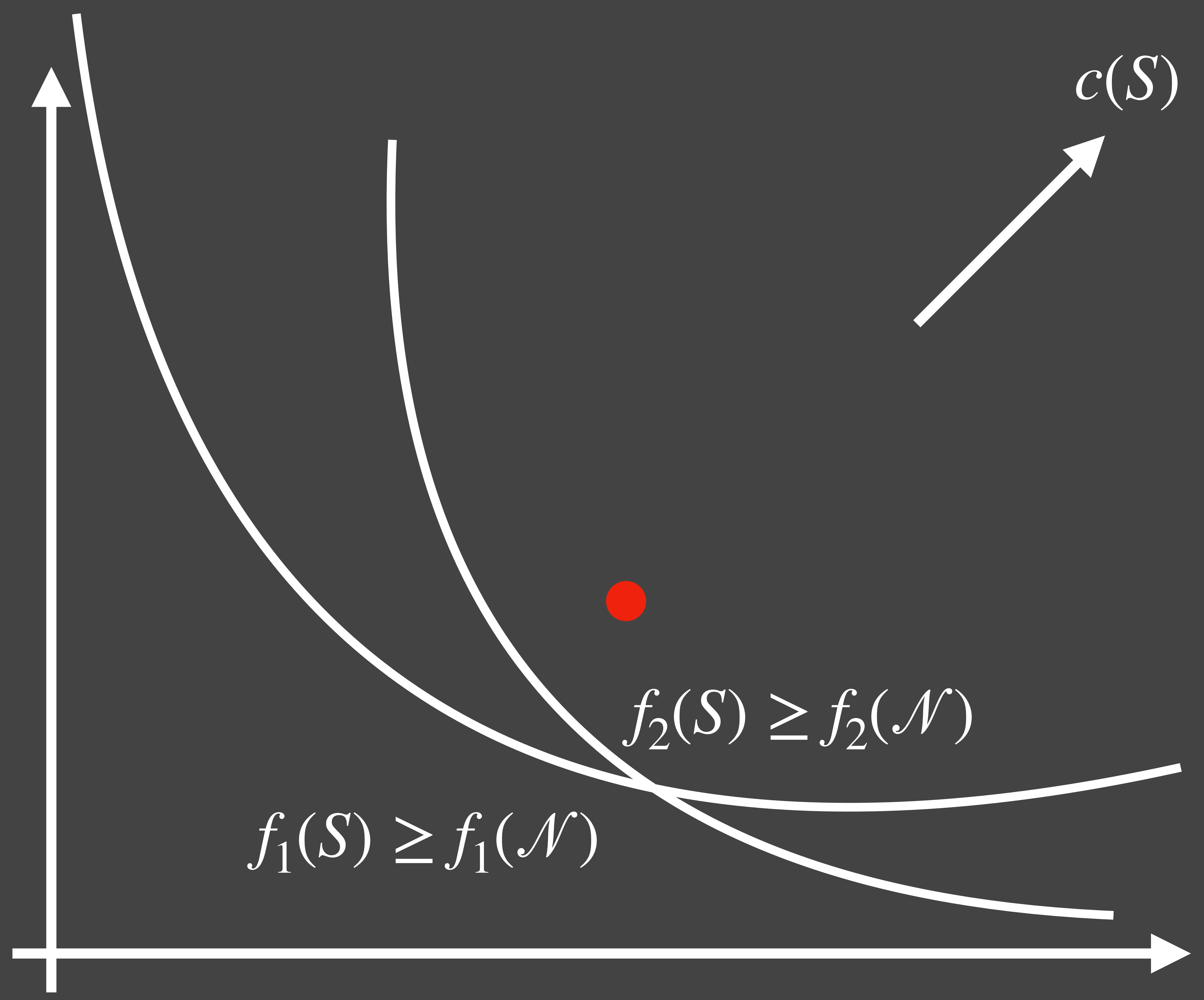

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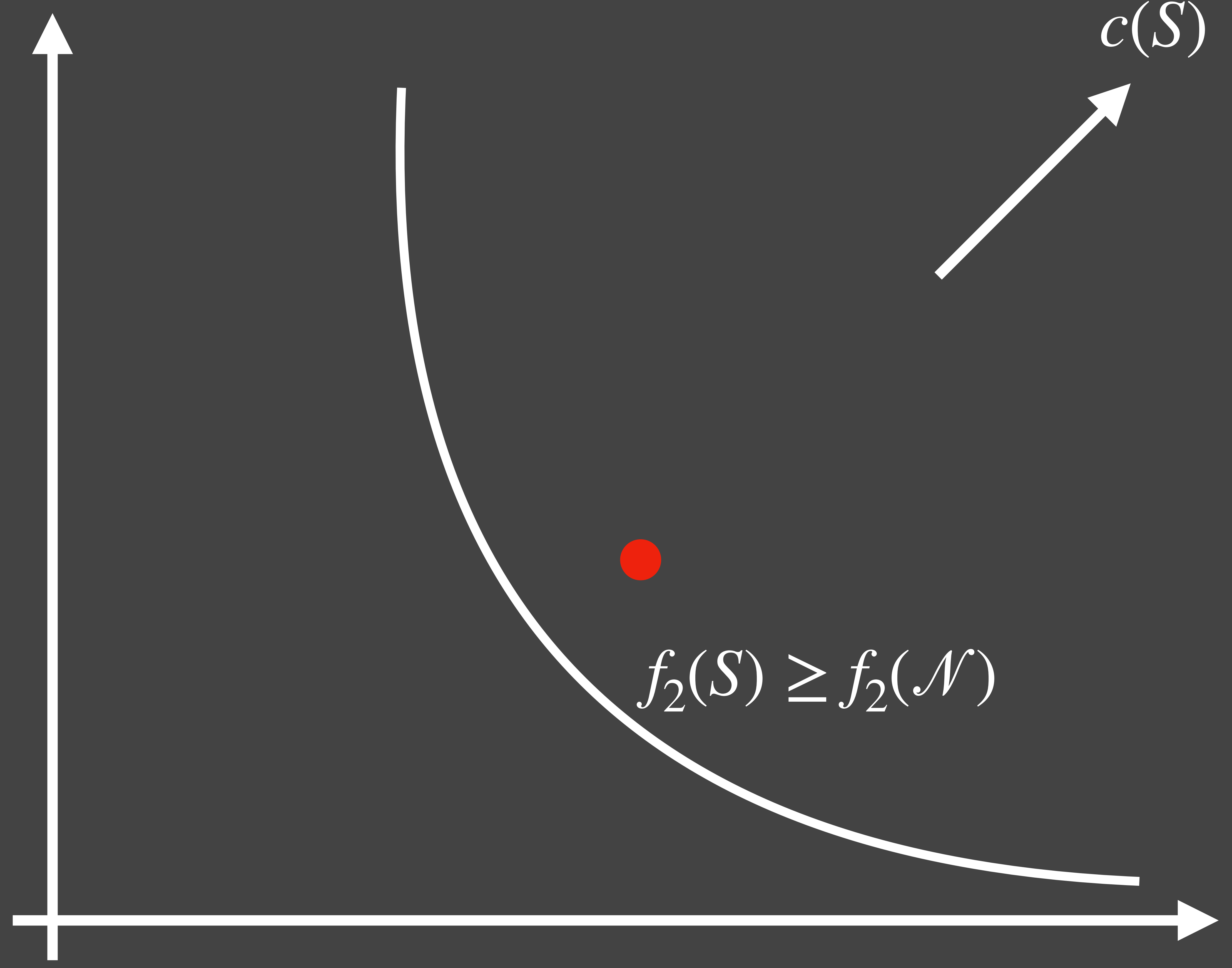

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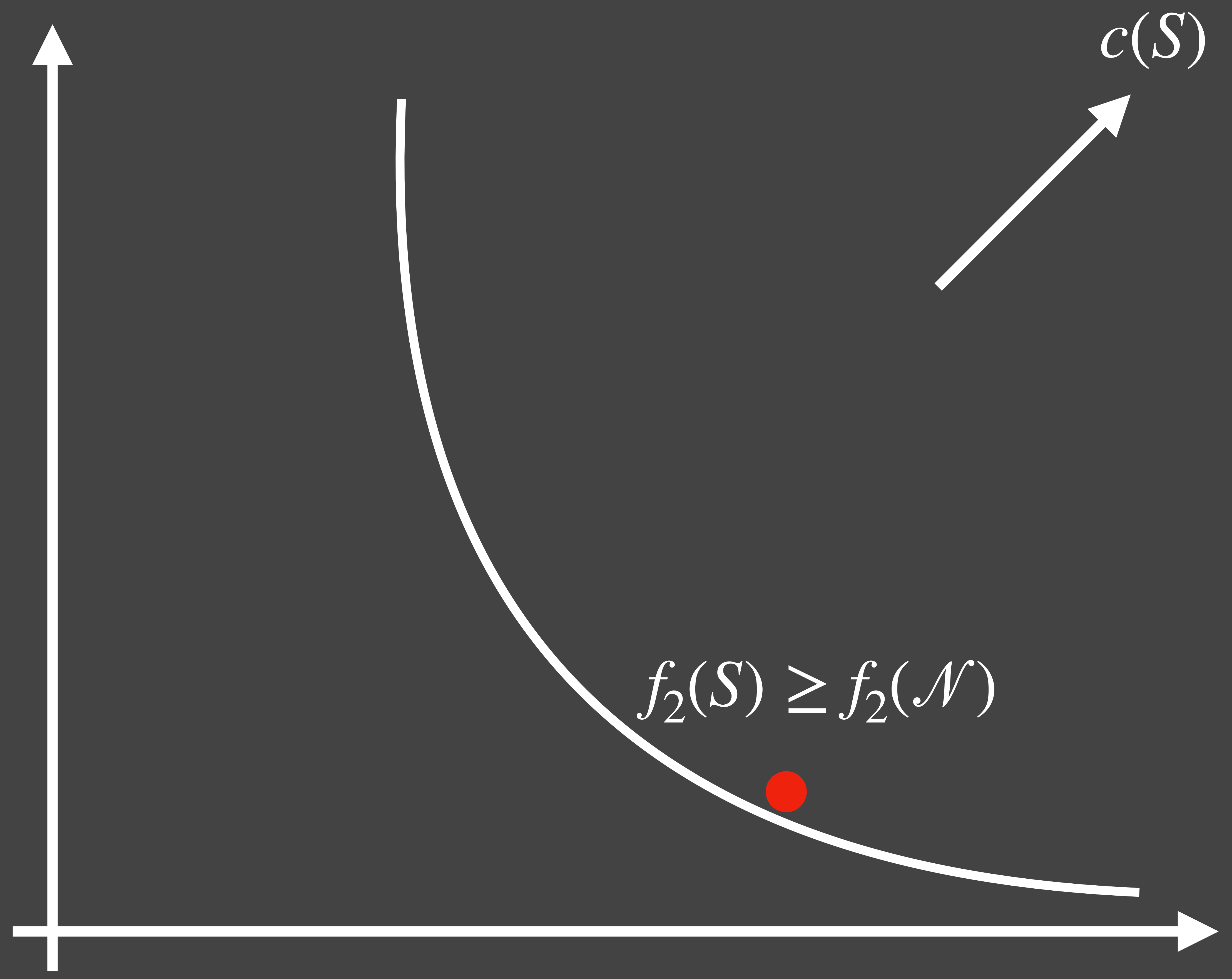

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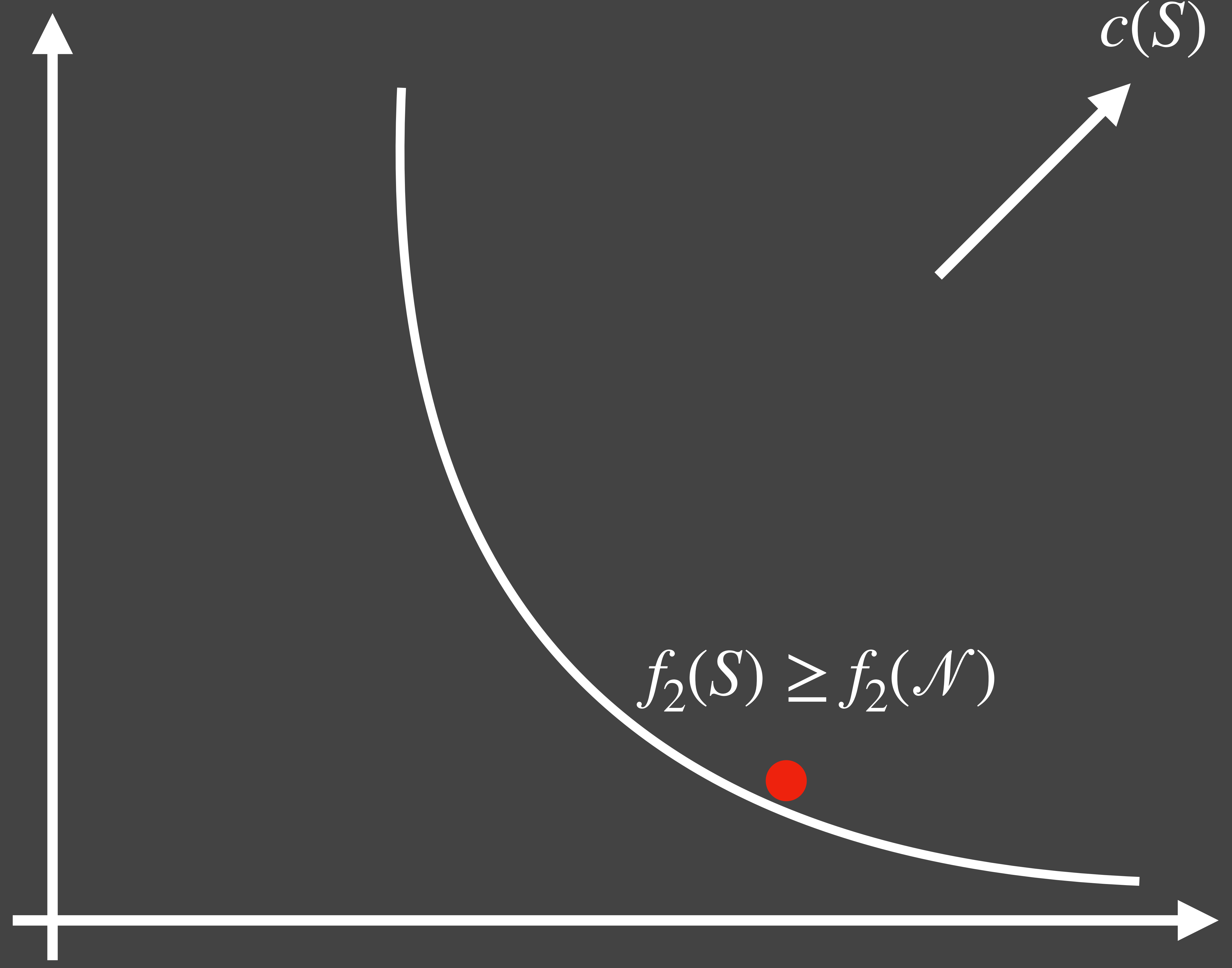

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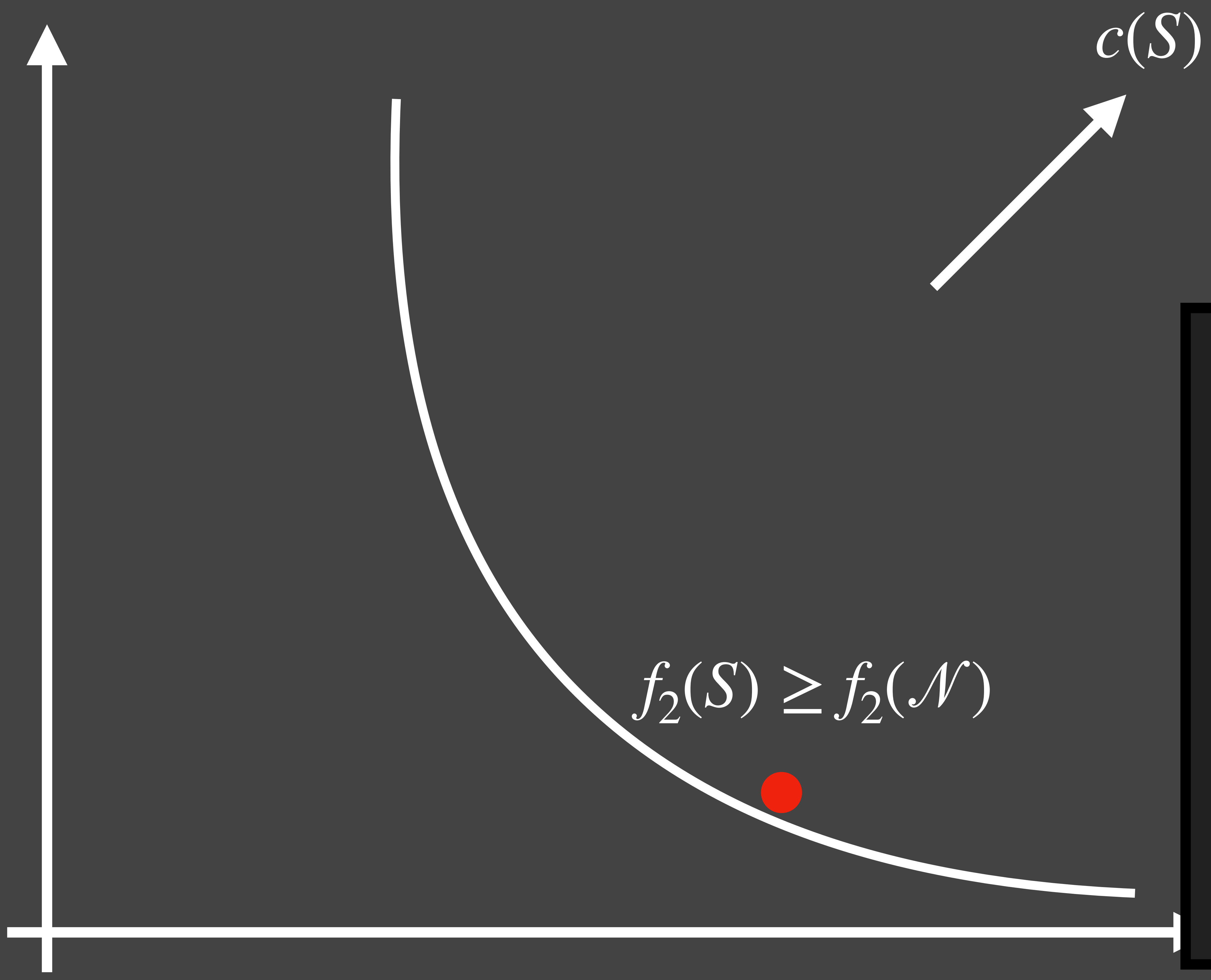
Definition: Recourse

$$\sum_t |S^t \triangle S^{t-1}|$$

Fully-Dynamic Submodular Cover



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Definition: Recourse

$$\sum_t |S^t \triangle S^{t-1}|$$

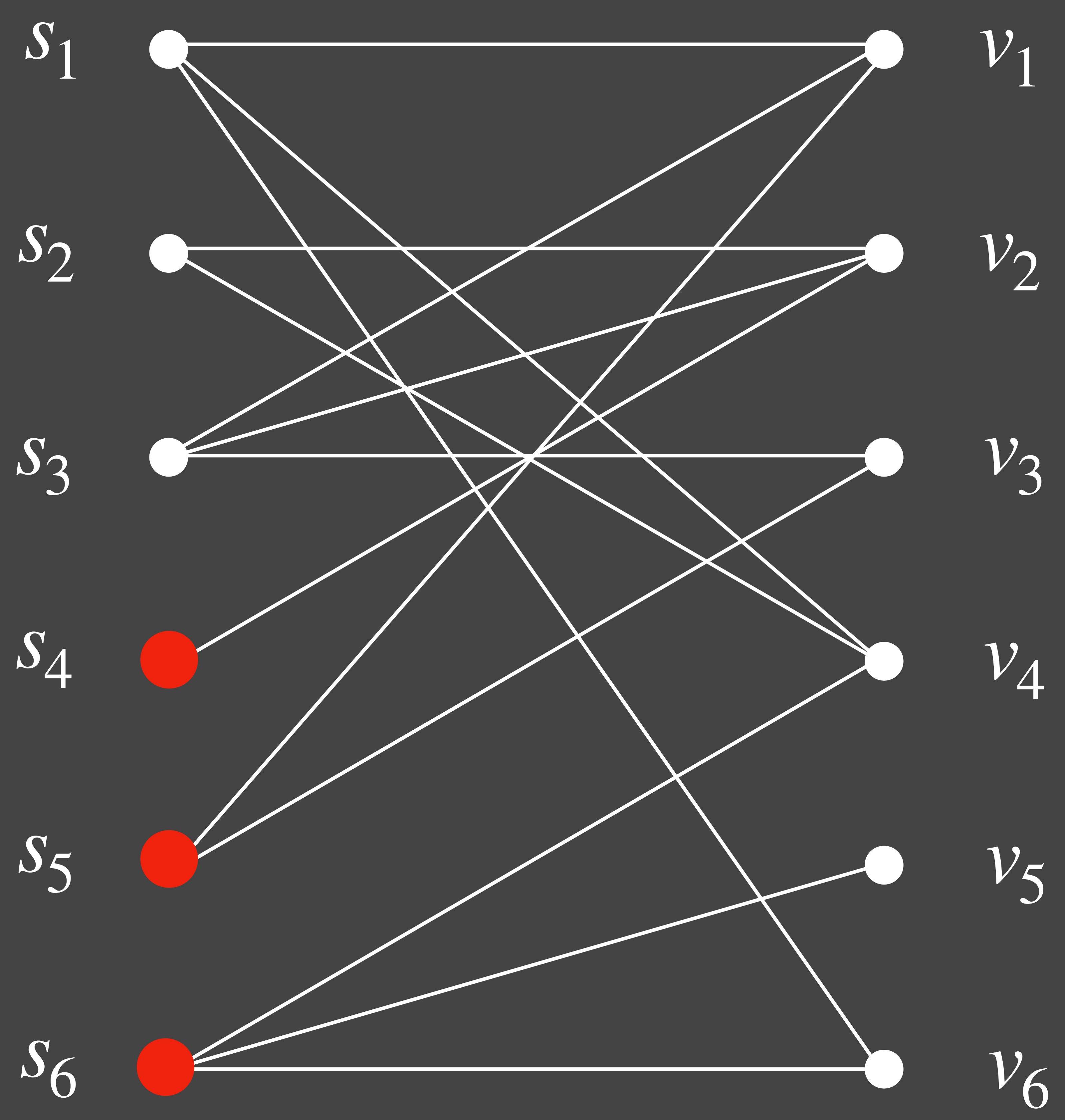
Theorem [Gupta **L.** FOCS 20]:

*There is a **deterministic poly time** algorithm for **Fully-Dynamic Submodular** Cover with:*

(i) *competitive ratio* $O(\log F(\mathcal{N}))$.

(ii) *average recourse* $\tilde{O}(f(\mathcal{N}))$.

Special Case: Dynamic Set Cover



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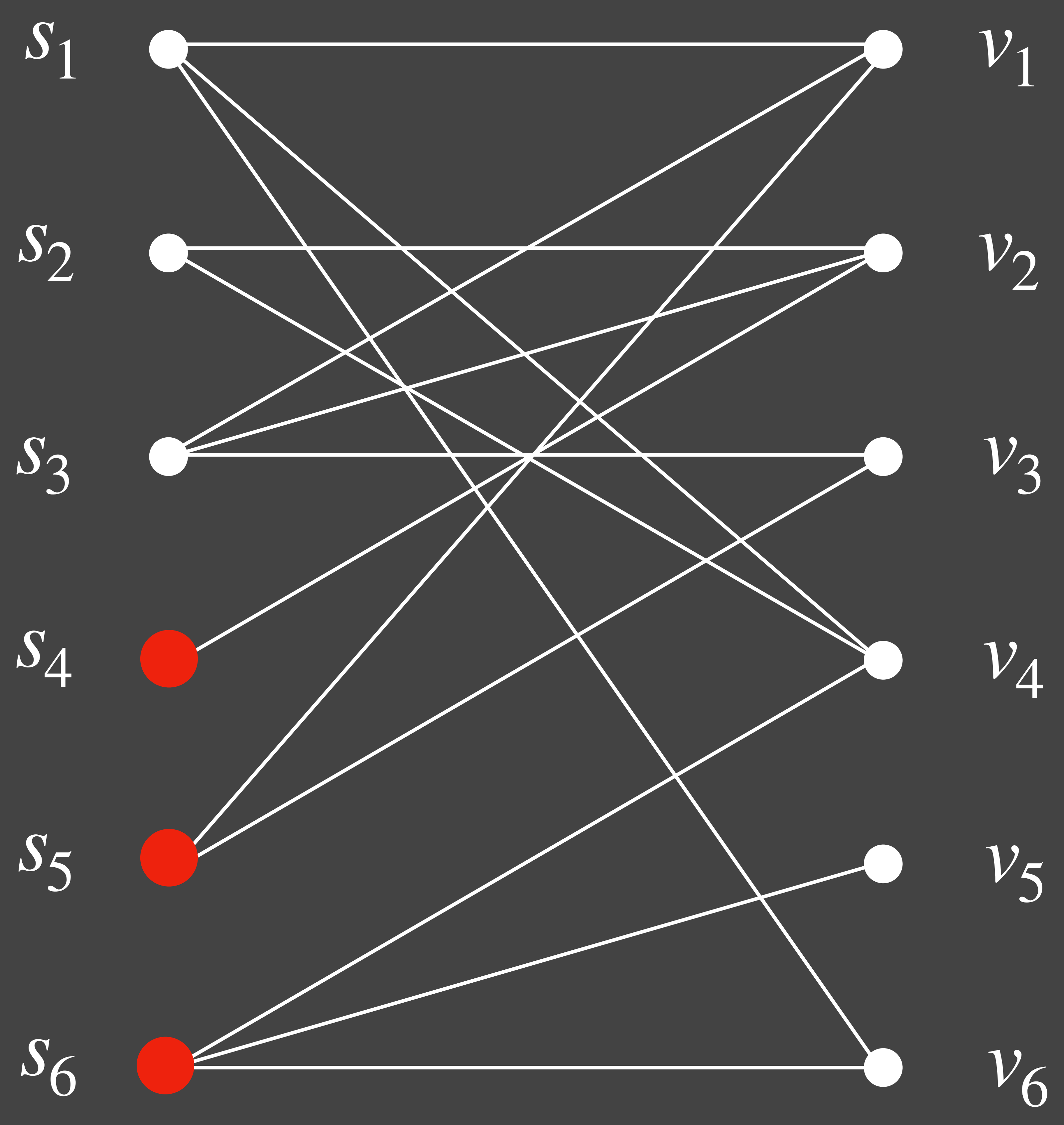

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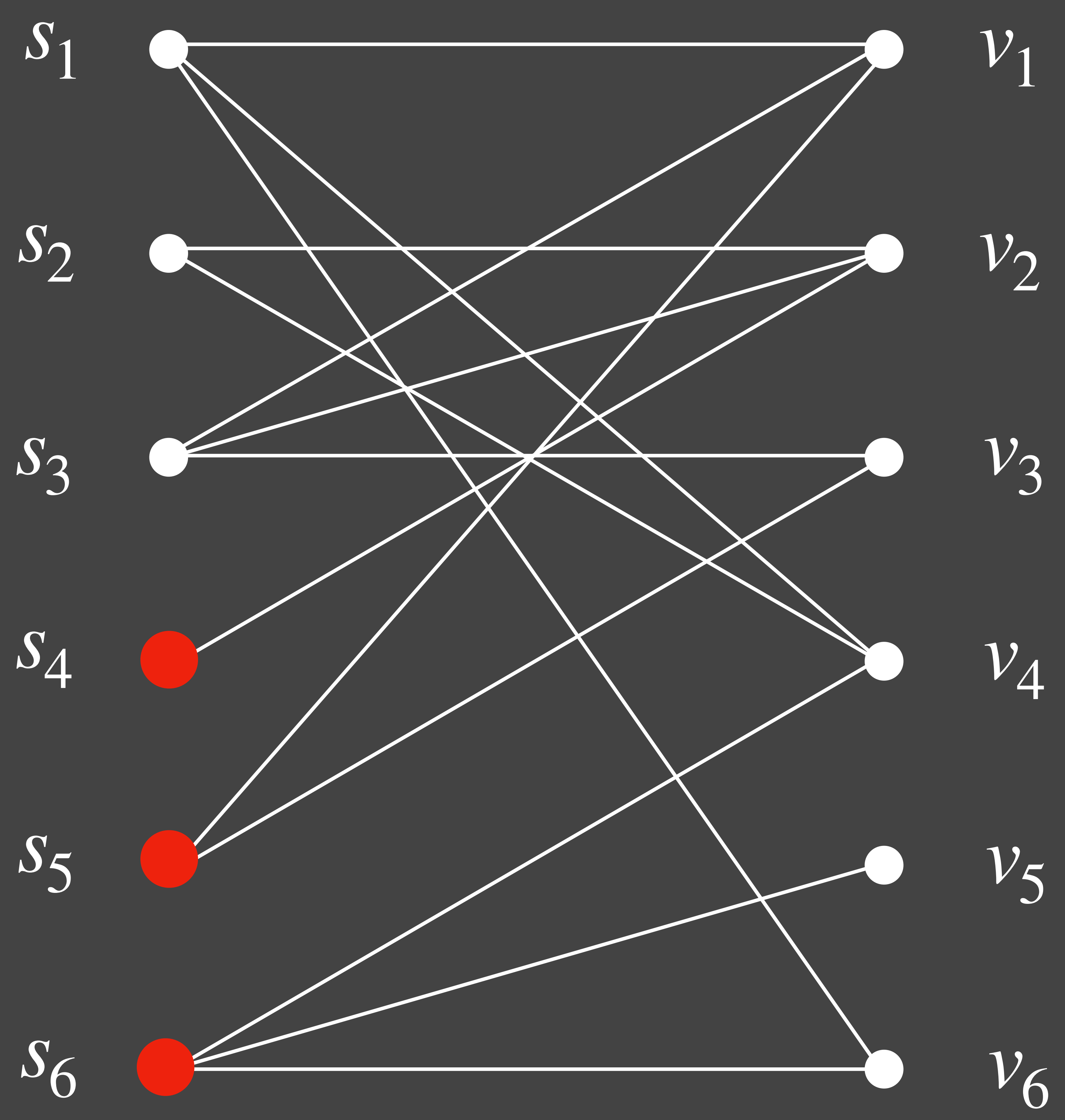
Theorem (Dynamic):

(i) *competitive ratio* $O(\log F(\mathcal{N}))$.

(ii) *average recourse* $O(f(\mathcal{N}))$.

Special Case: Dynamic Set Cover



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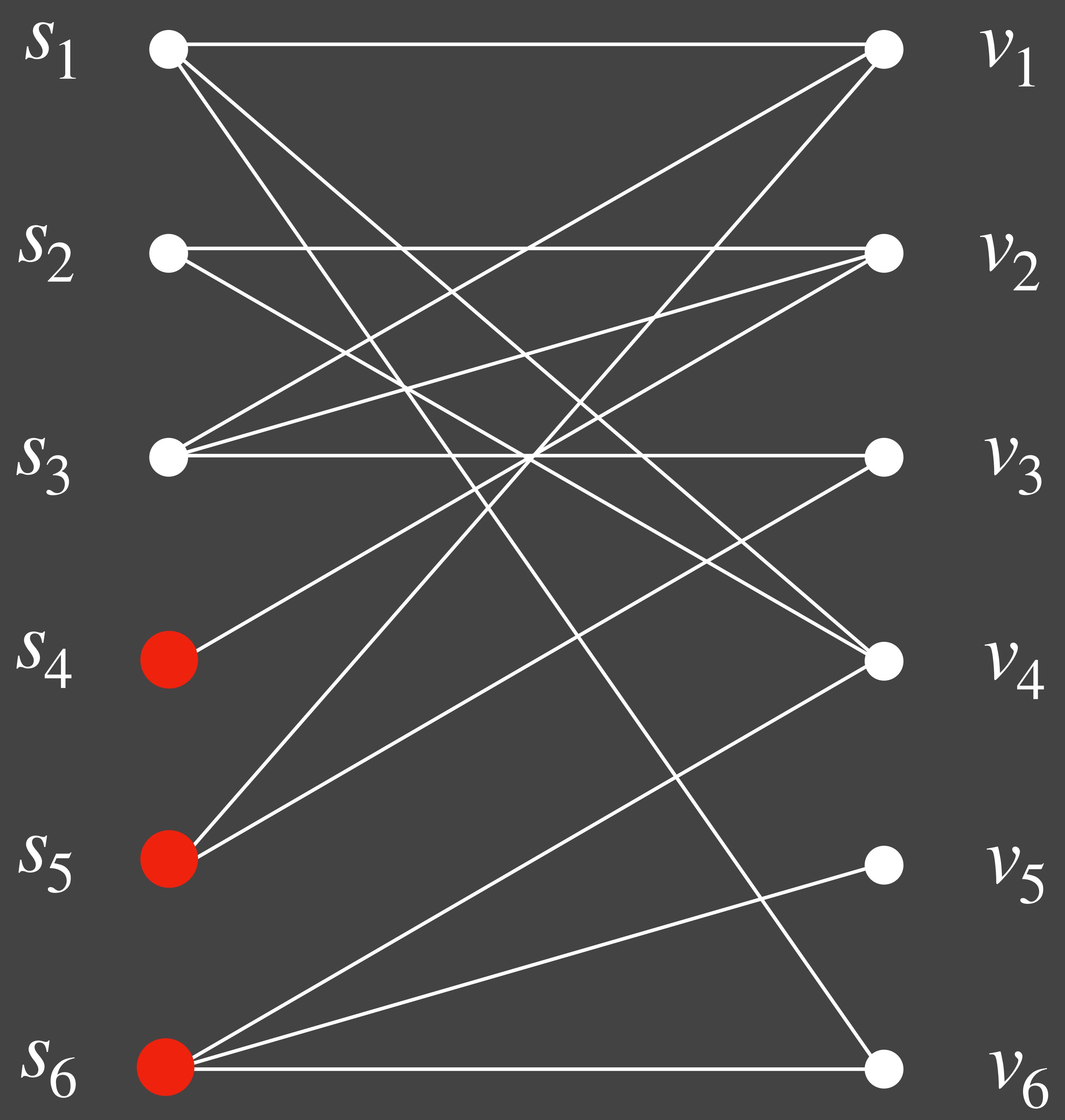

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Generalizes [Gupta Kumar Krishnaswamy Panigrahi 17]

Recap so far

Part I: Online

- Inserts Only
- Decisions are **irrevocable**

Part II: Fully- Dynamic

- Inserts + Deletes
- Want minimum # edits, a.k.a. **recourse.**

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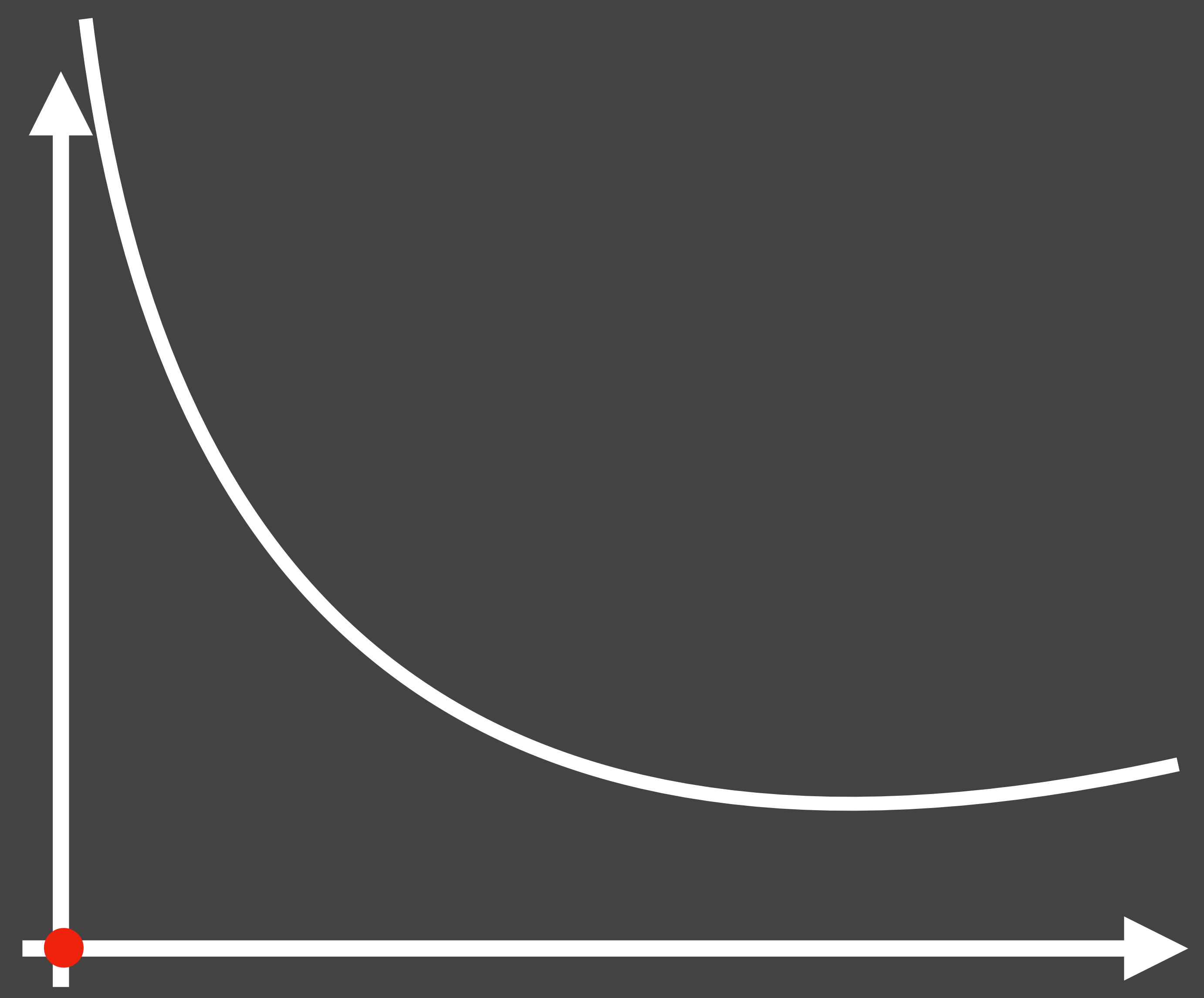

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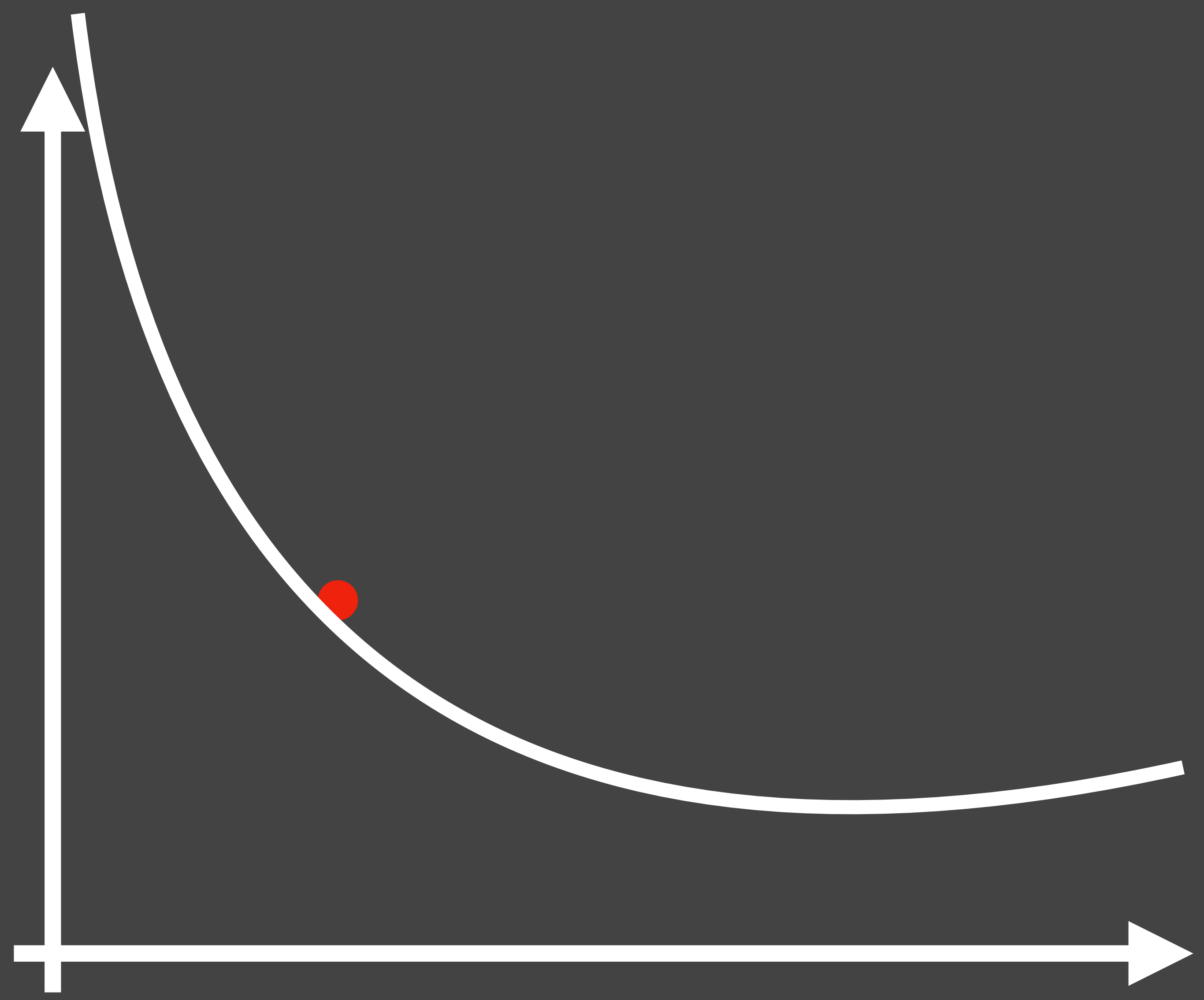

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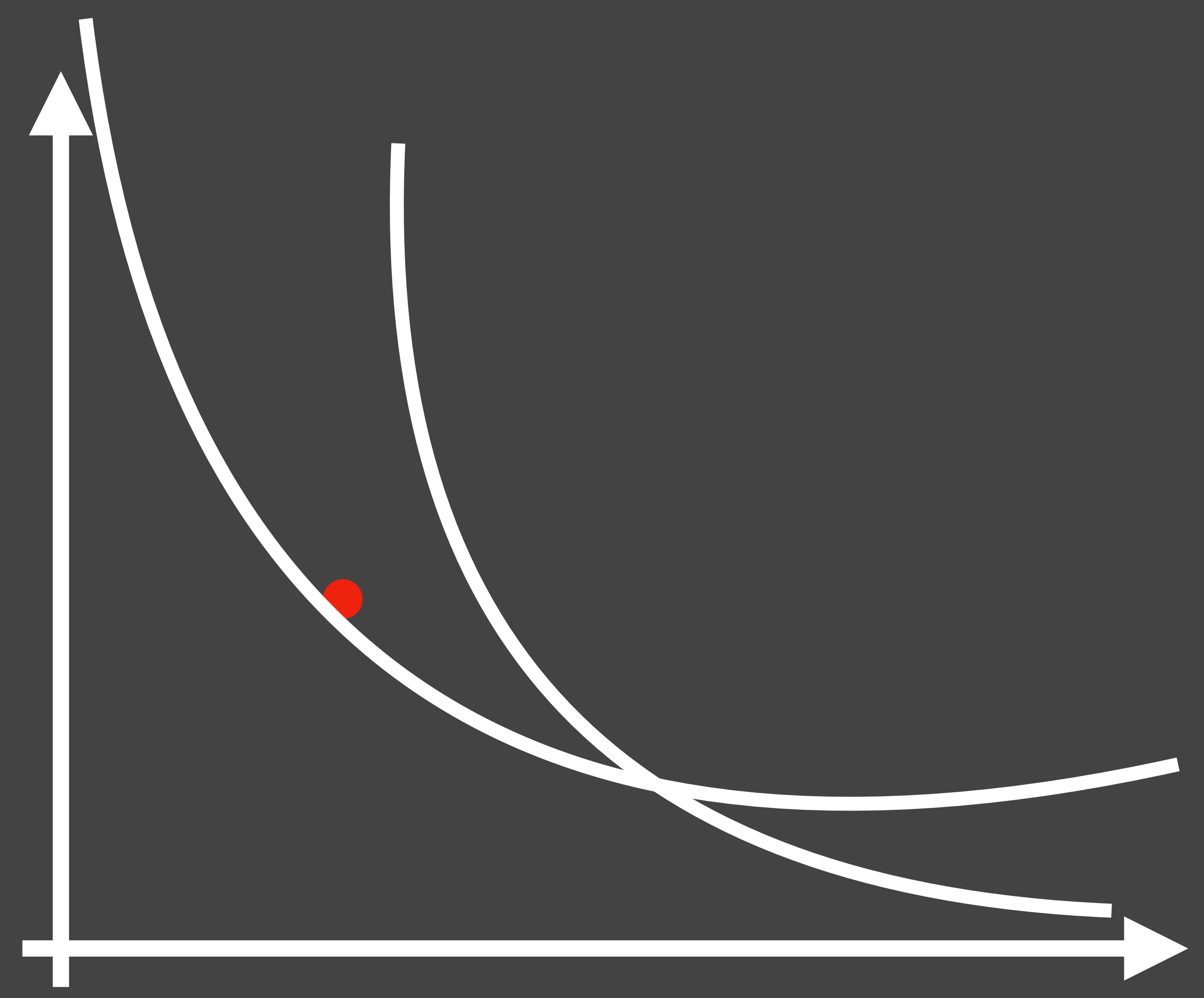

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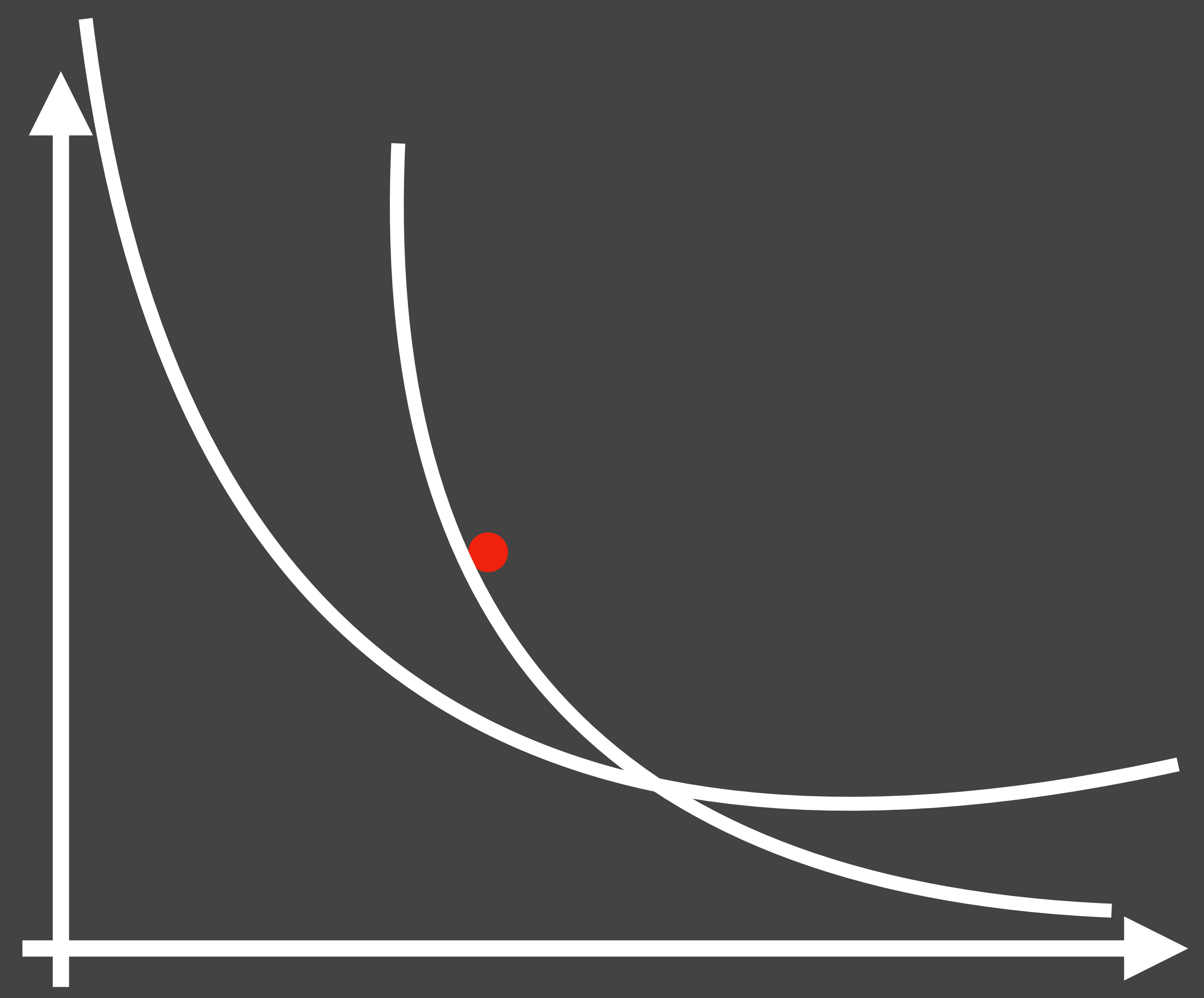

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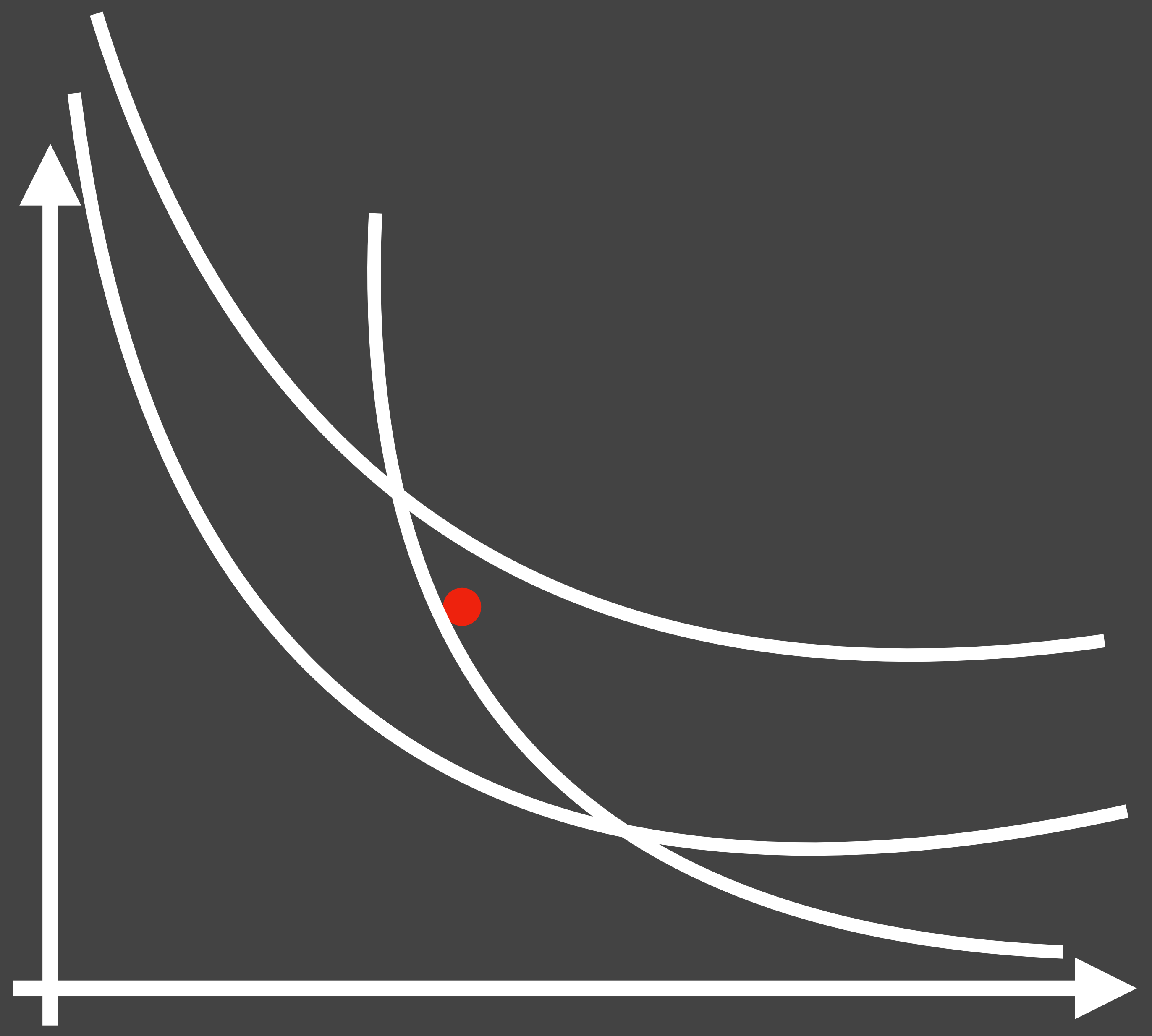

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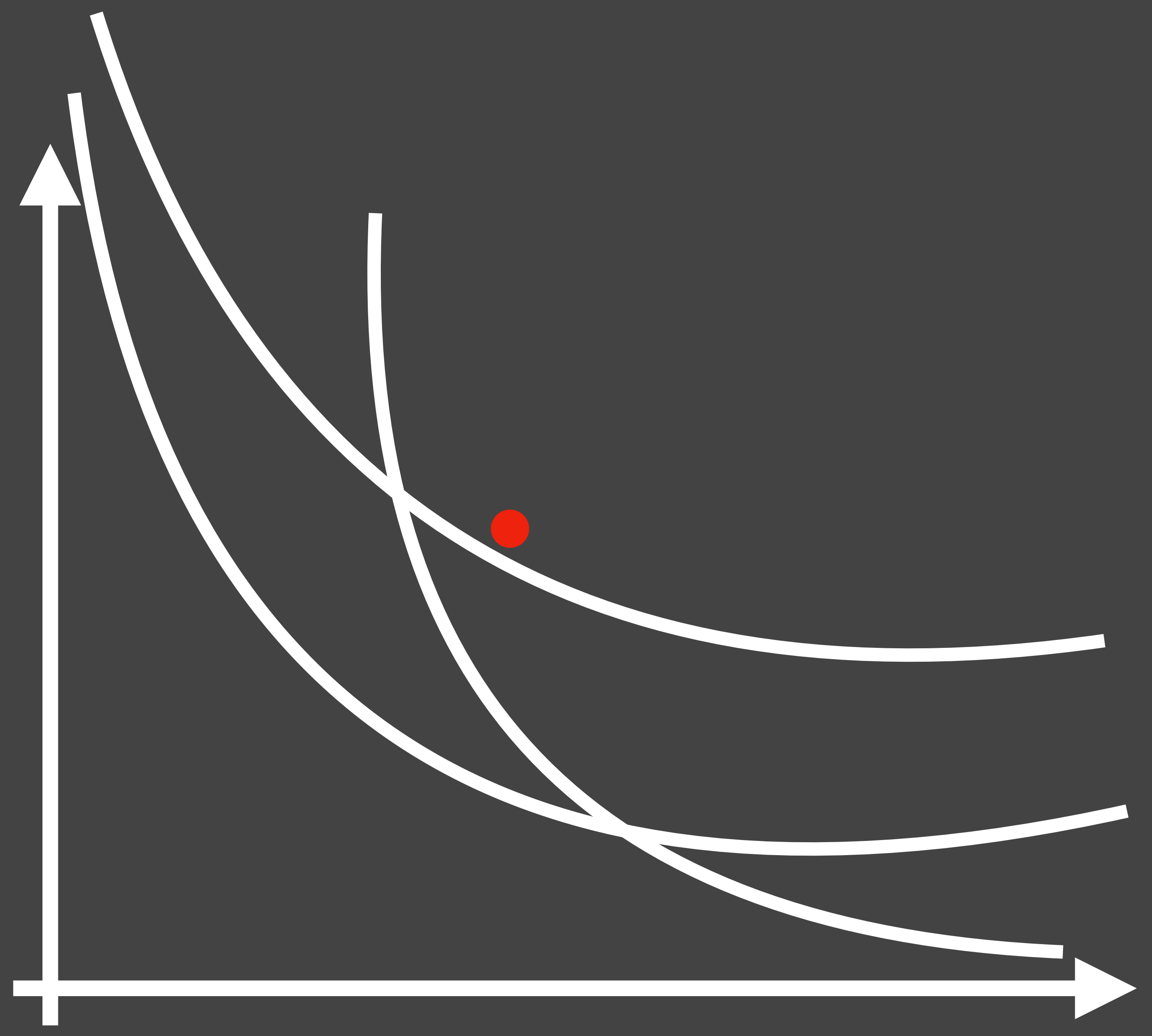

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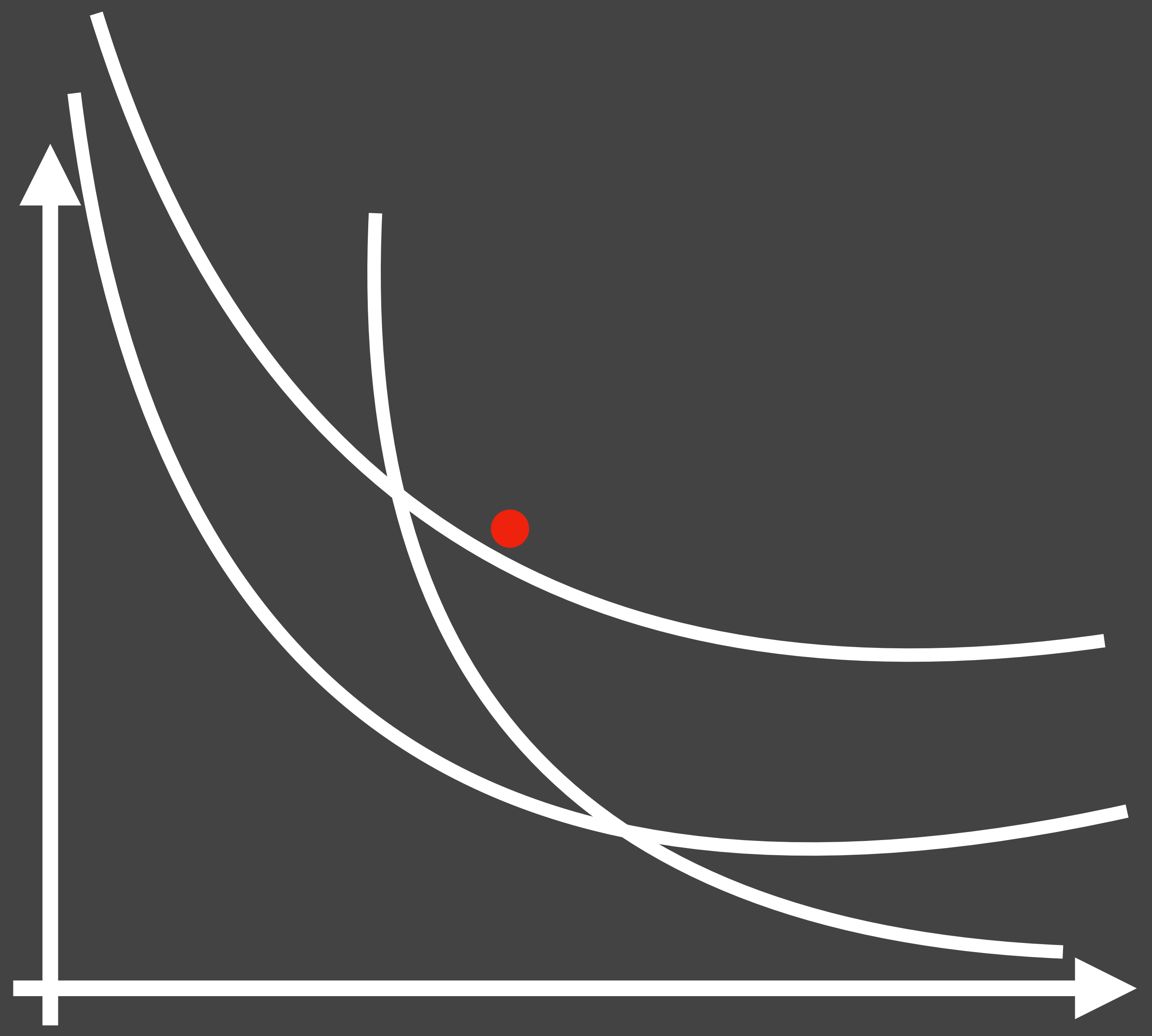

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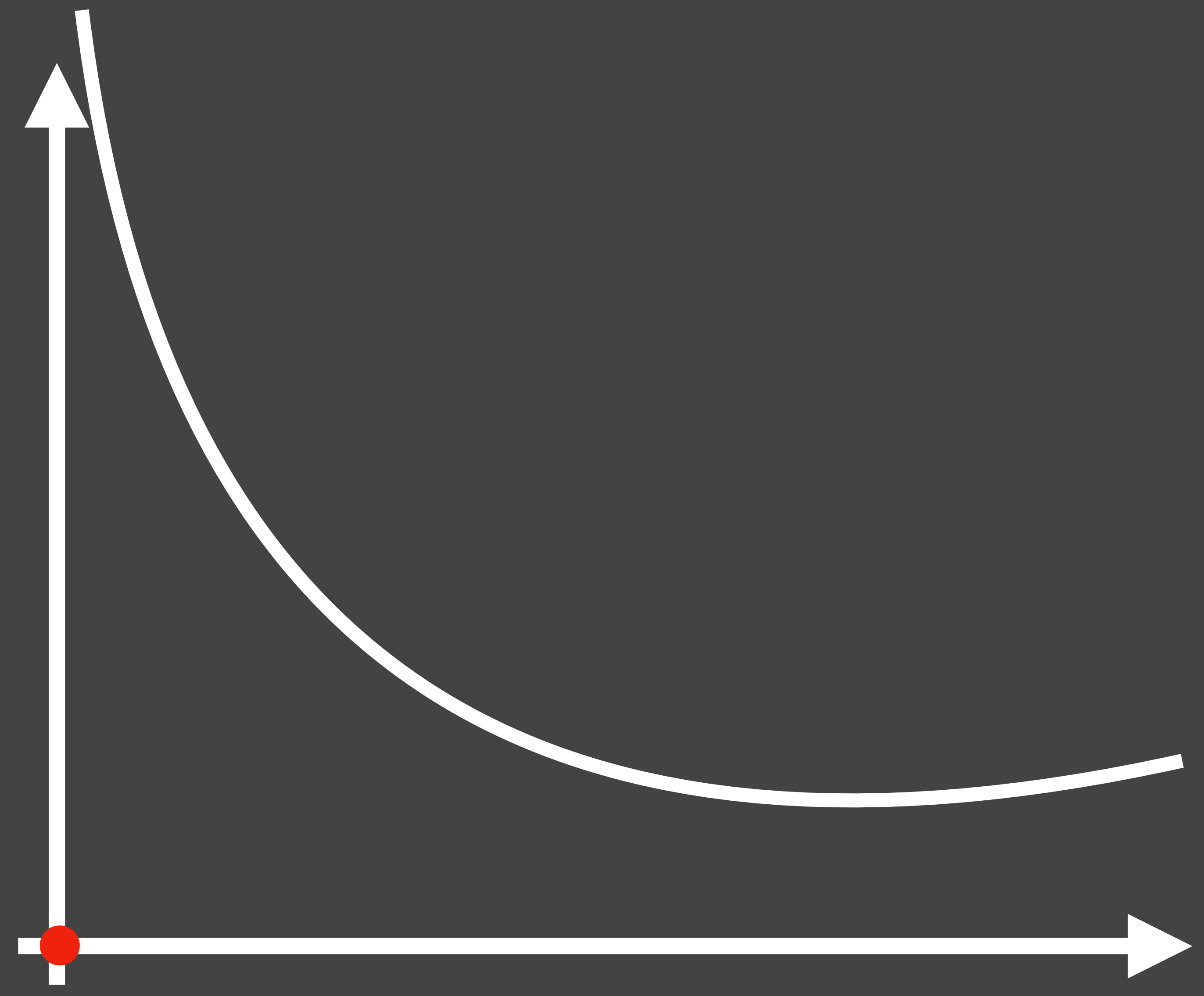

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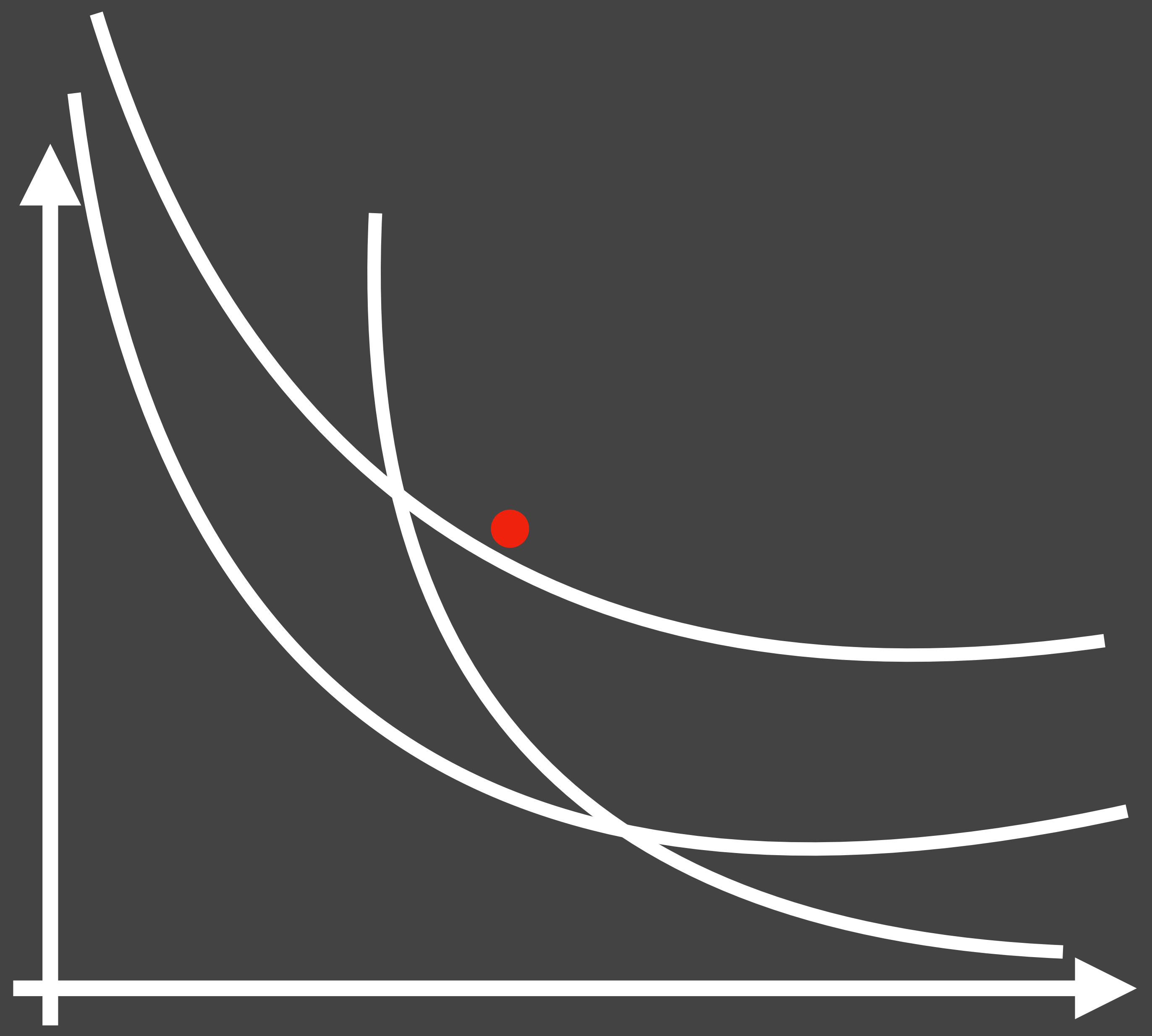

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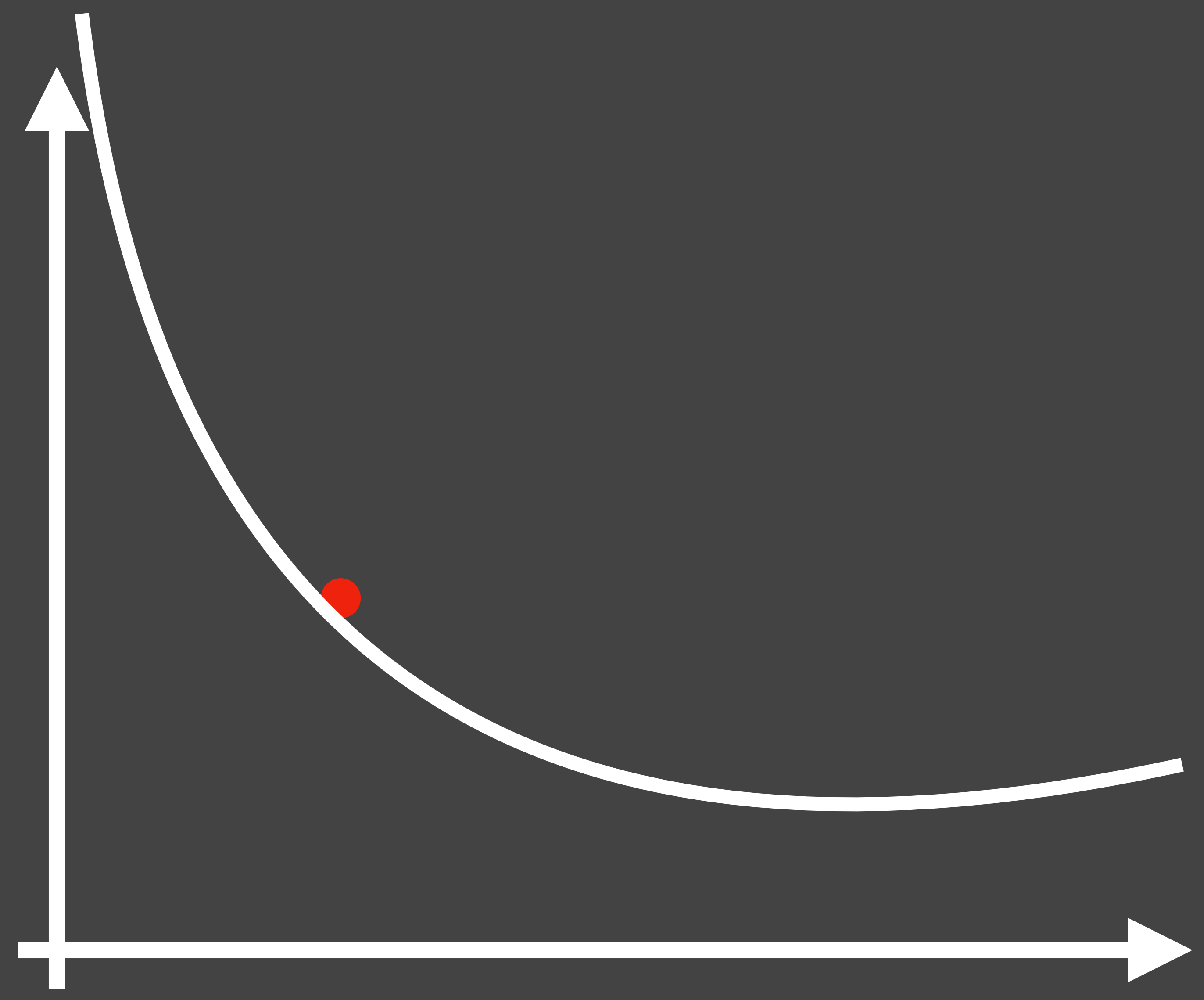

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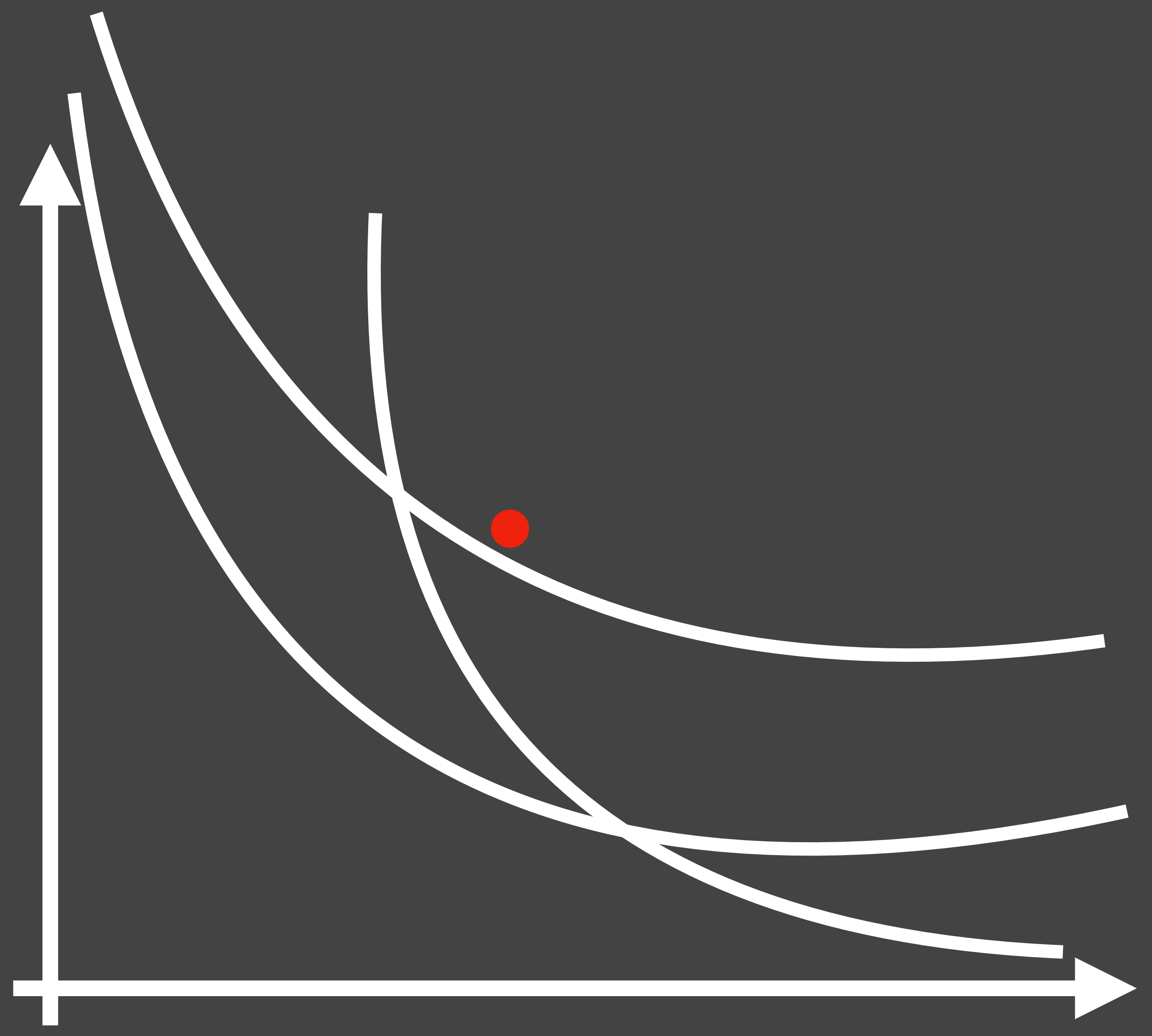

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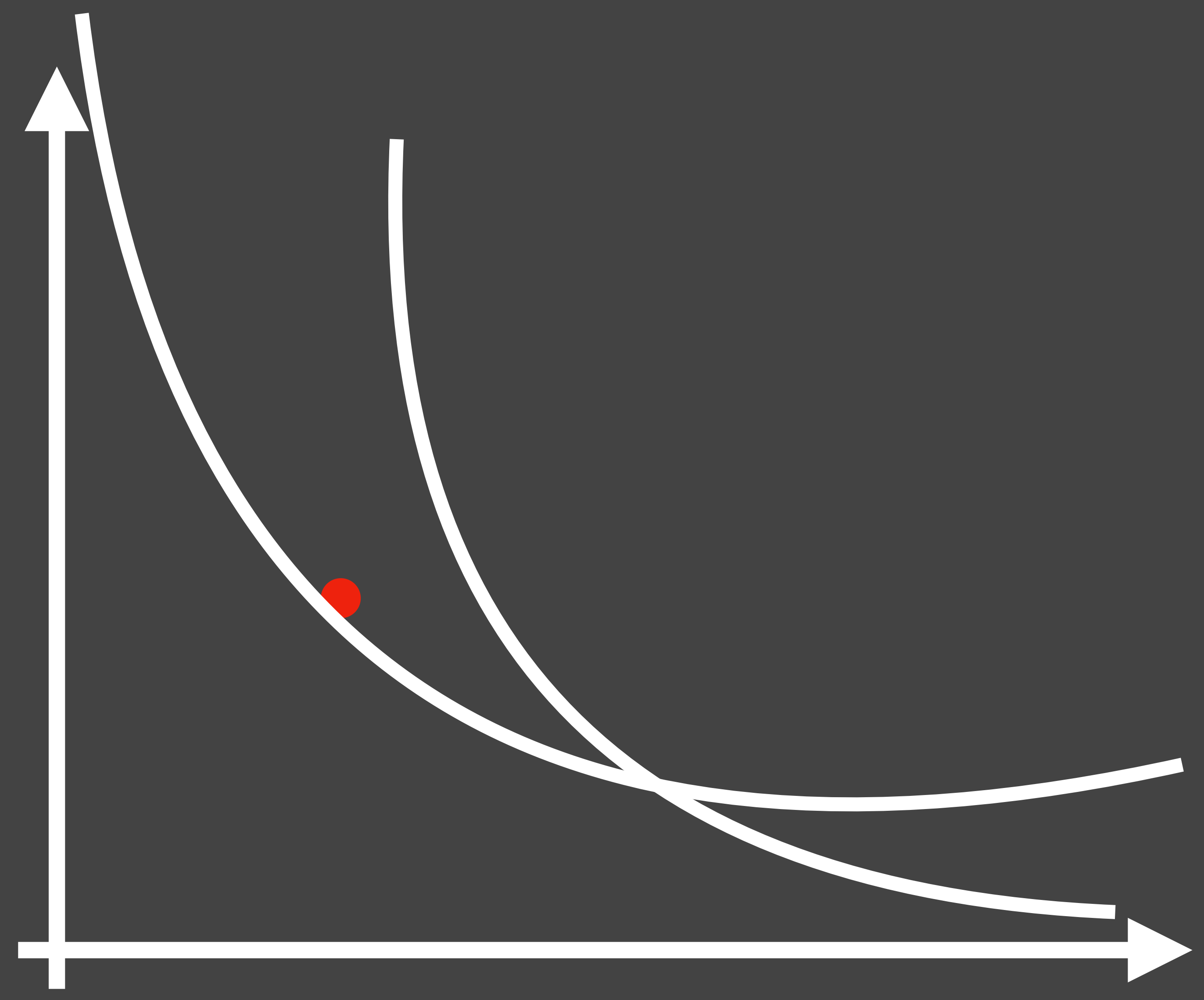

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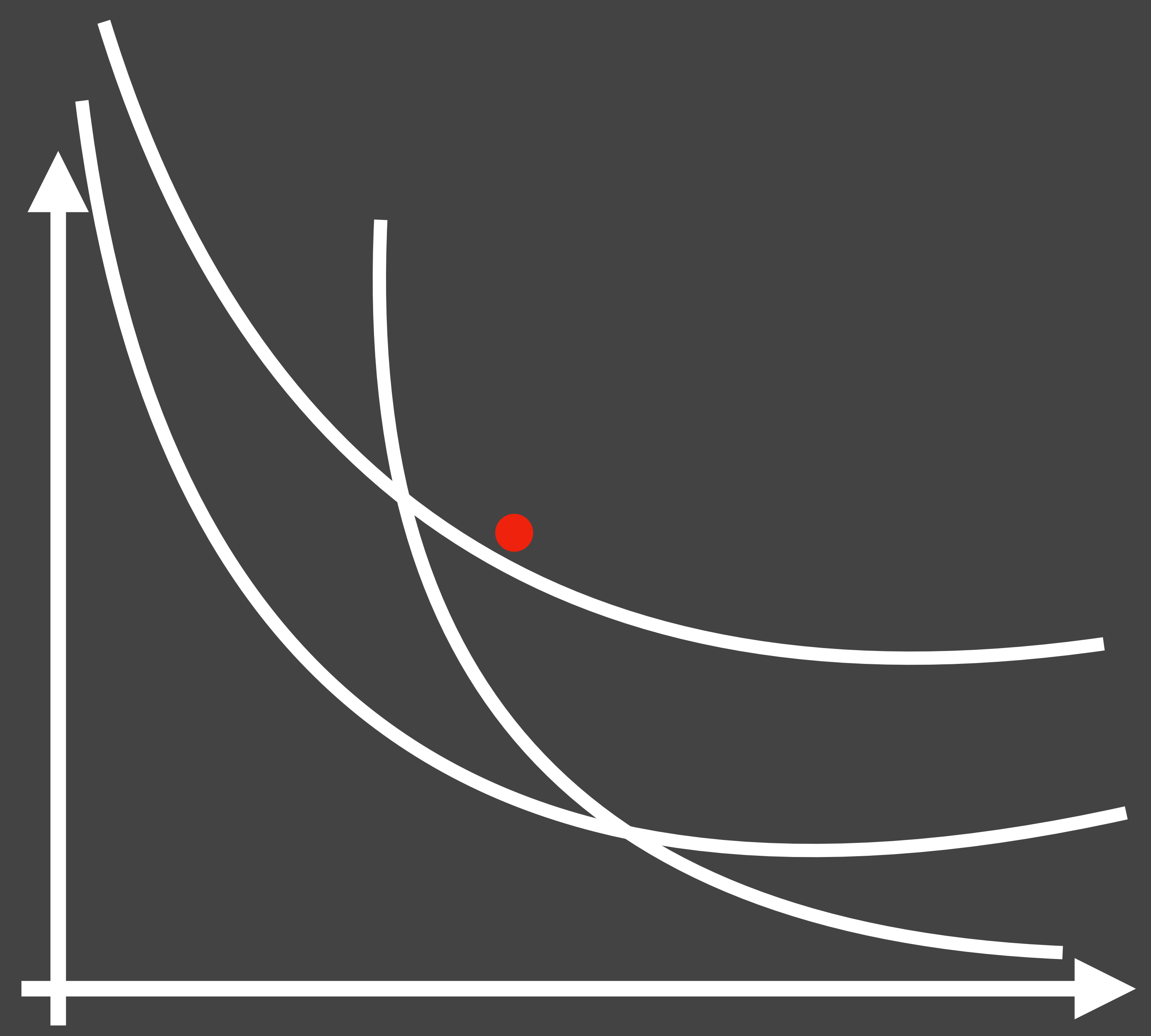

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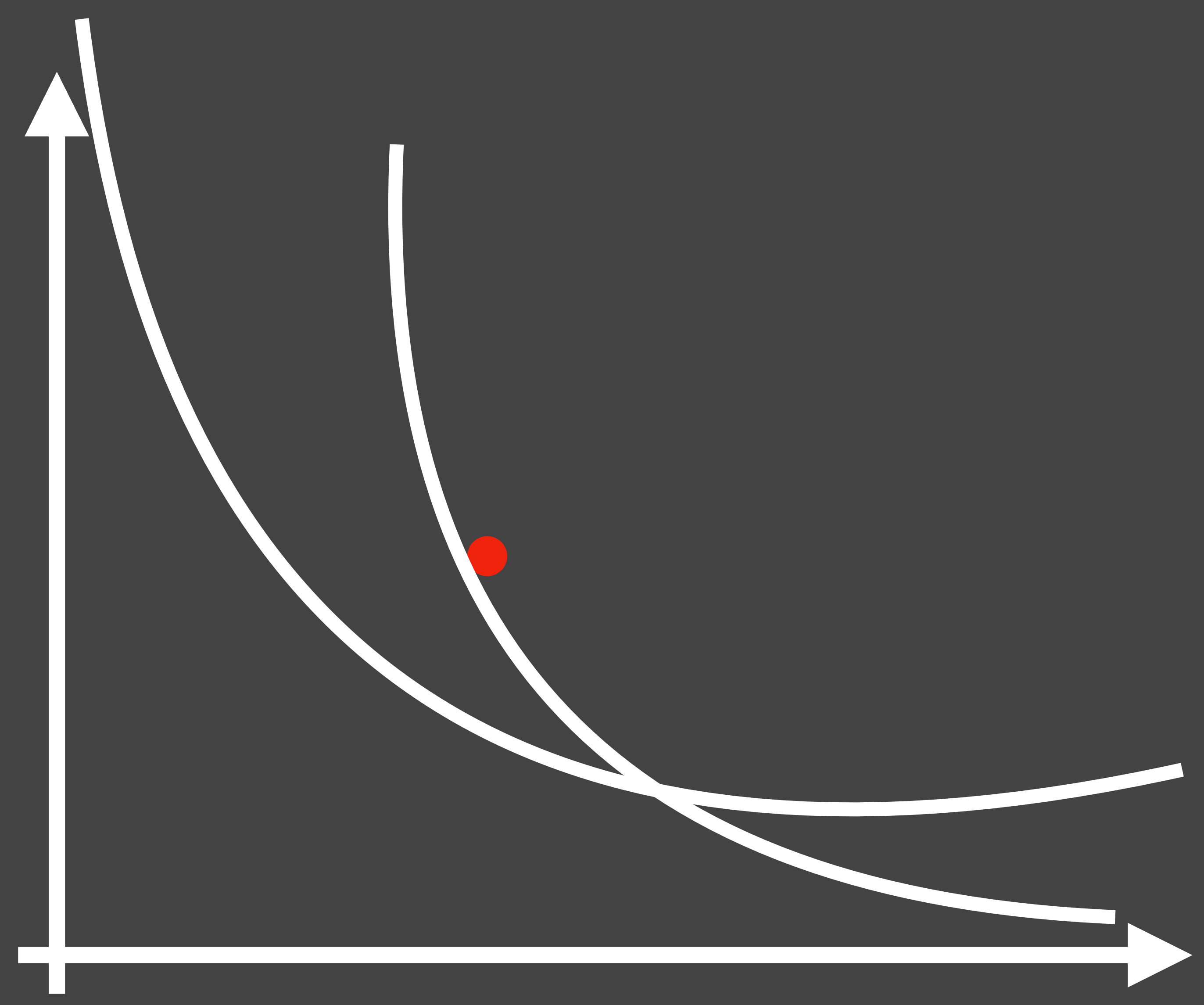

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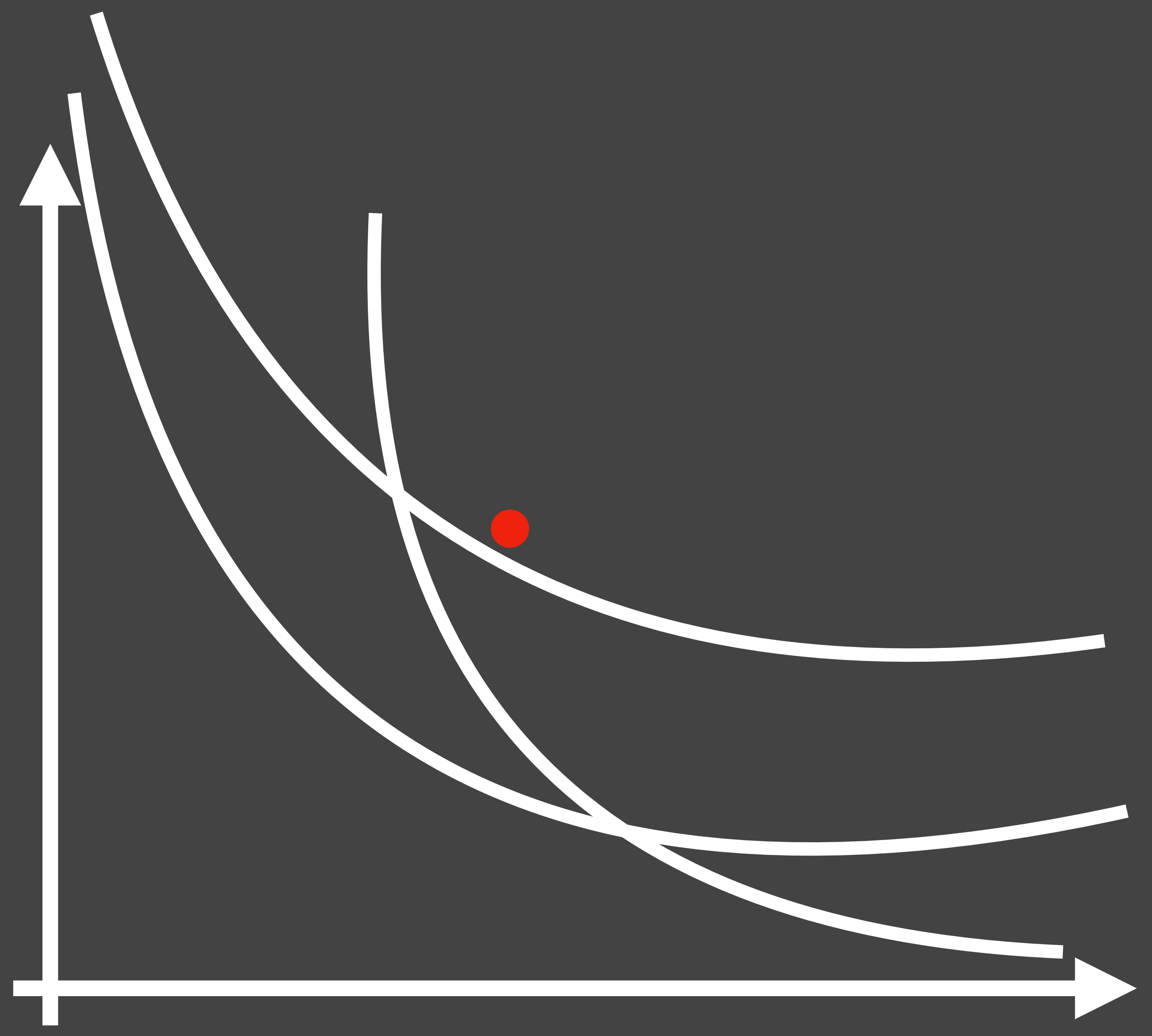

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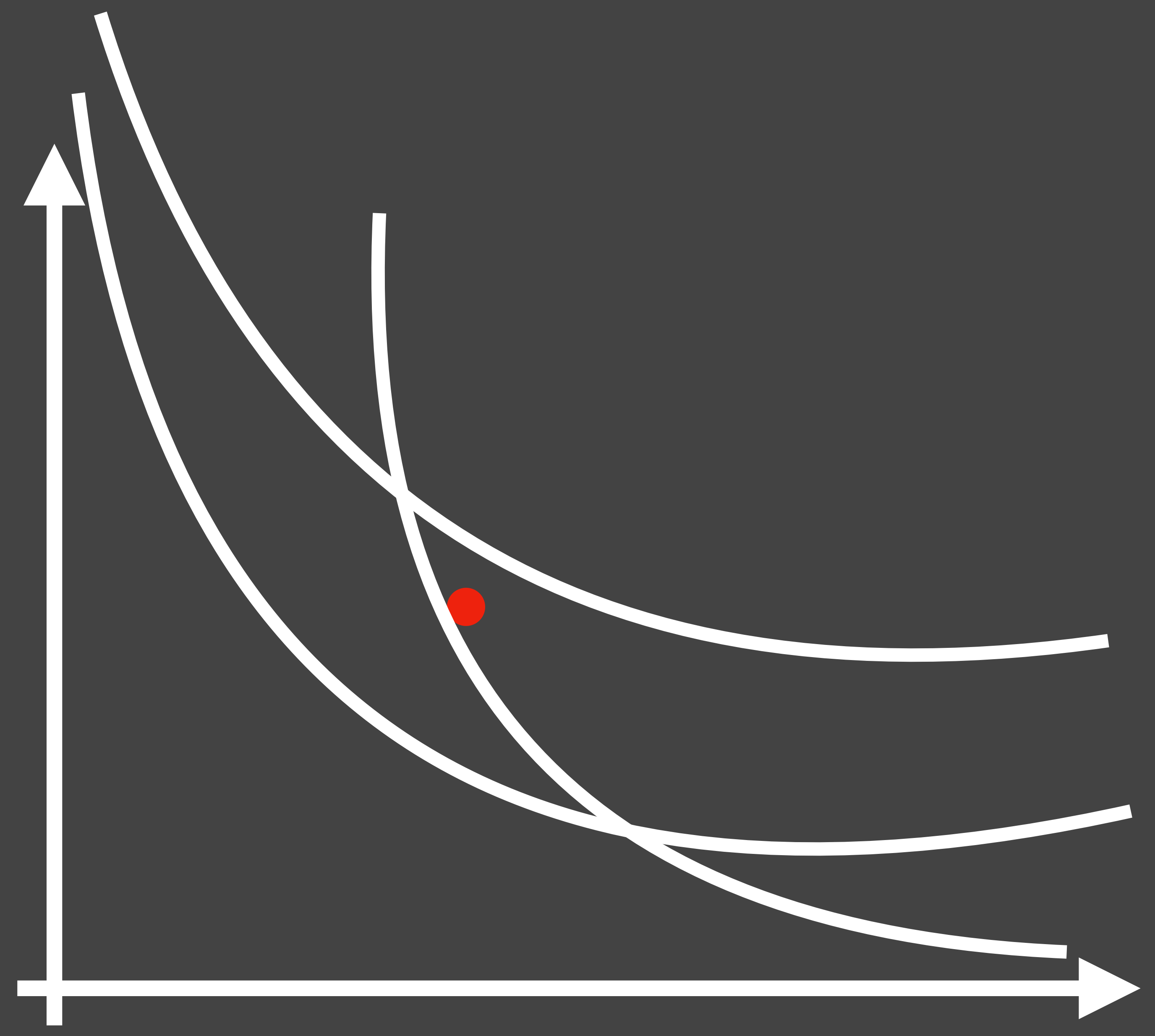

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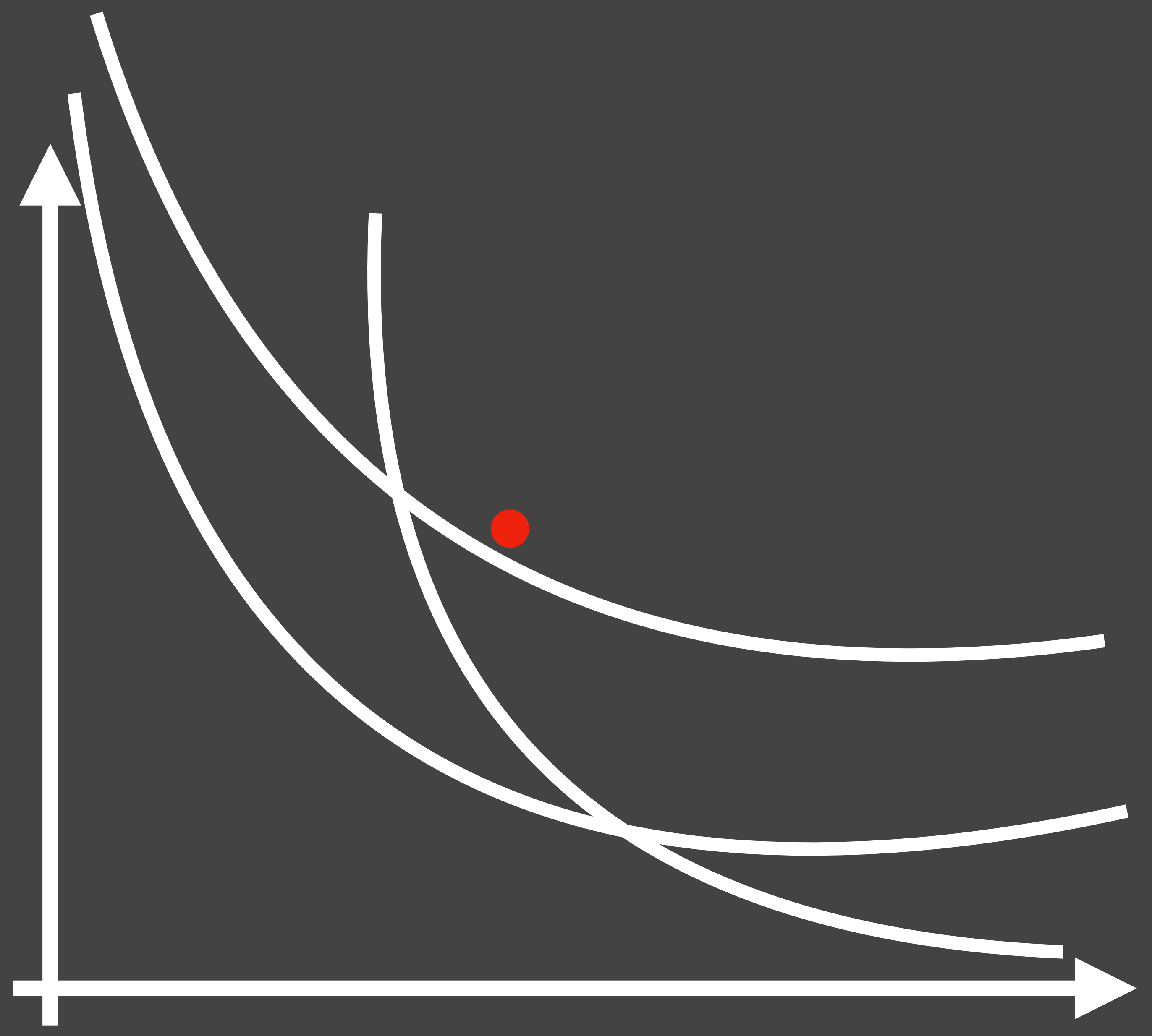

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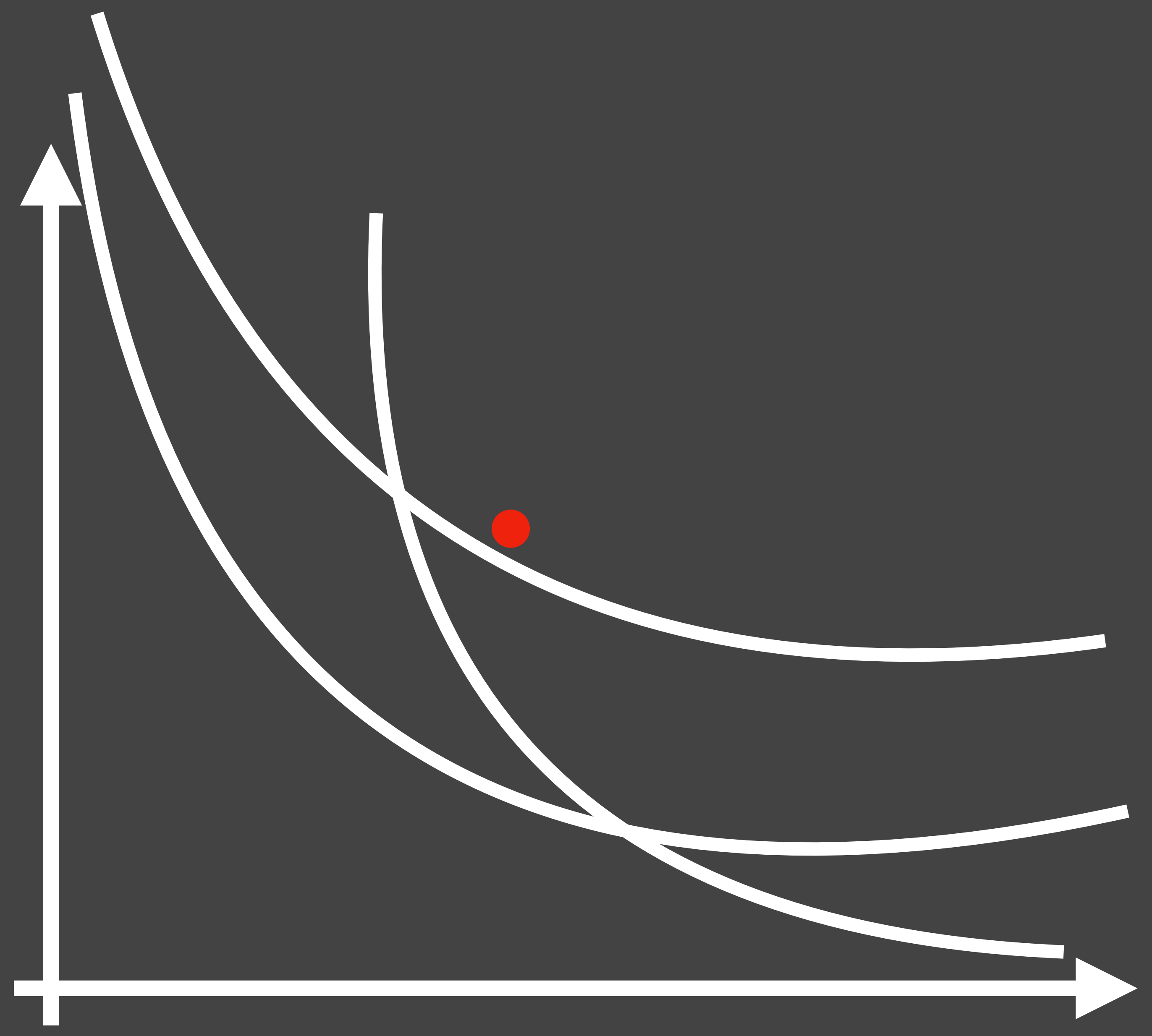

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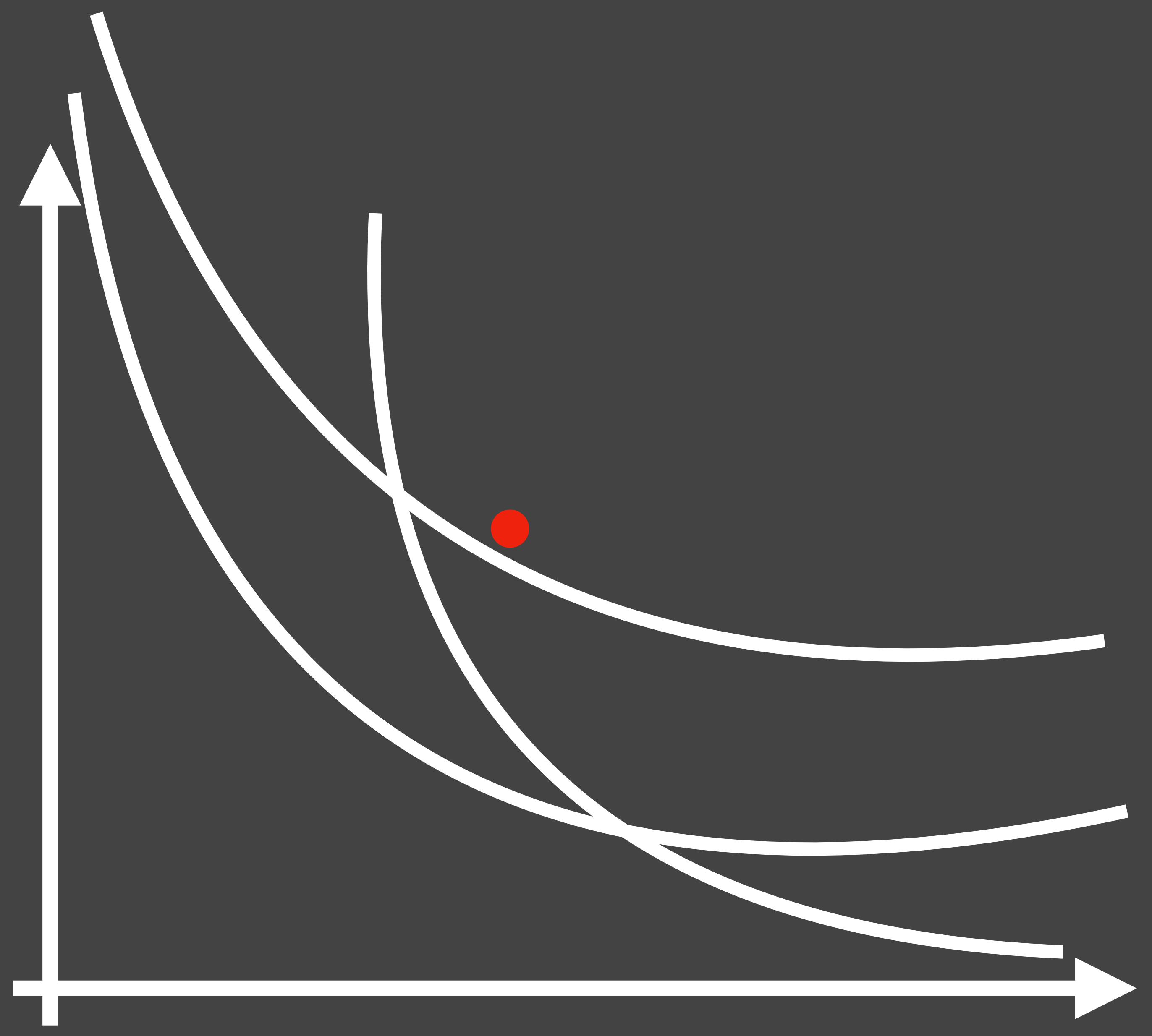
- Decisions are **irrevocable**



Part II: Fully- Dynamic

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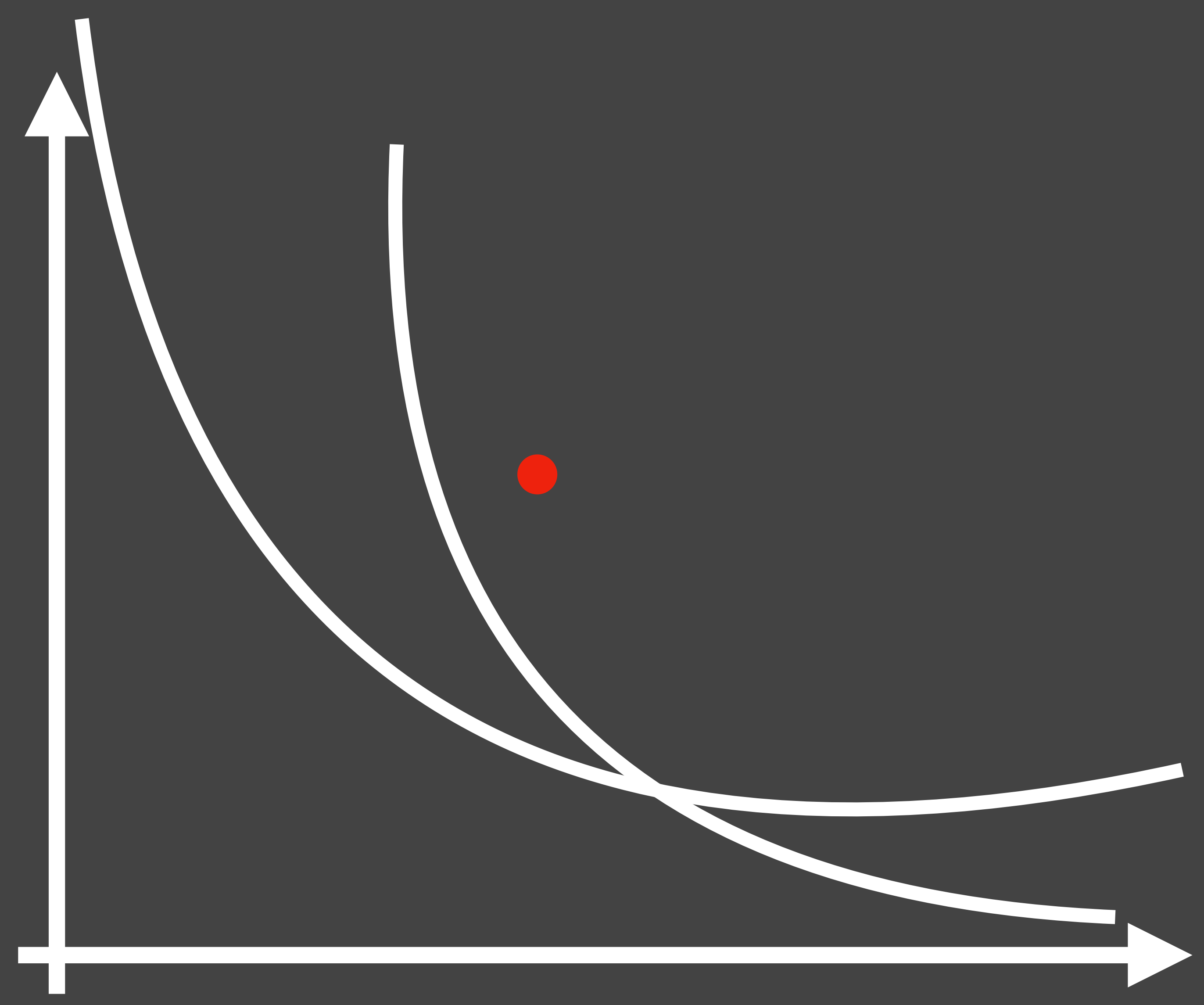
- Want minimum # edits, a.k.a. **recourse.**



Recap so far

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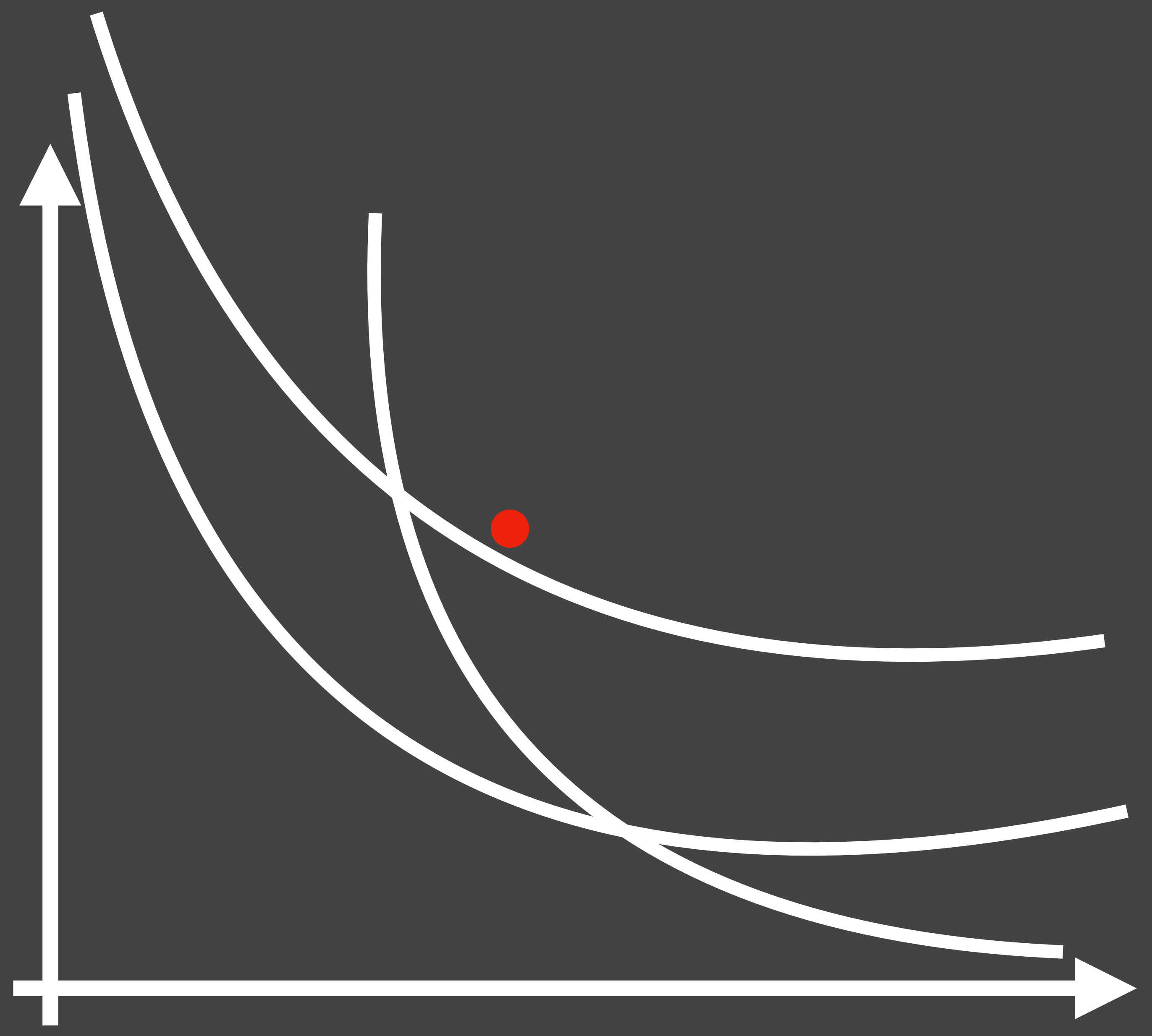

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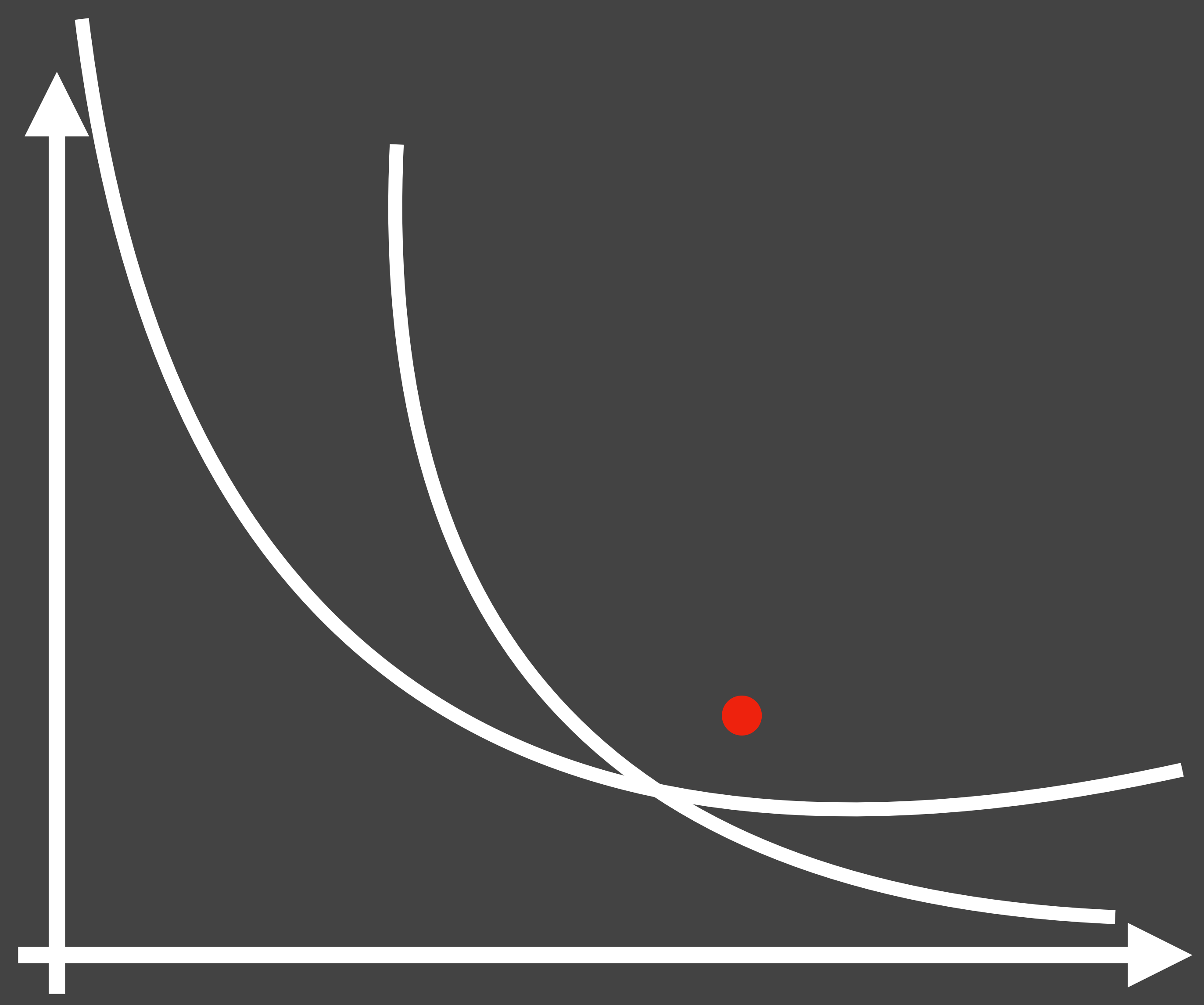

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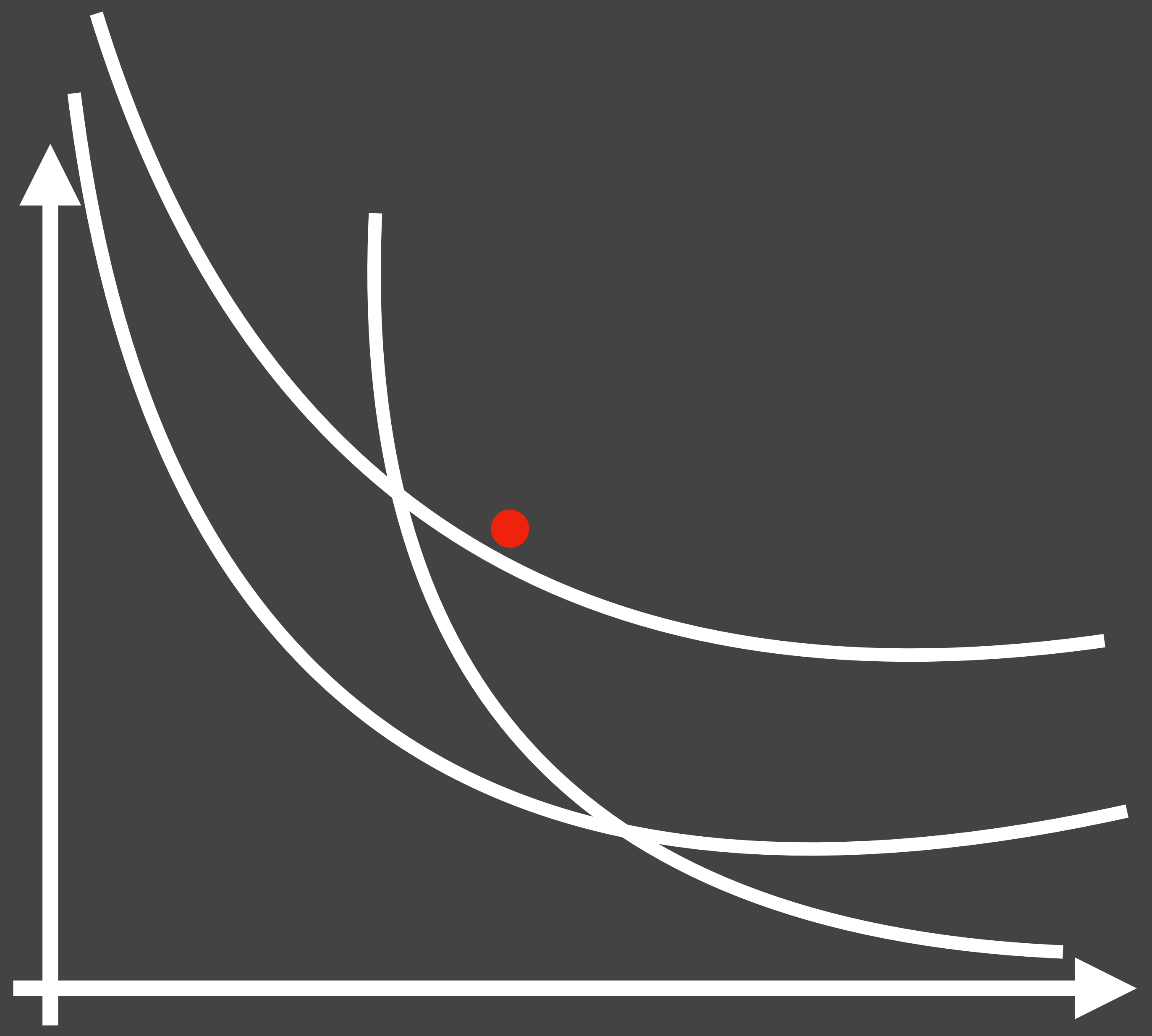

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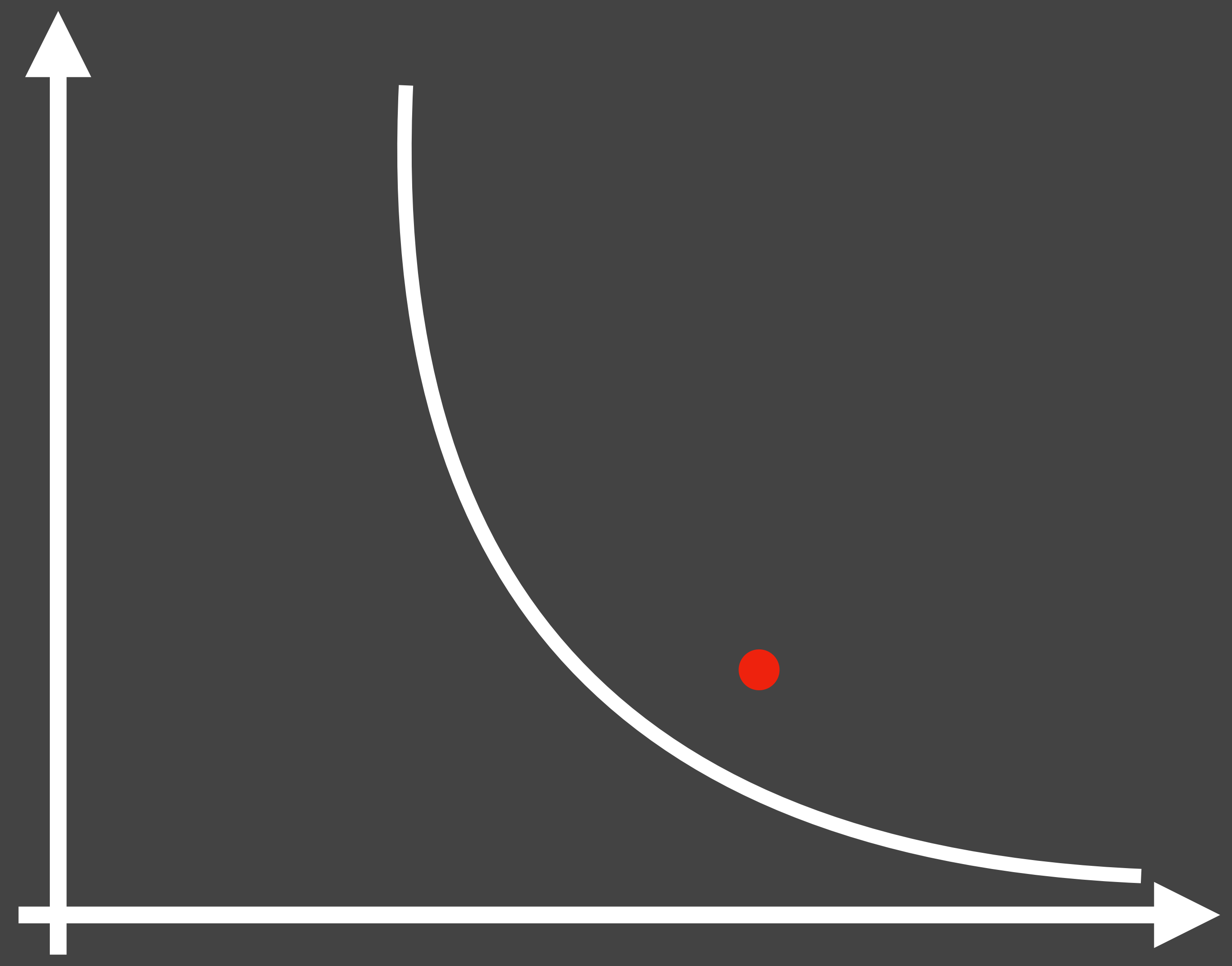

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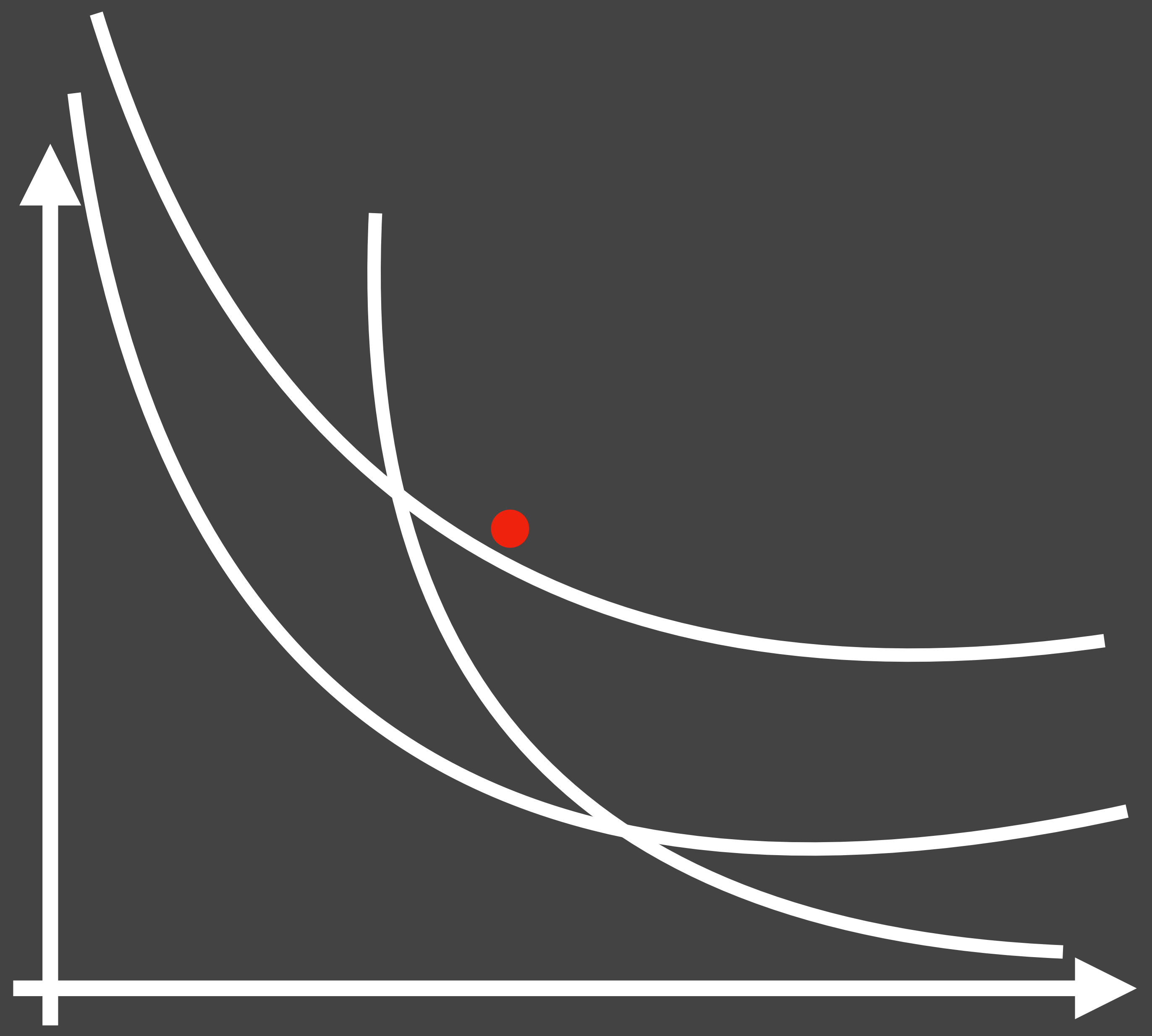

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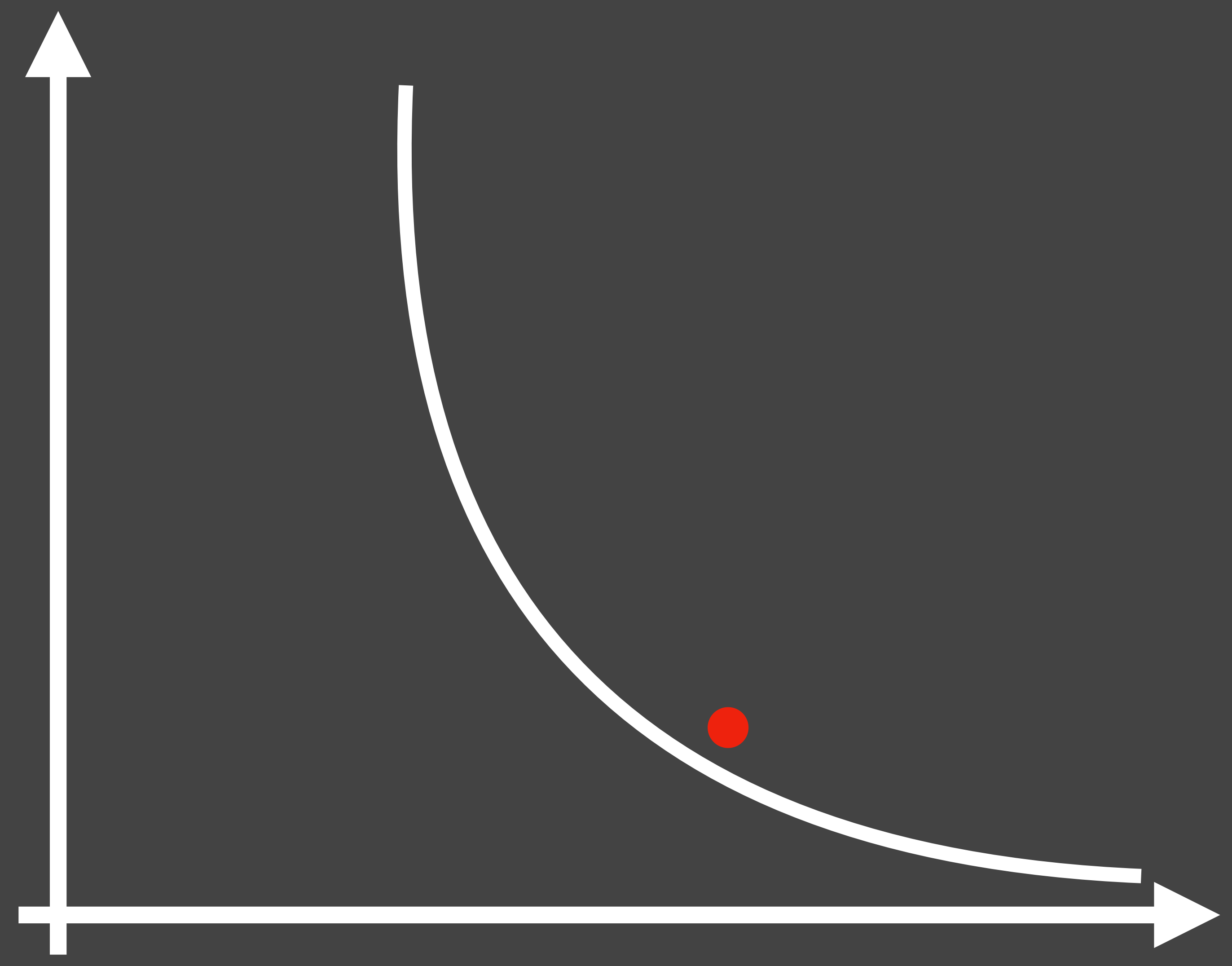

Recap so far

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Theorem (Online) [Gupta L. SODA 20]:

Competitive ratio $O(\log n \ \log F(\mathcal{N}))$.

Part II: Fully- Dynamic

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Theorem (Dynamic) [Gupta L. FOCS 20]:

(i) *Competitive ratio* $O(\log F(\mathcal{N}))$.

(ii) *Average recourse* $\tilde{O}(f(\mathcal{N}))$.

Recap so far

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Modeling power of Submod Cover + robustness to uncertainty of Online/Dynamic algos.

Talk Outline

Intro

Part I — Online/Dynamic Submodular Cover

Part II — Application: Block-Aware Caching

Part III — Random Order Online Set Cover

Conclusion

Talk Outline

Intro

Part I — Online/Dynamic Submodular Cover

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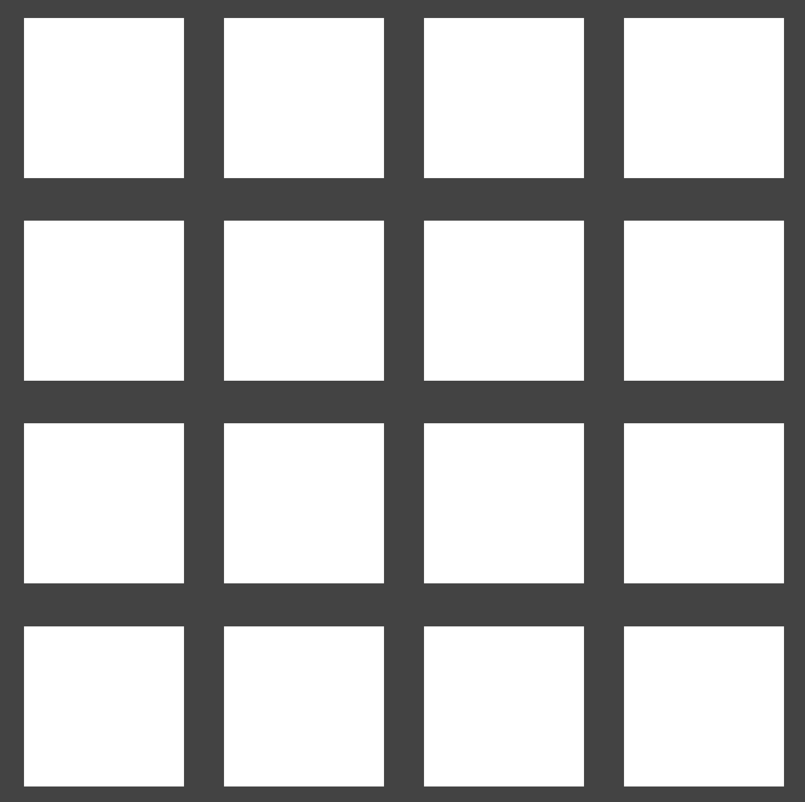
Conclusion

Part II — Application: Block-Aware Caching

with Christian Coester, Seffi Naor, Ohad Talmon

Classic Caching

Cache of size k



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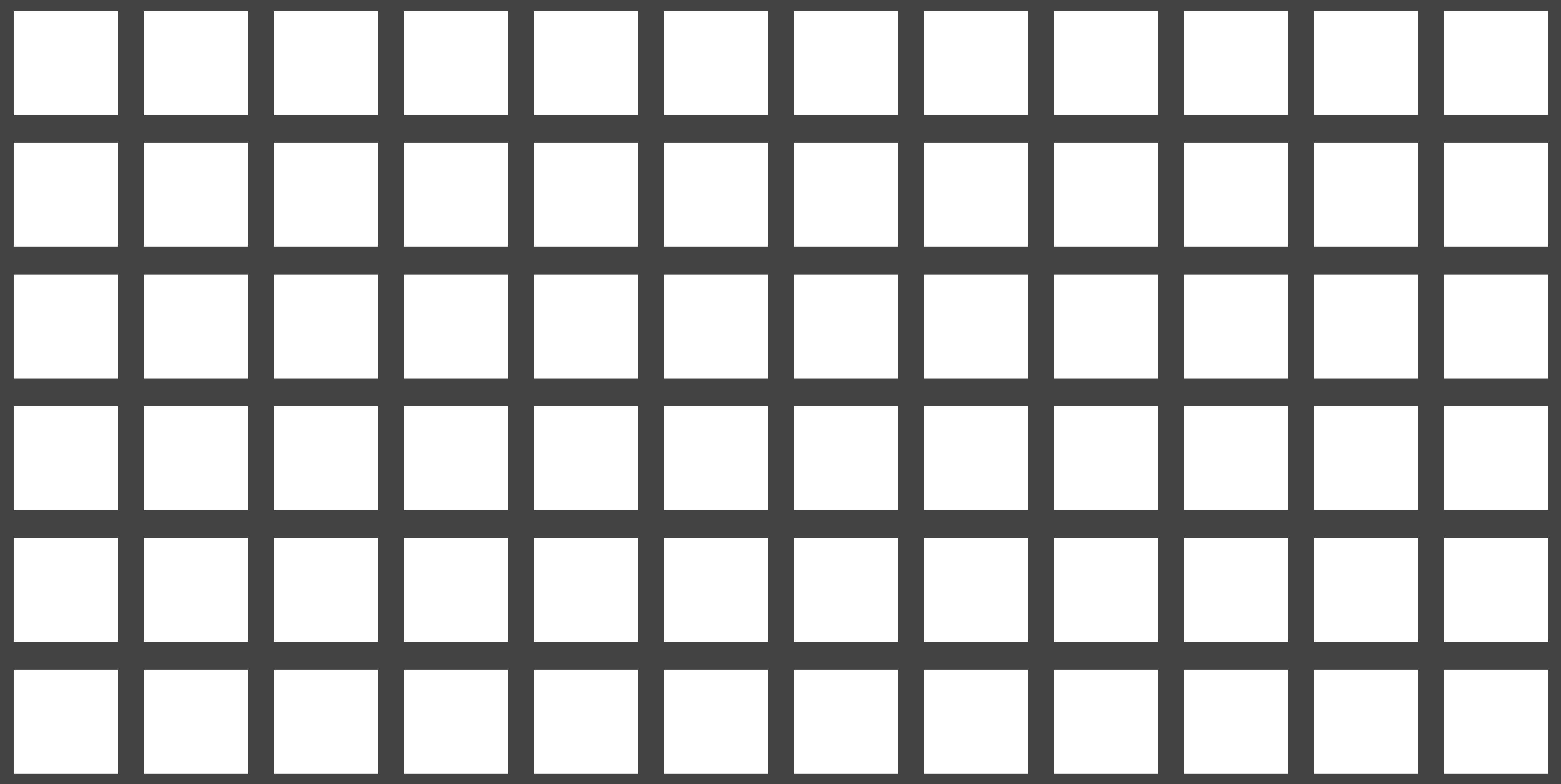

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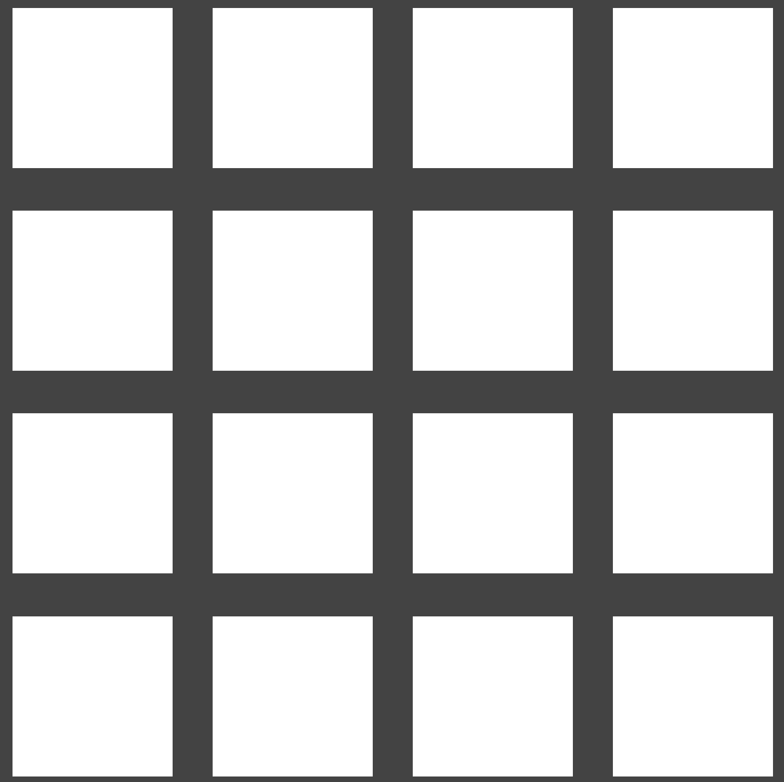

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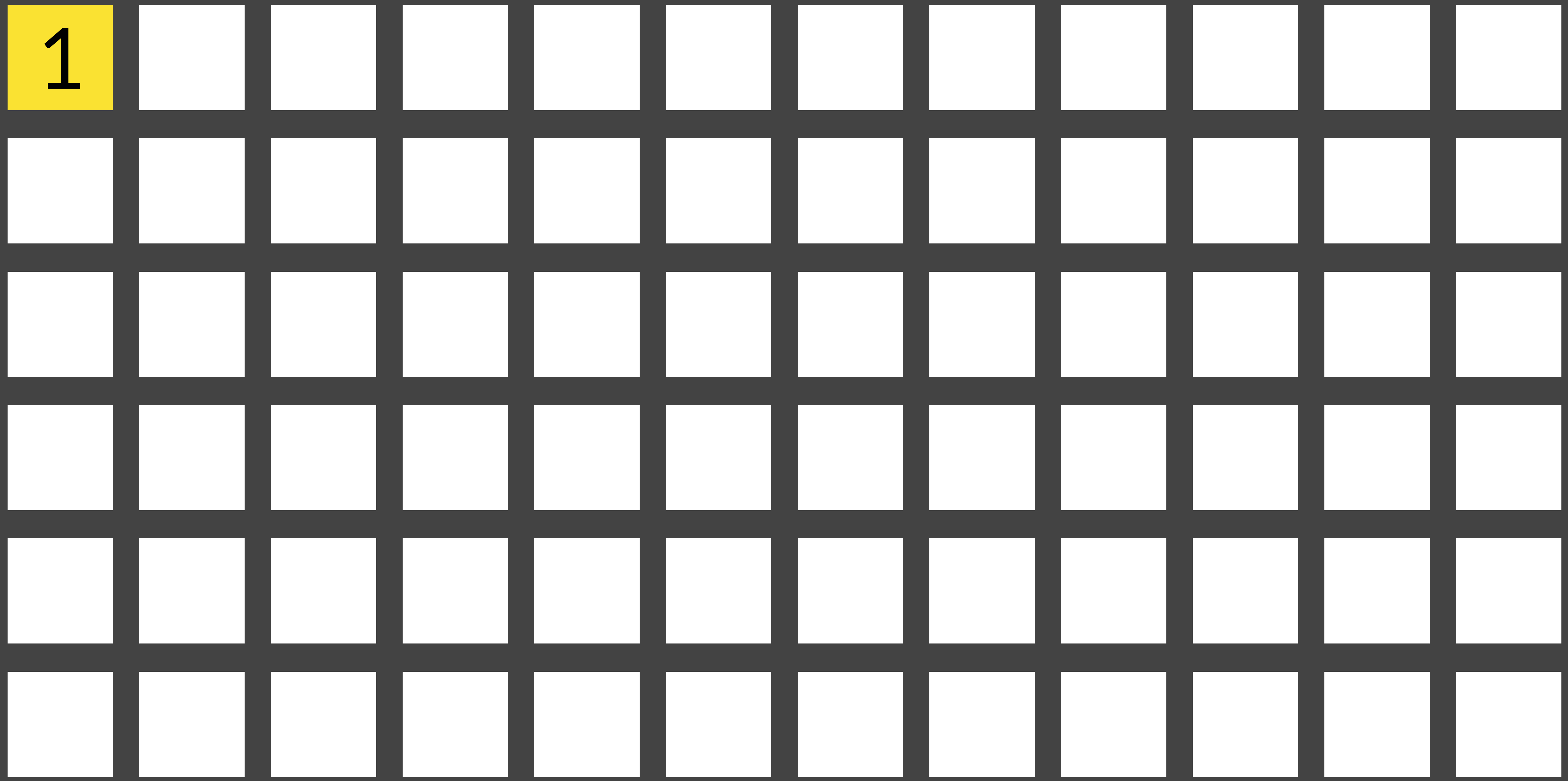

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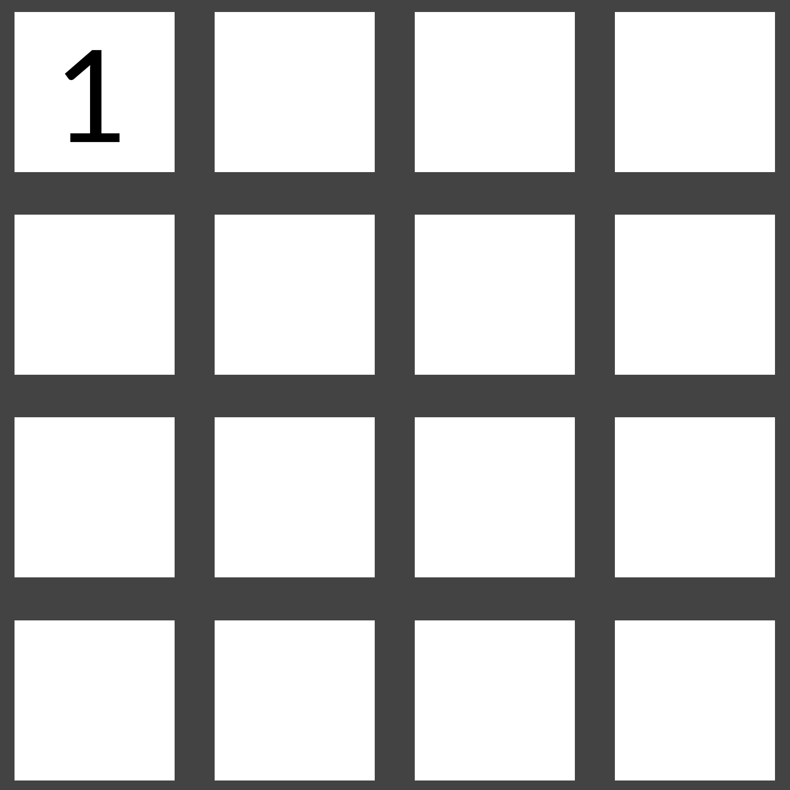

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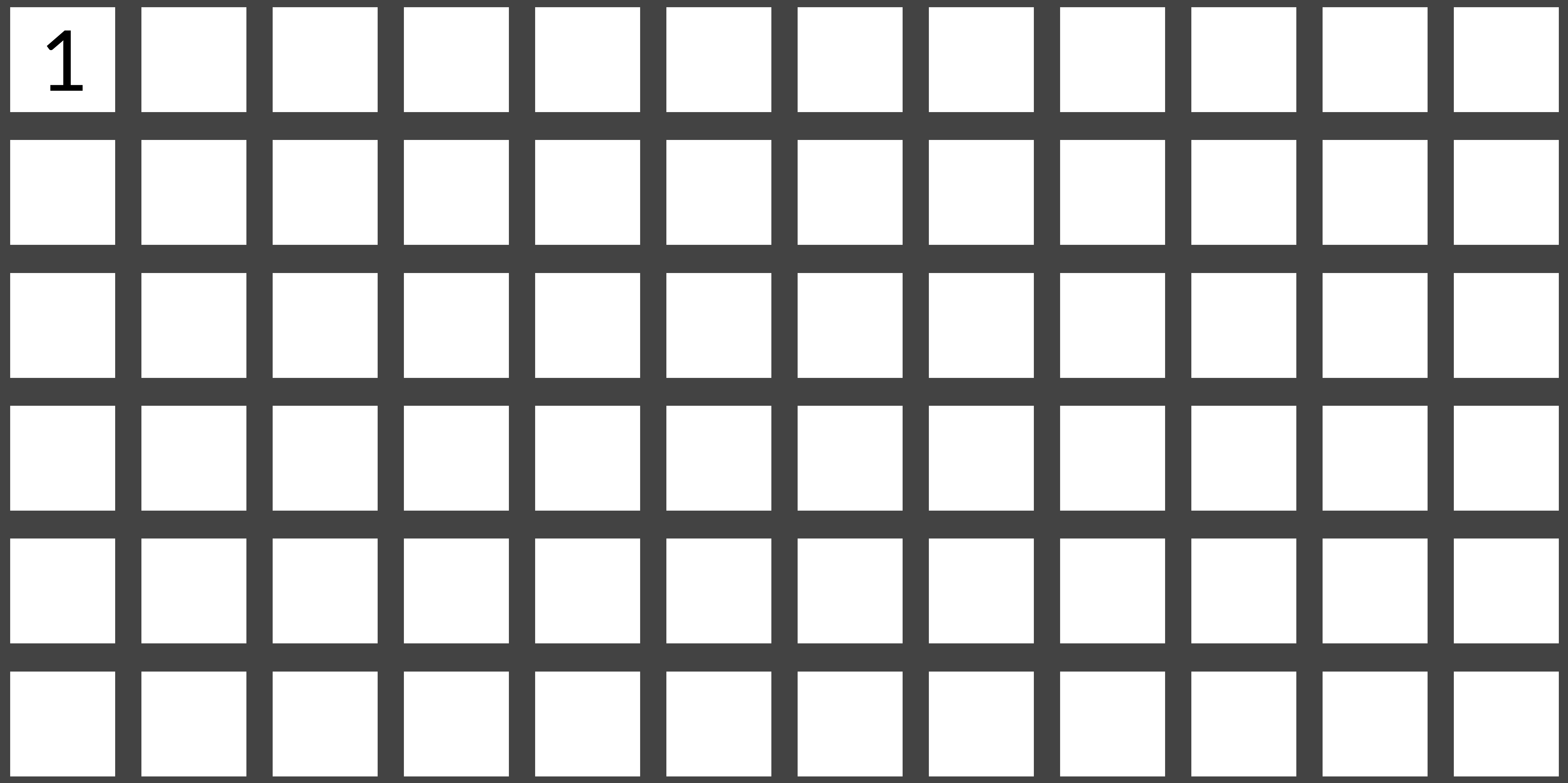

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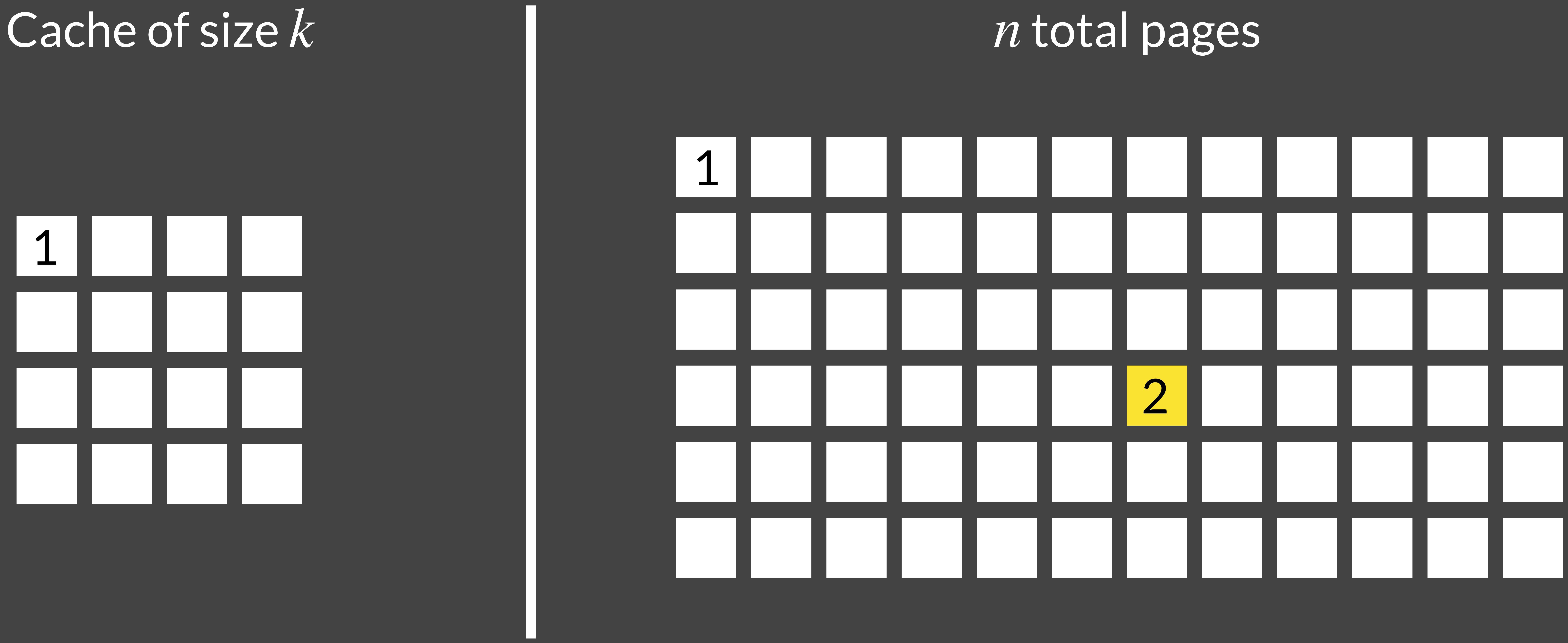

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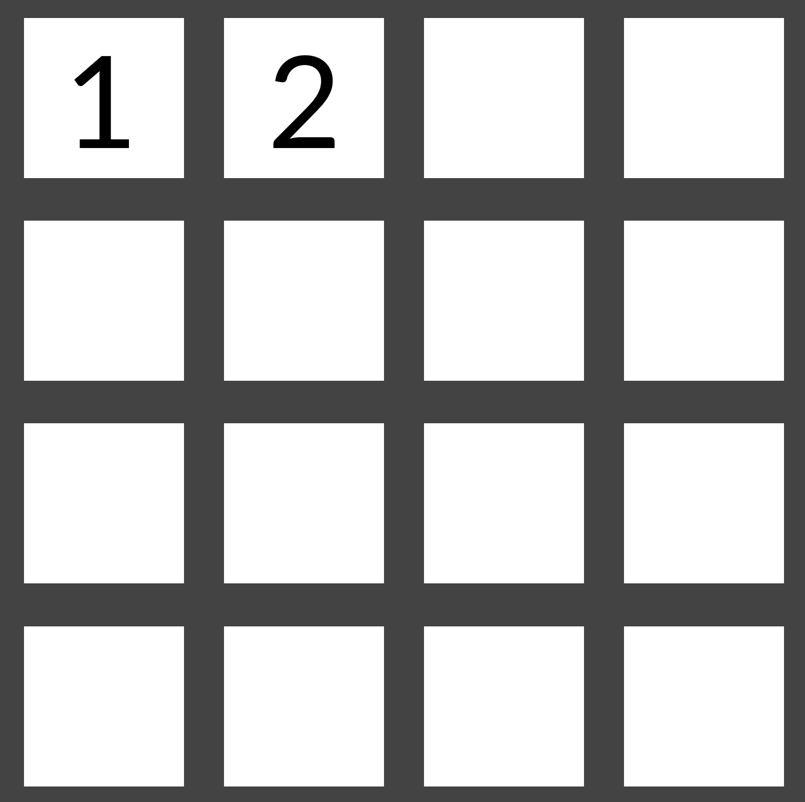

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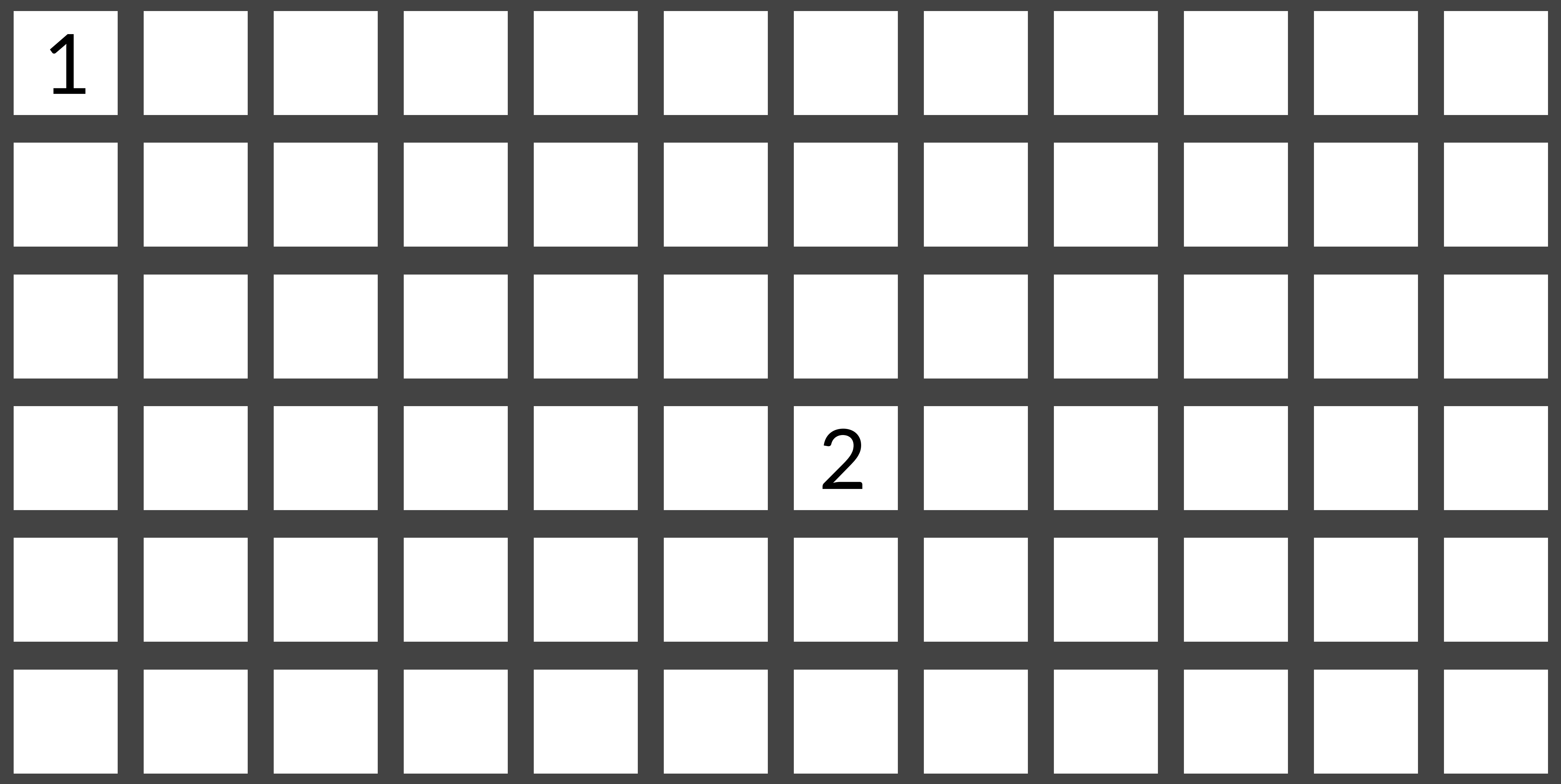

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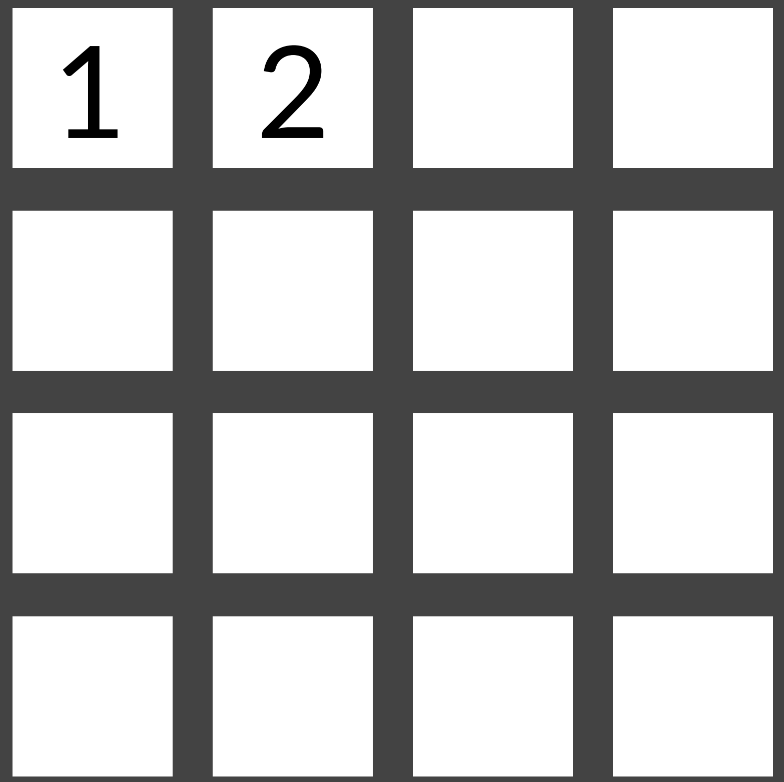

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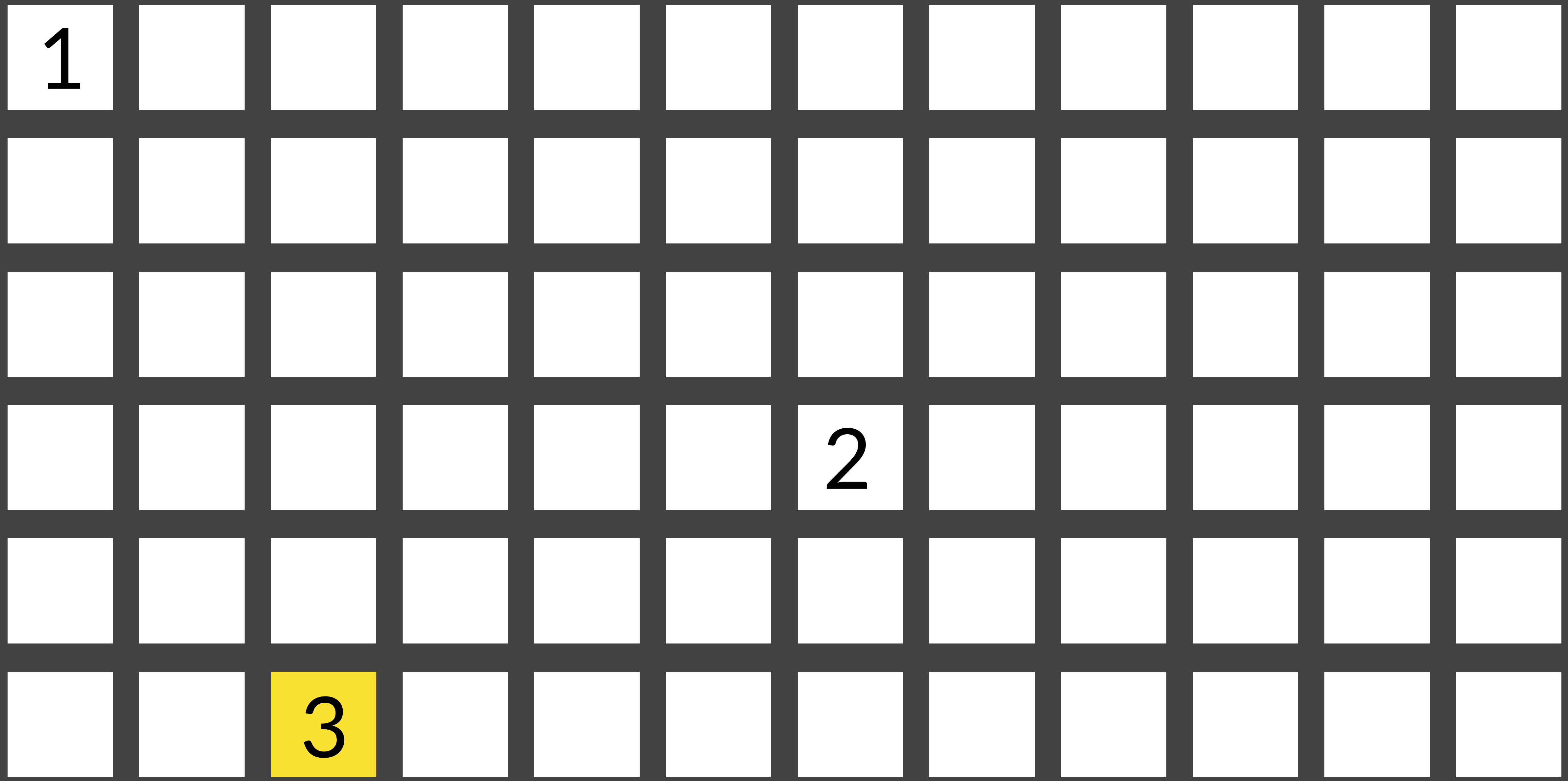

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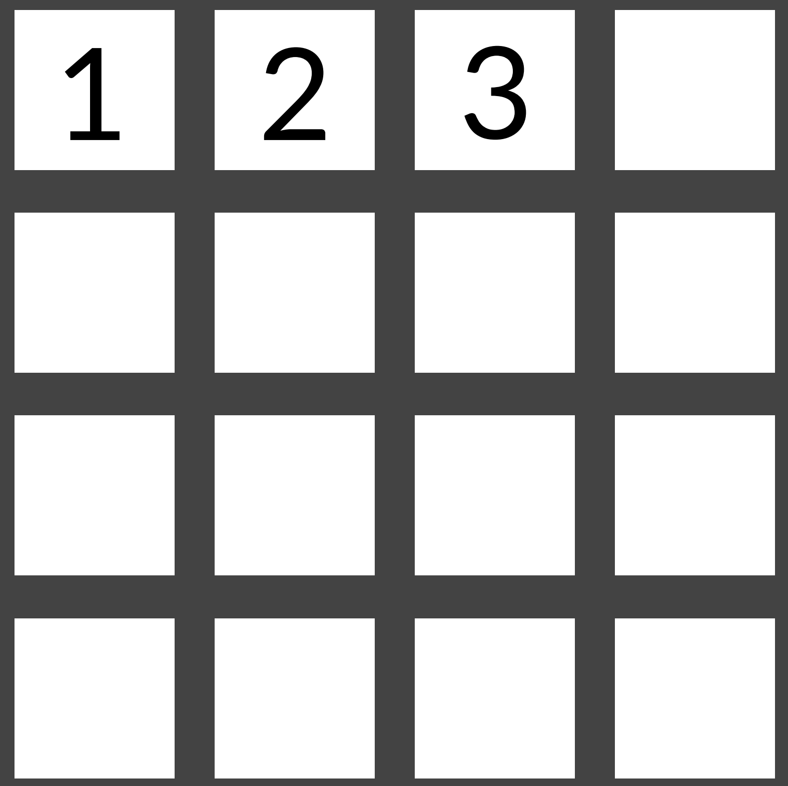

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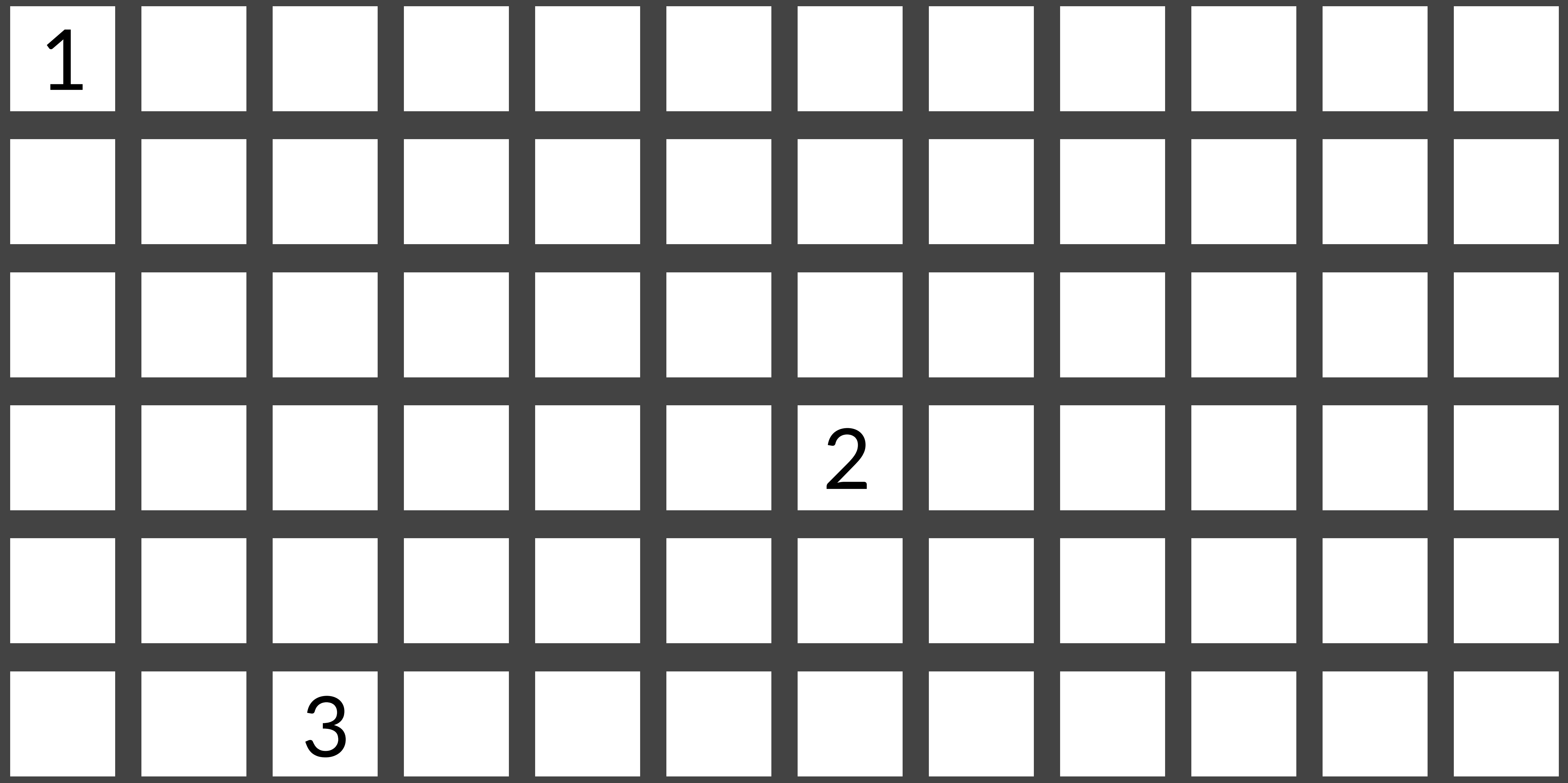

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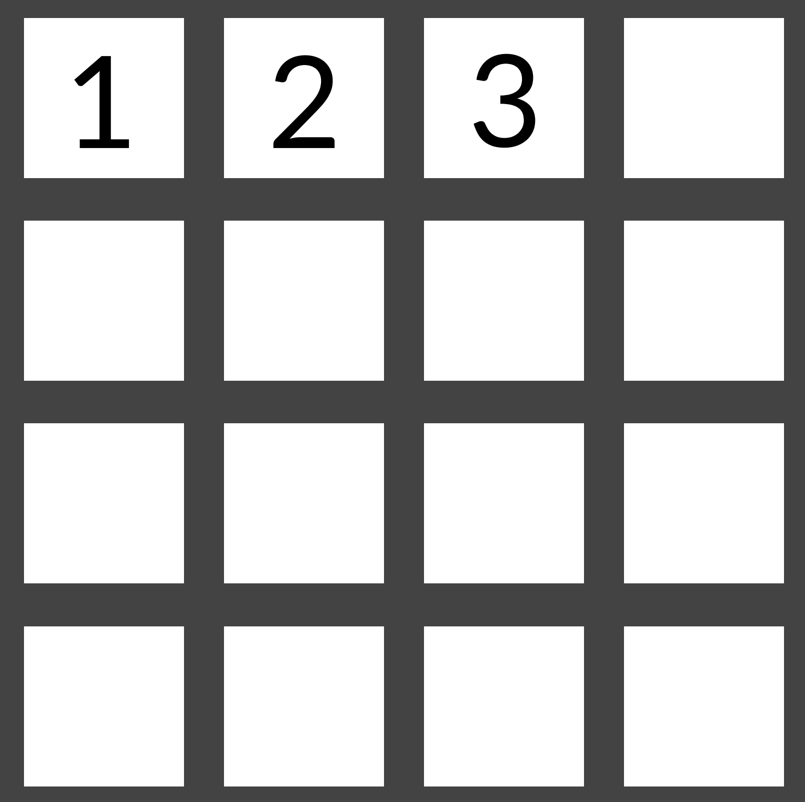

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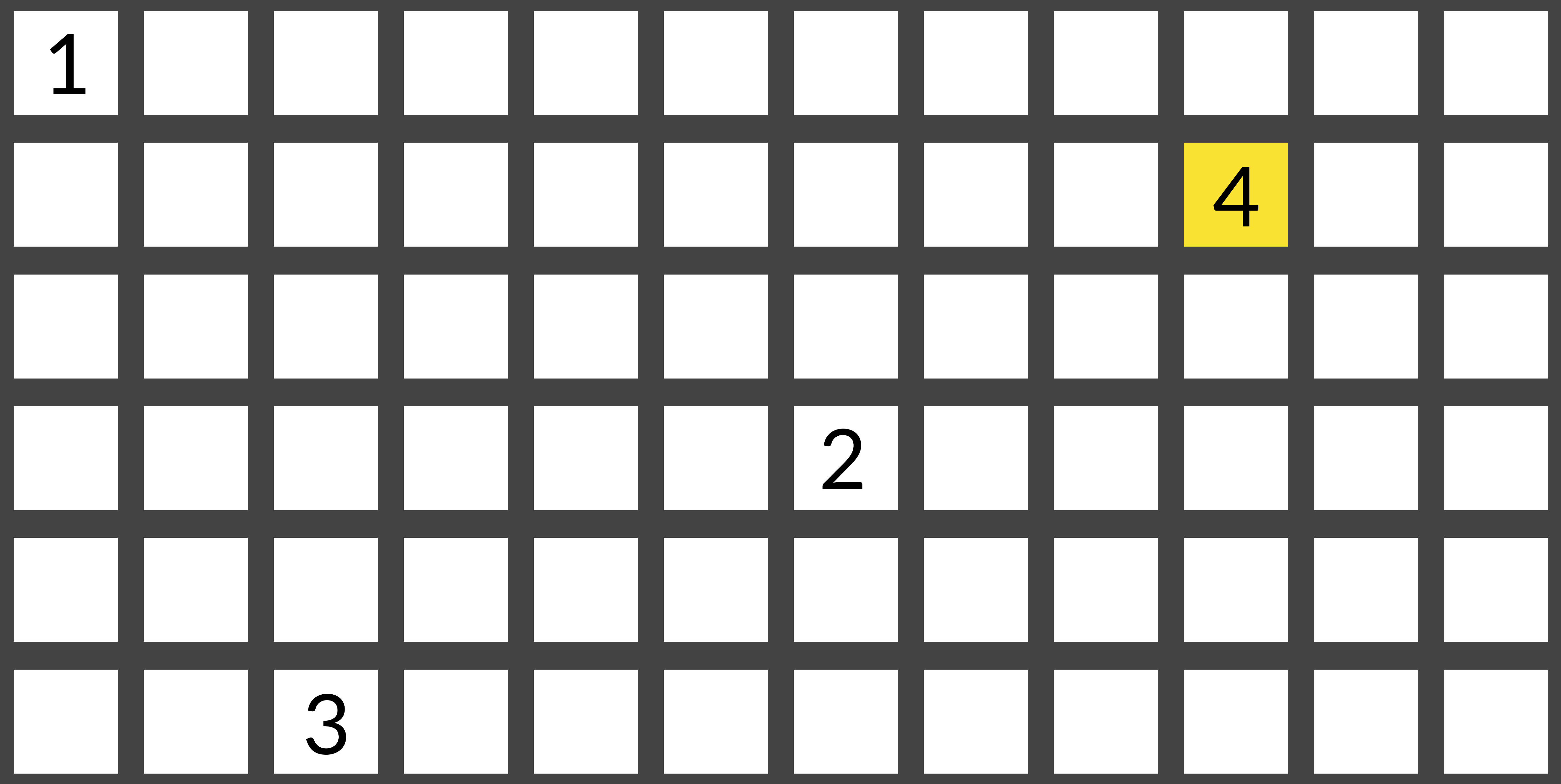

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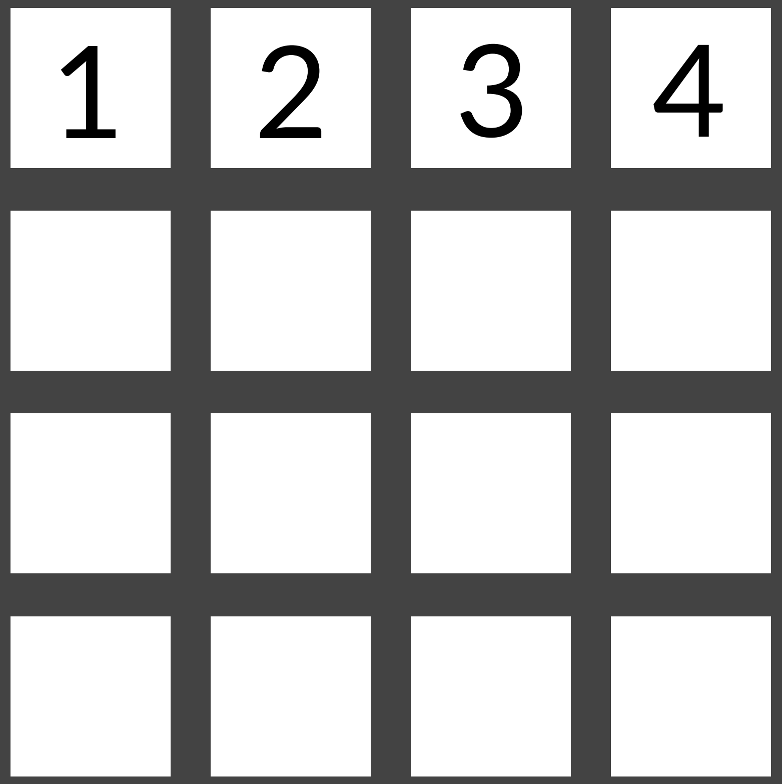

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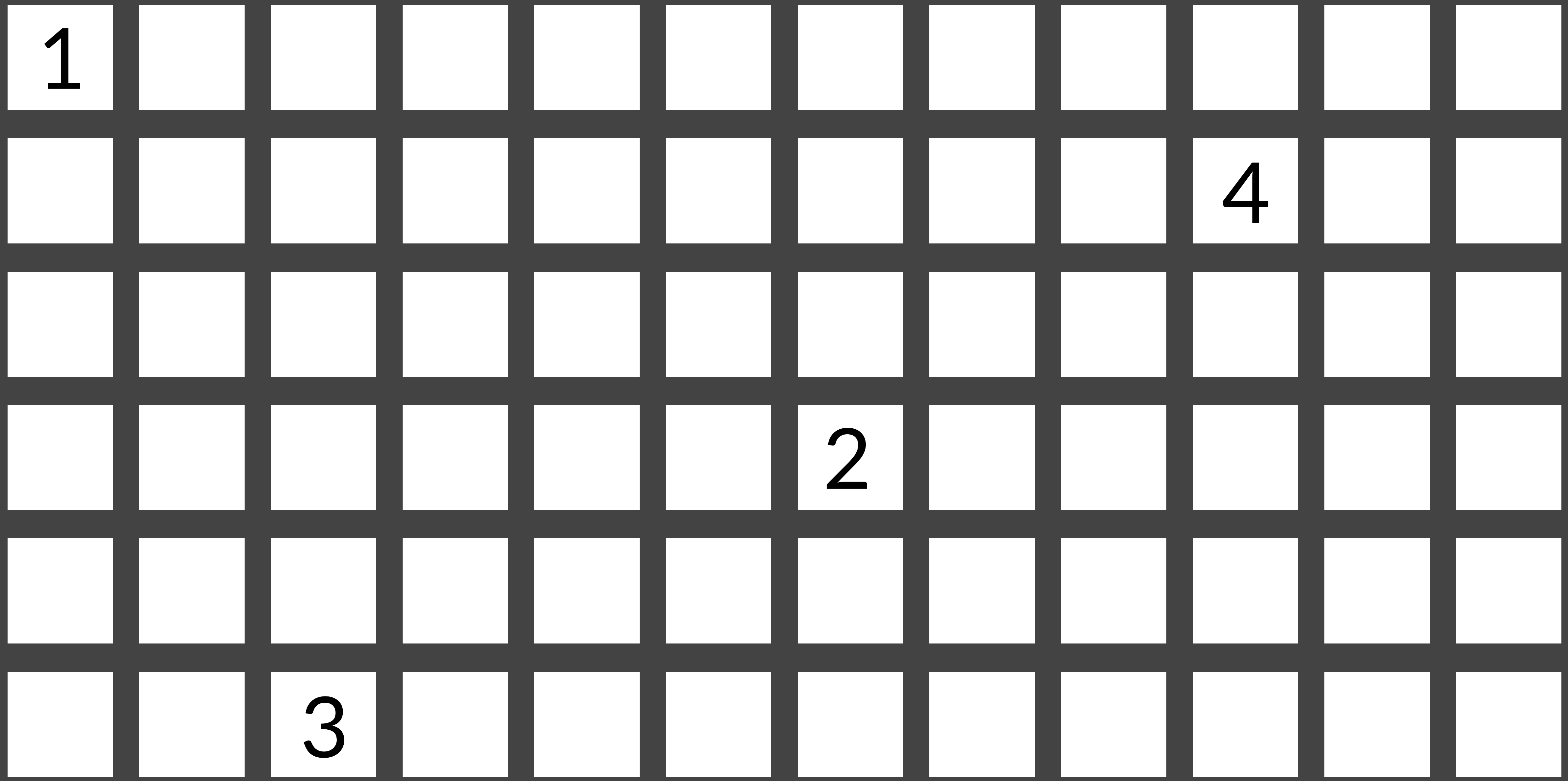

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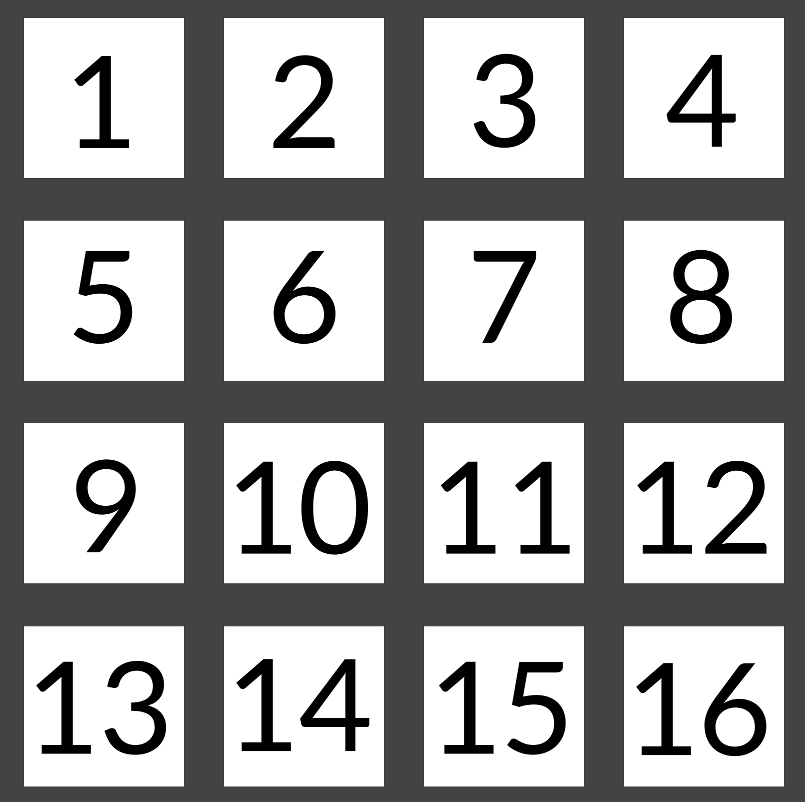

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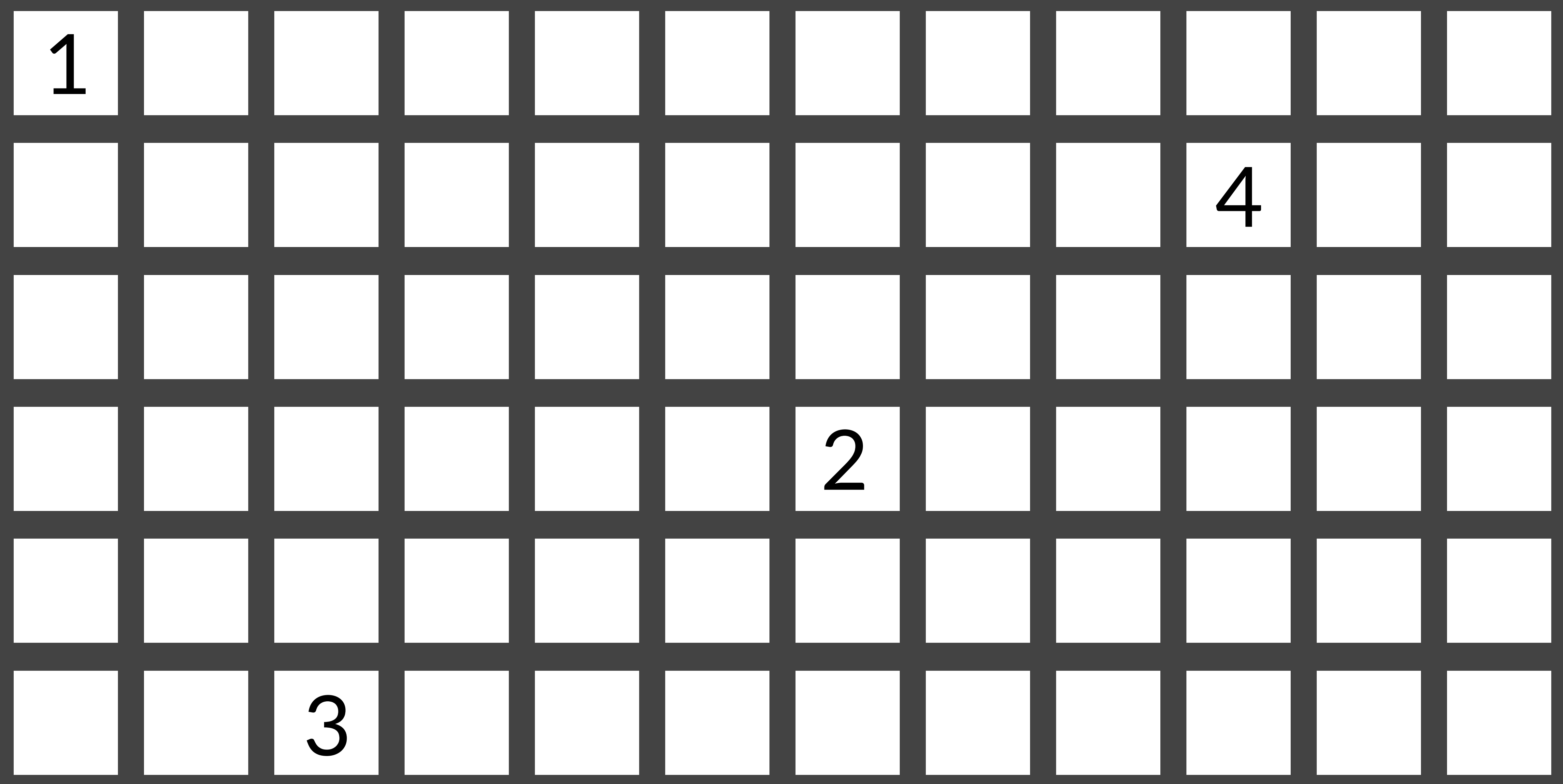

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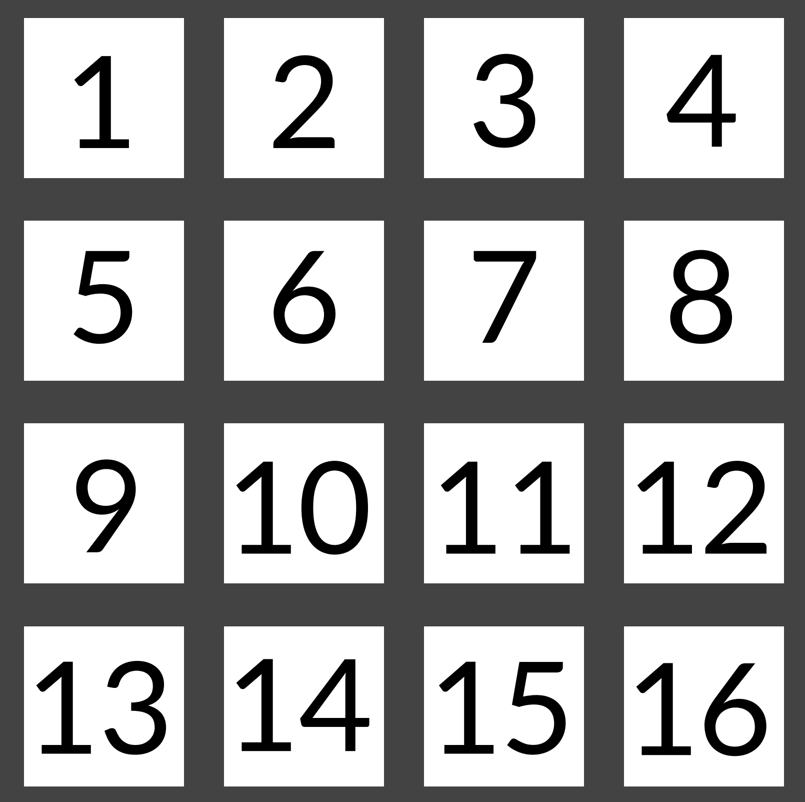

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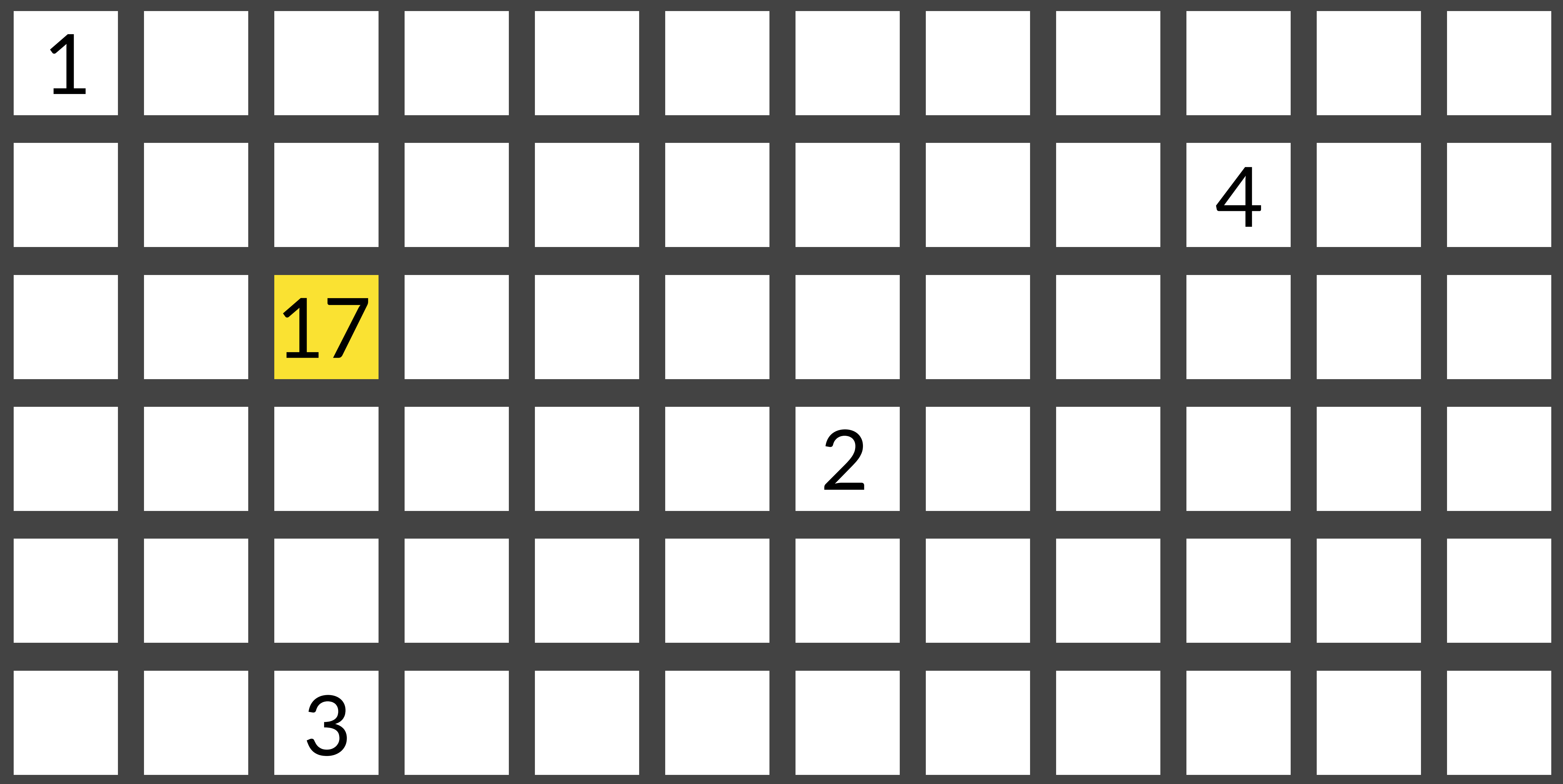

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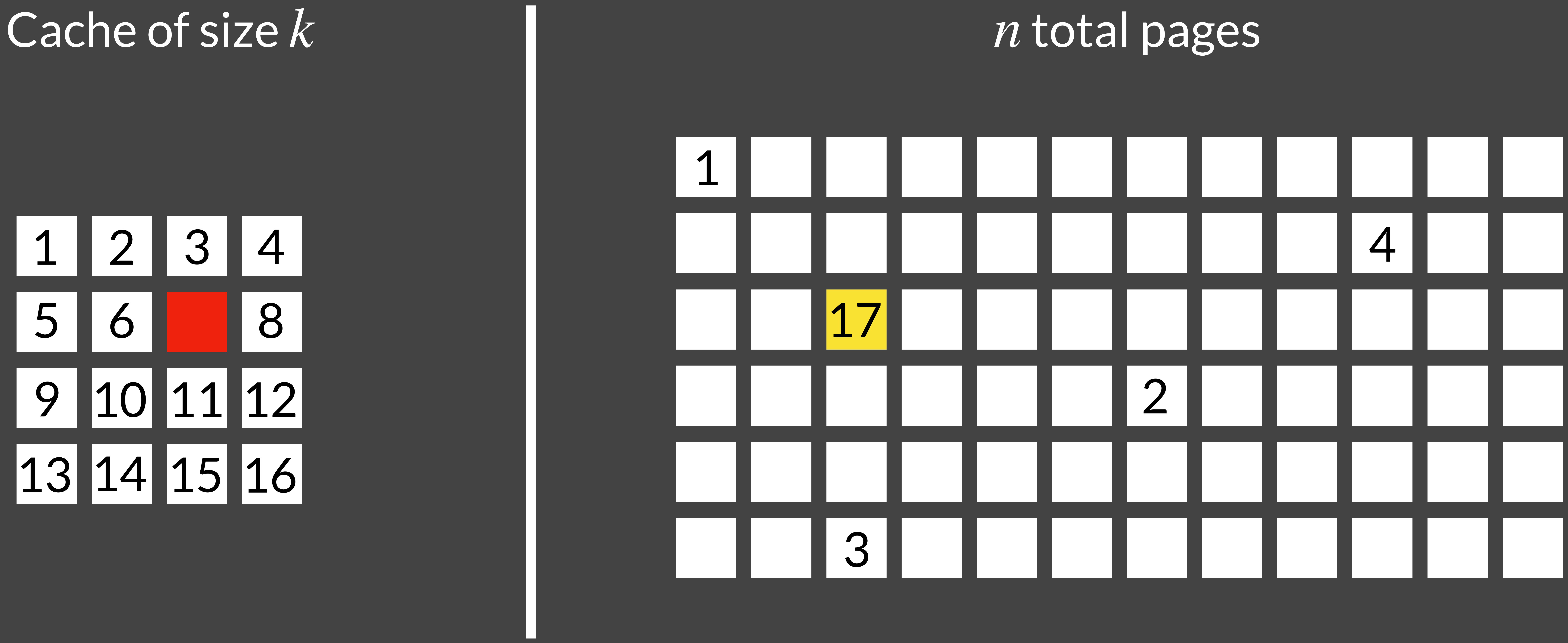

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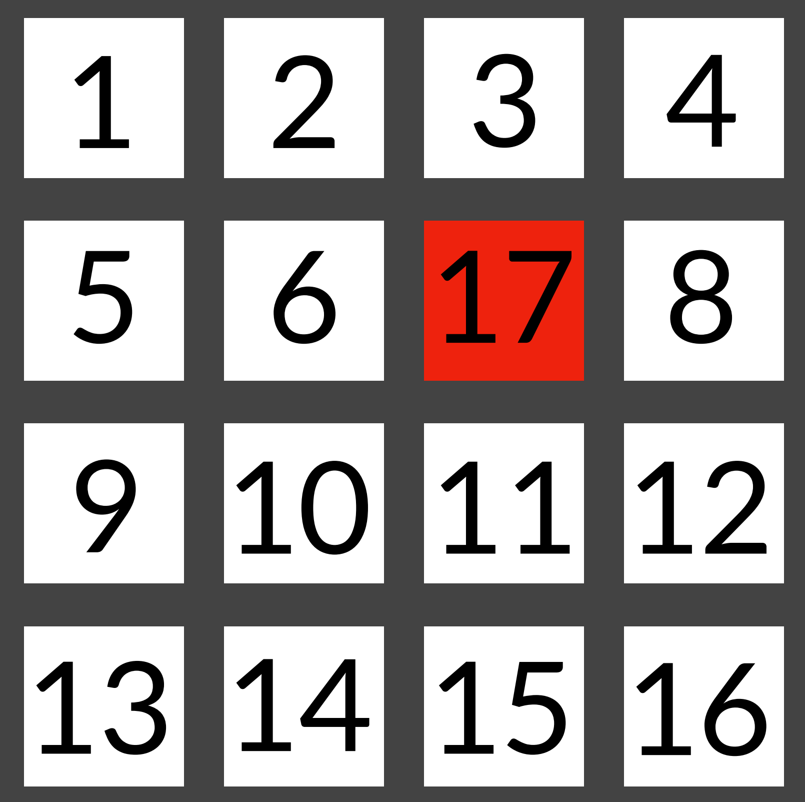

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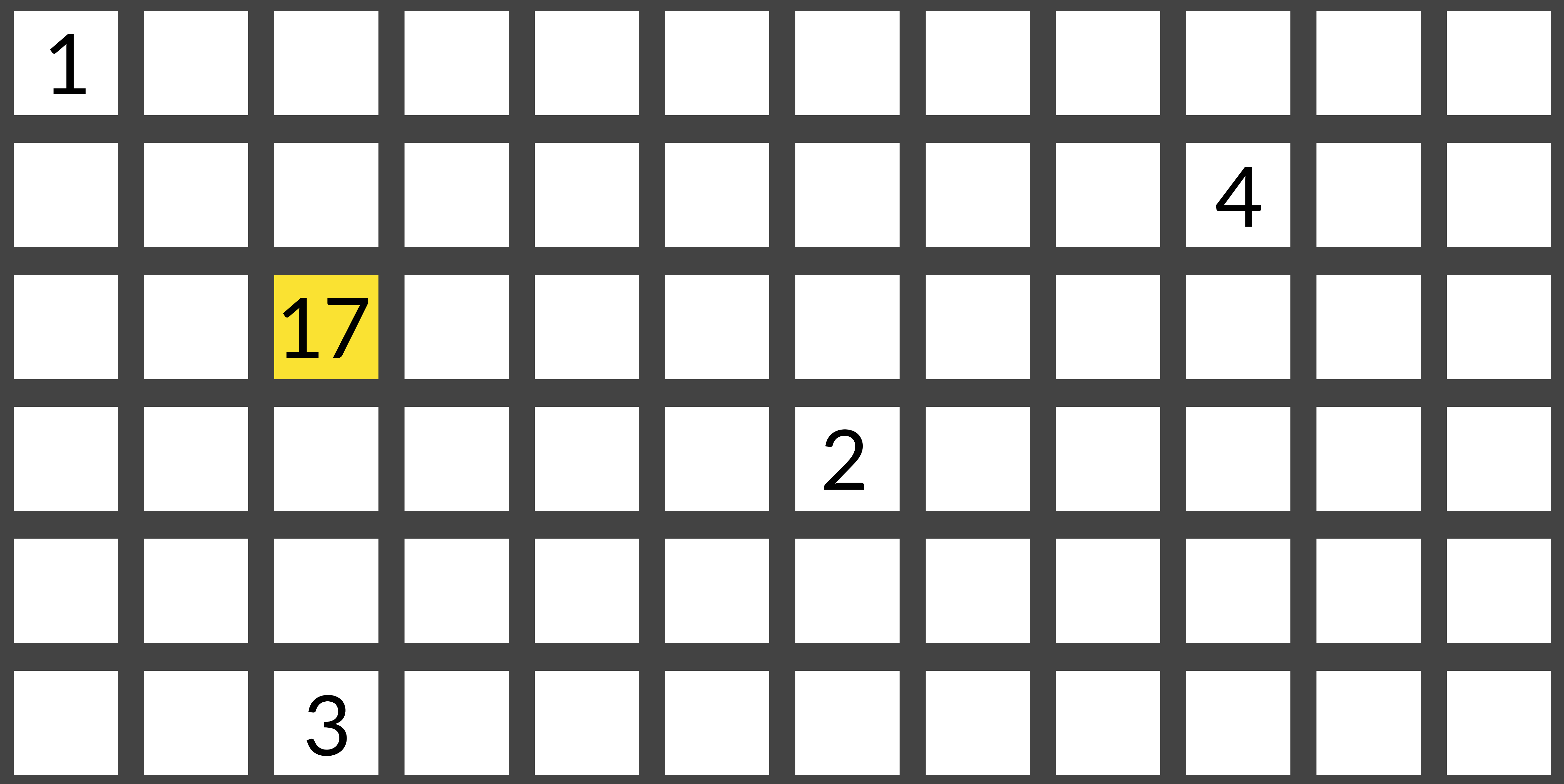

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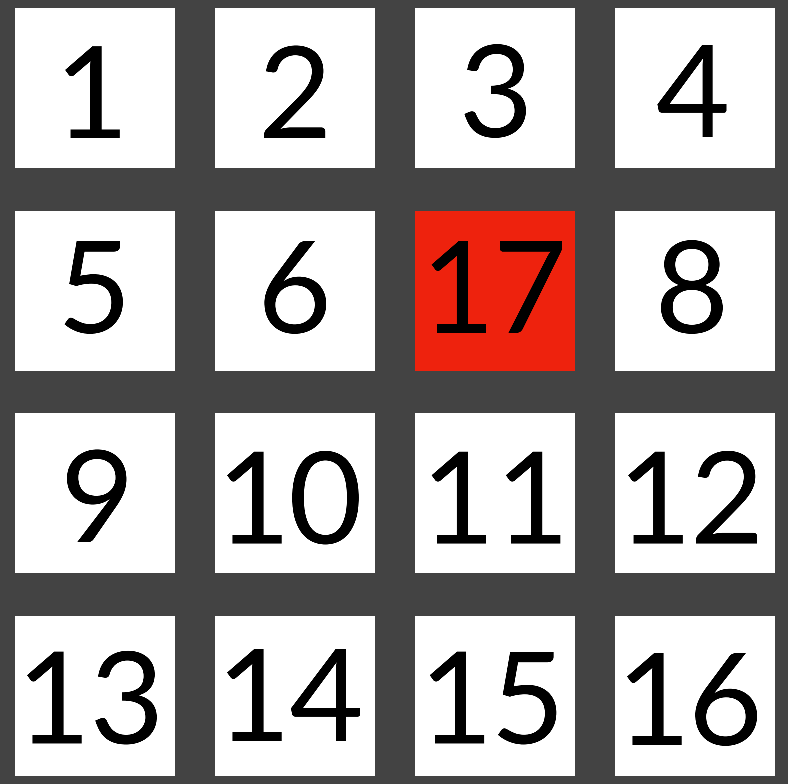

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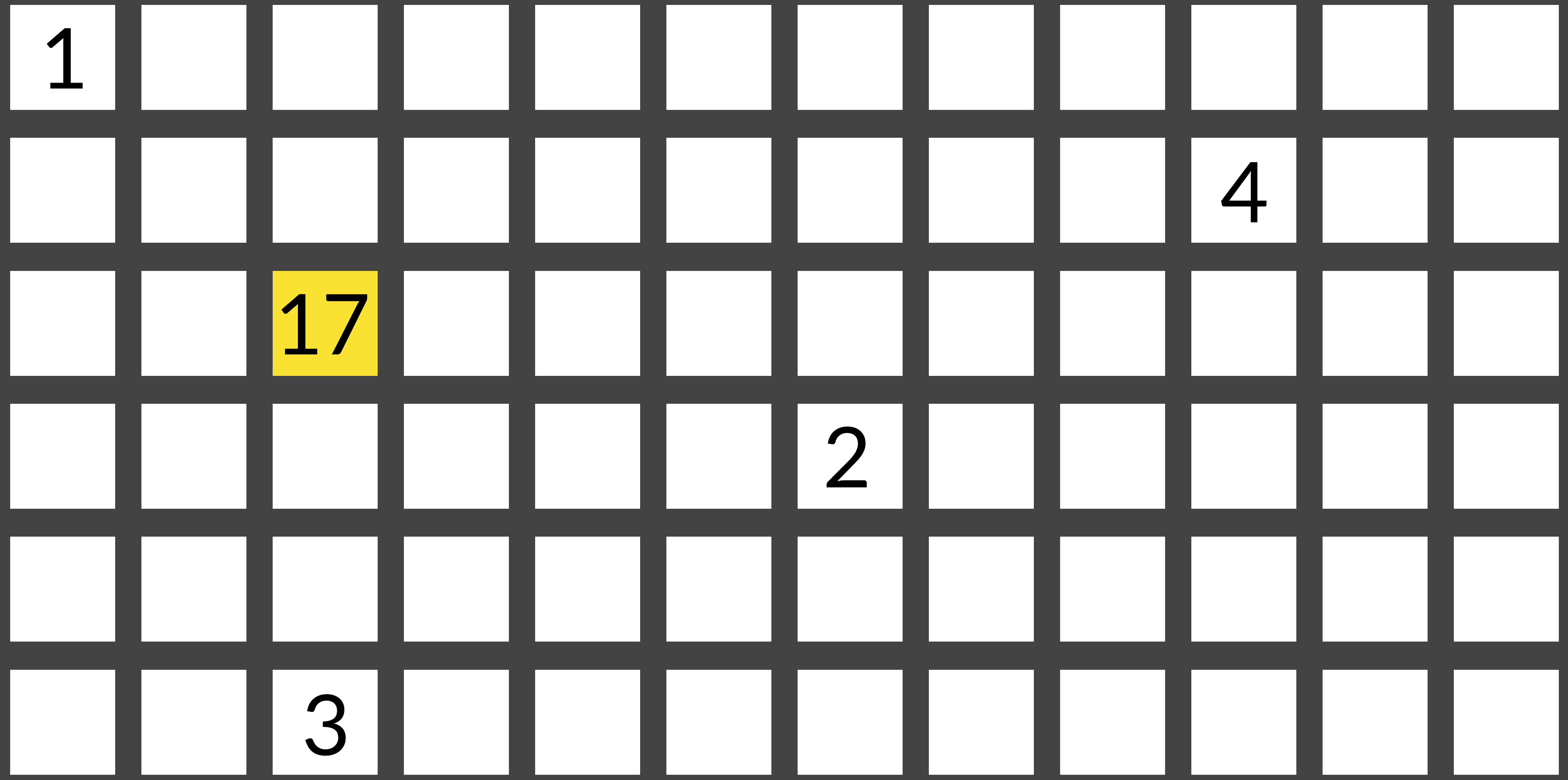
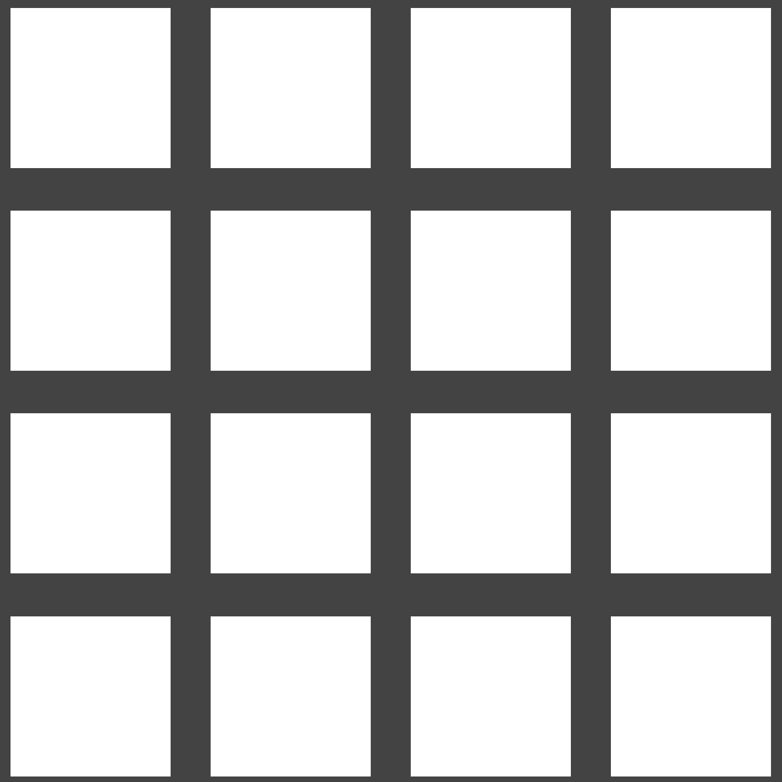


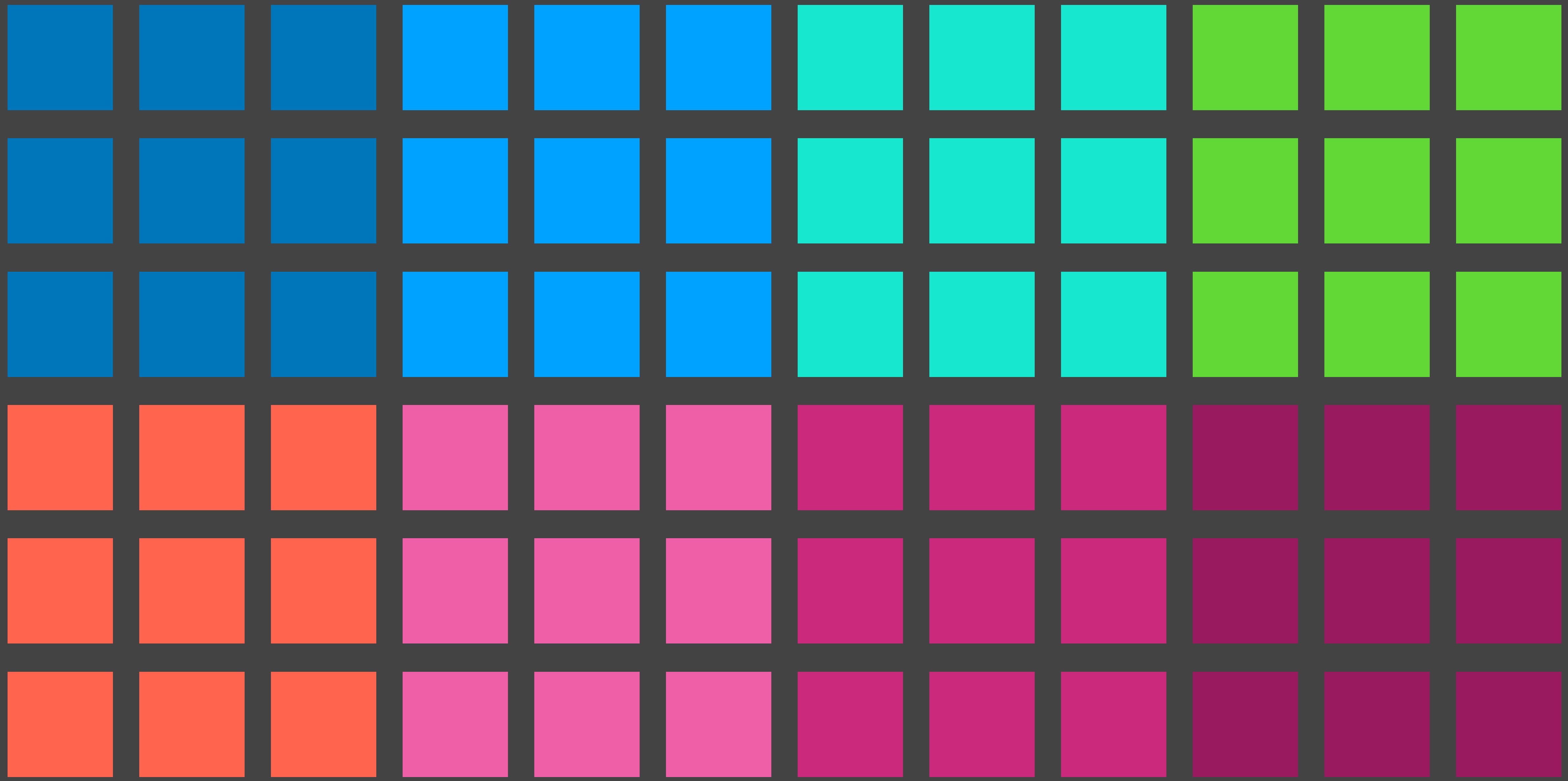
Goal is to minimize number of evictions!

Block-Aware Caching [Beckmann+ 2021]

Cache of size k



n total pages, **in blocks of size β**



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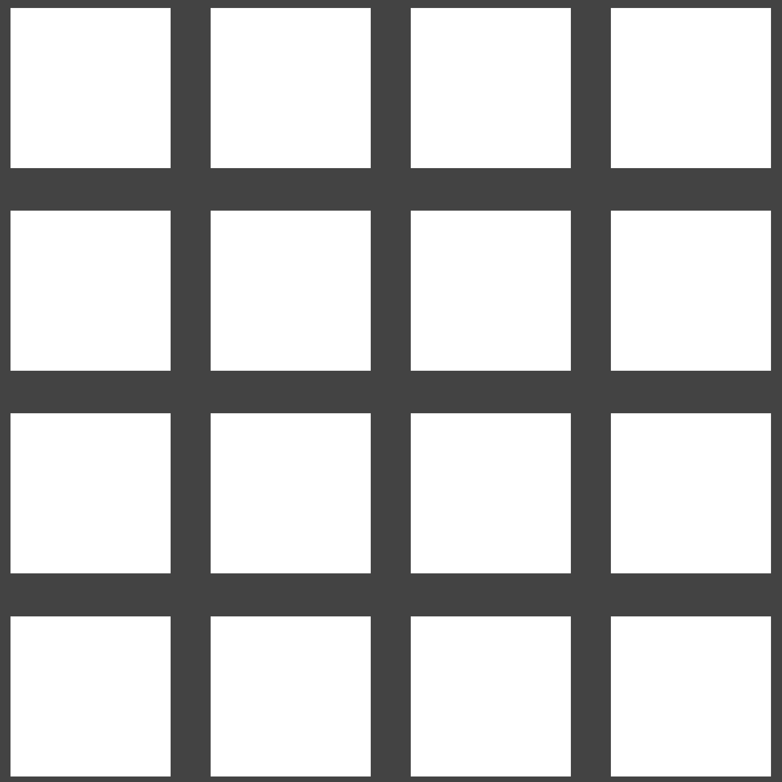

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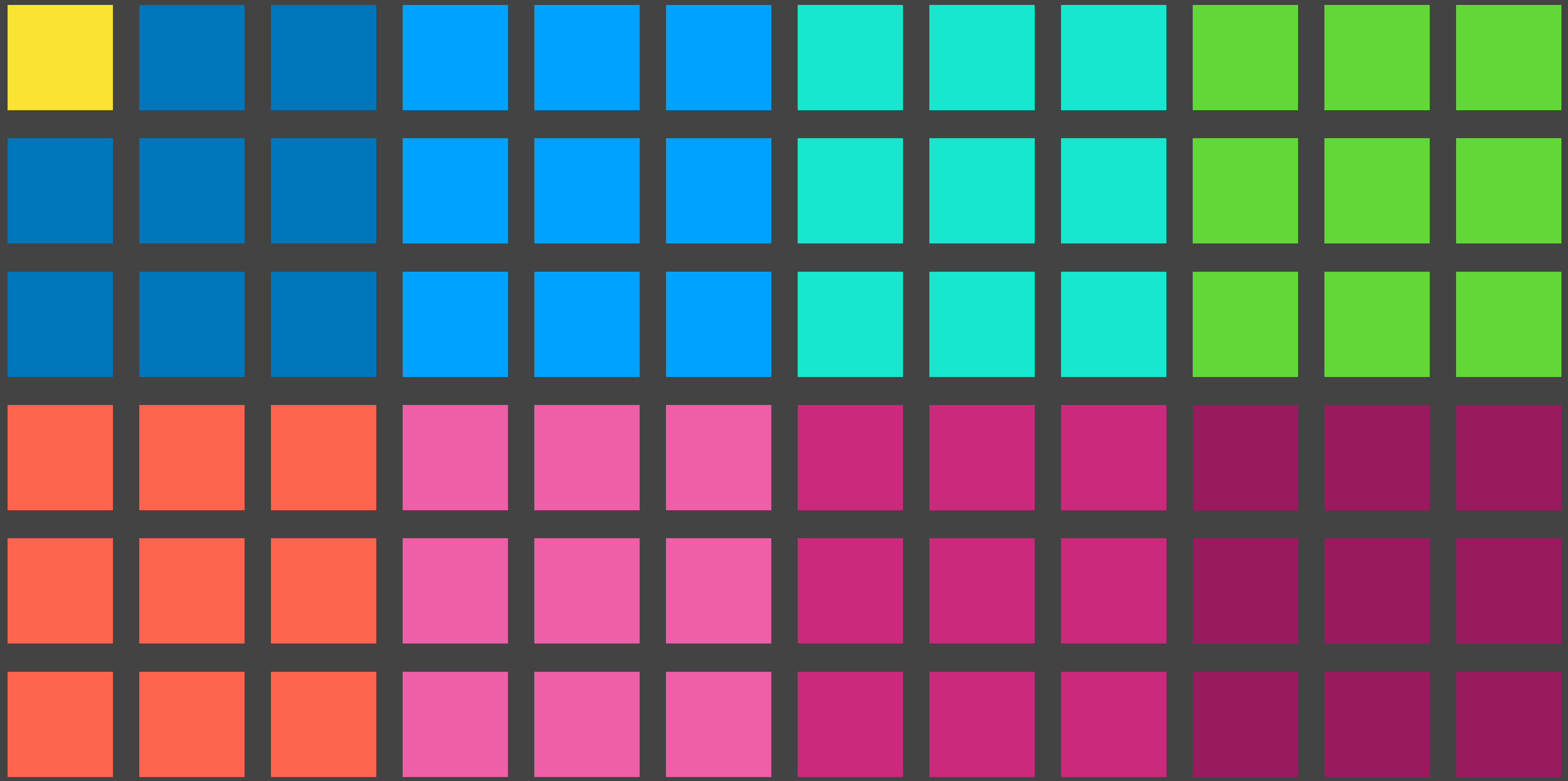

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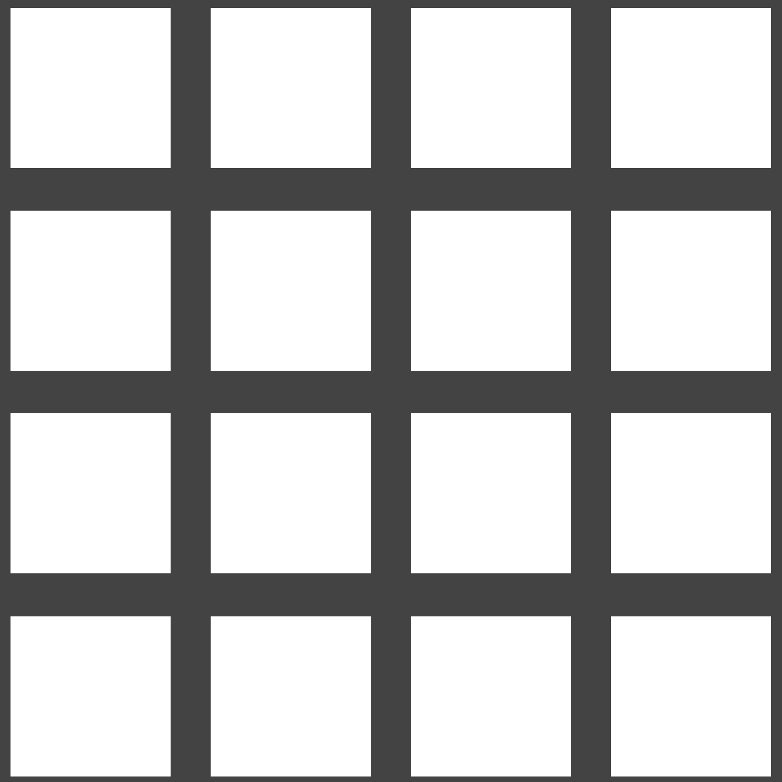

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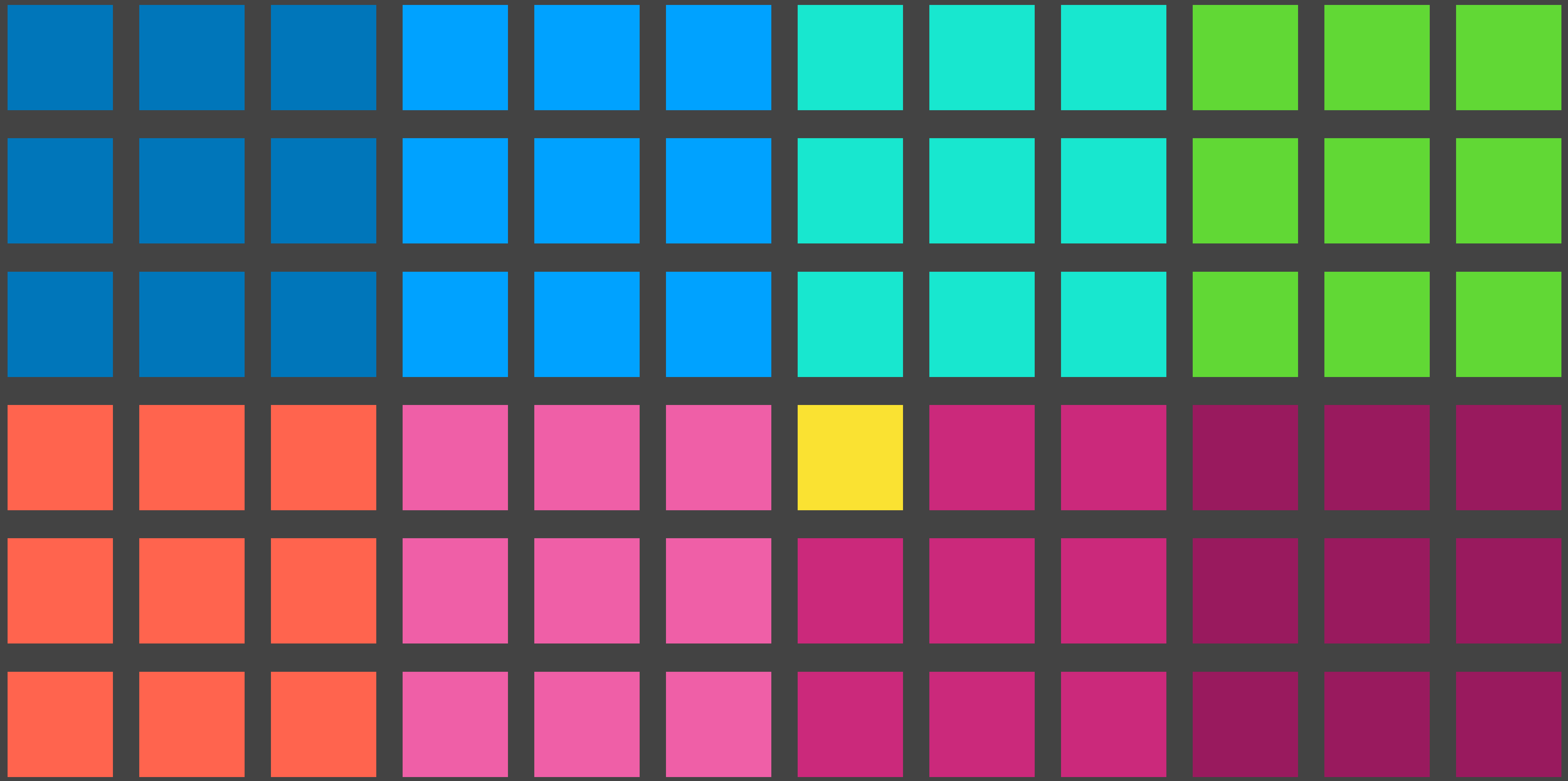

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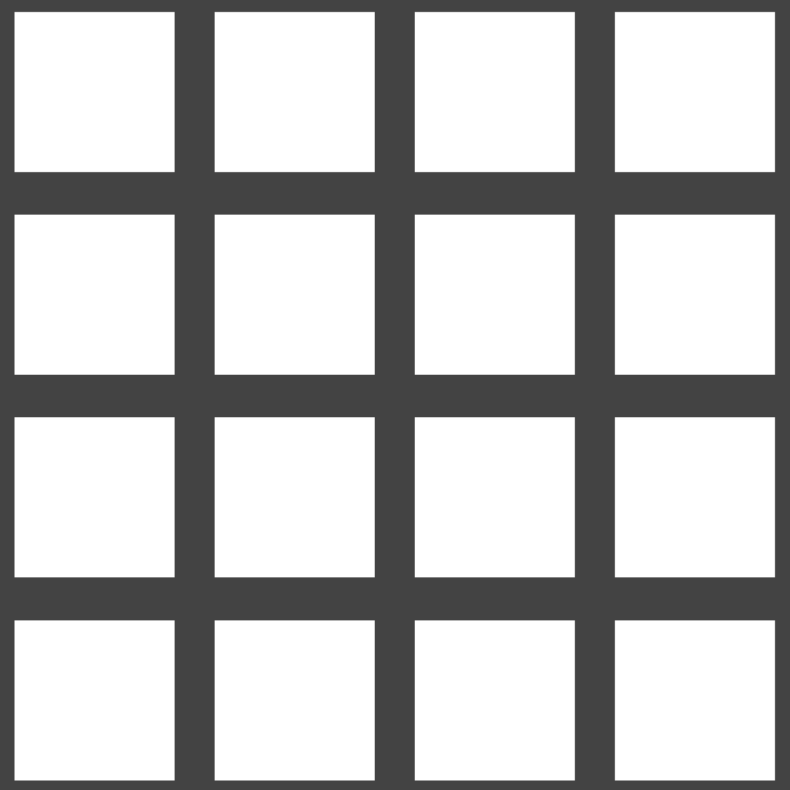

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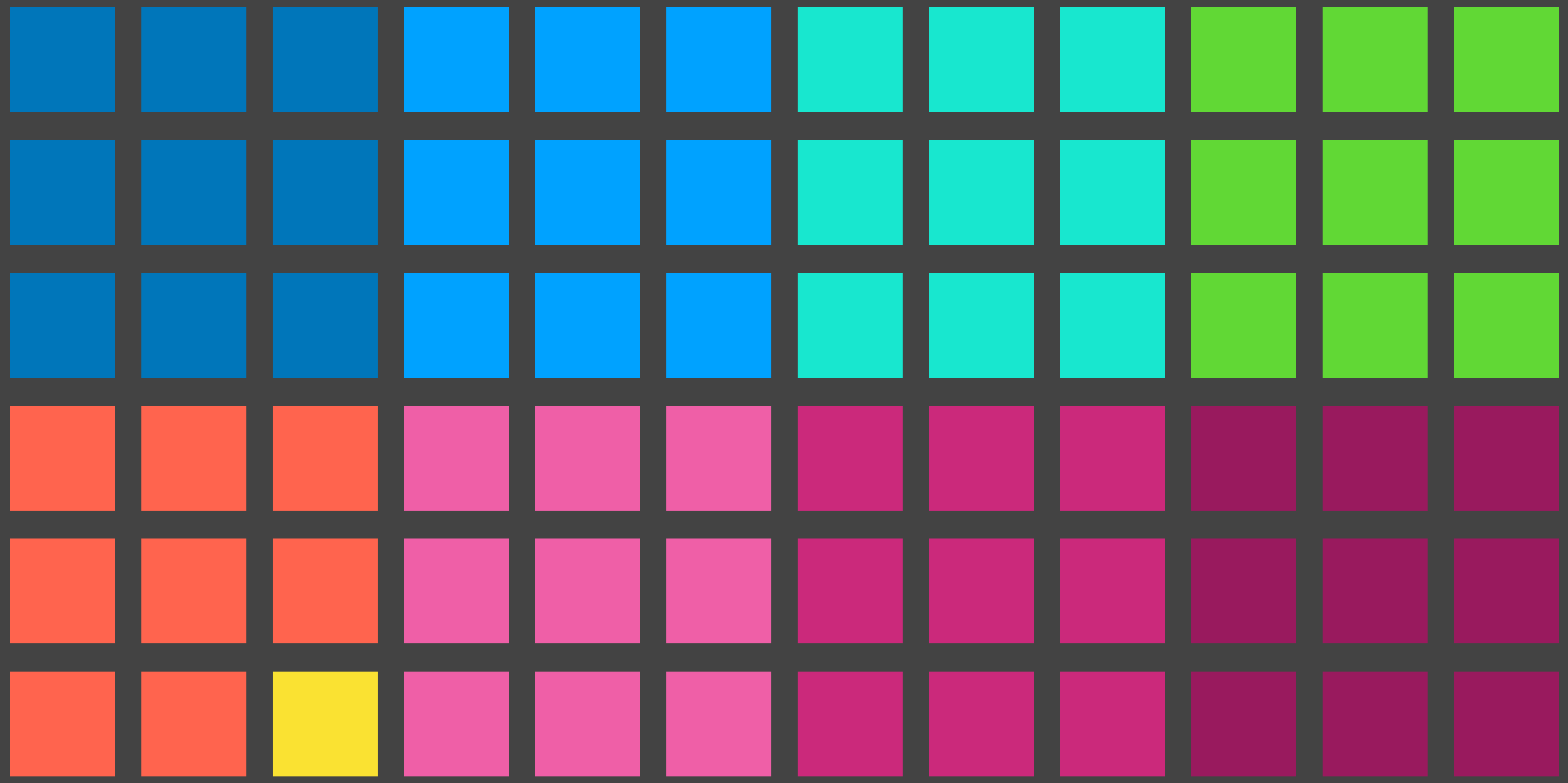

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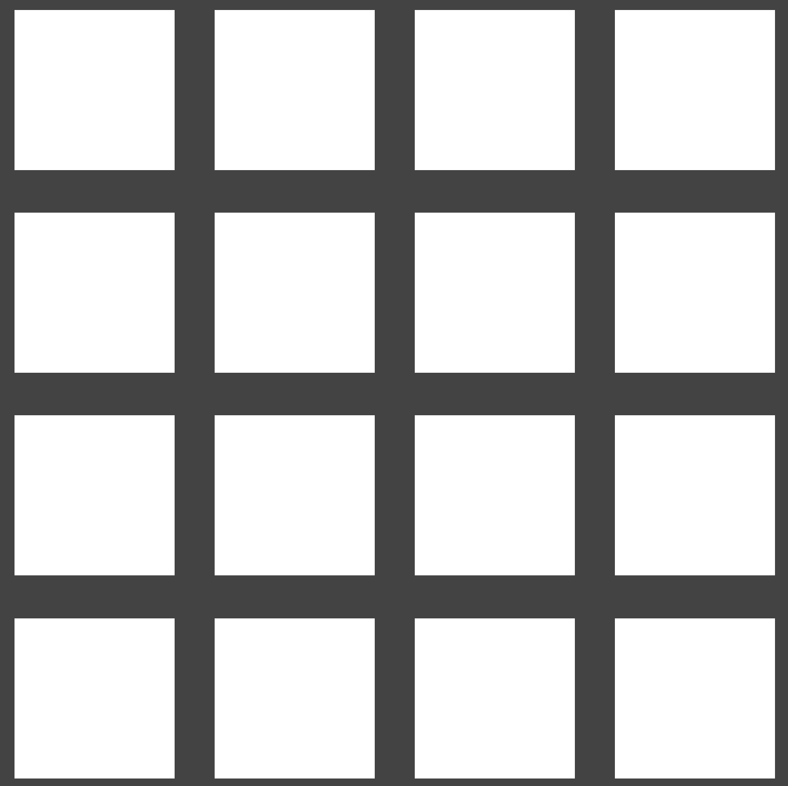

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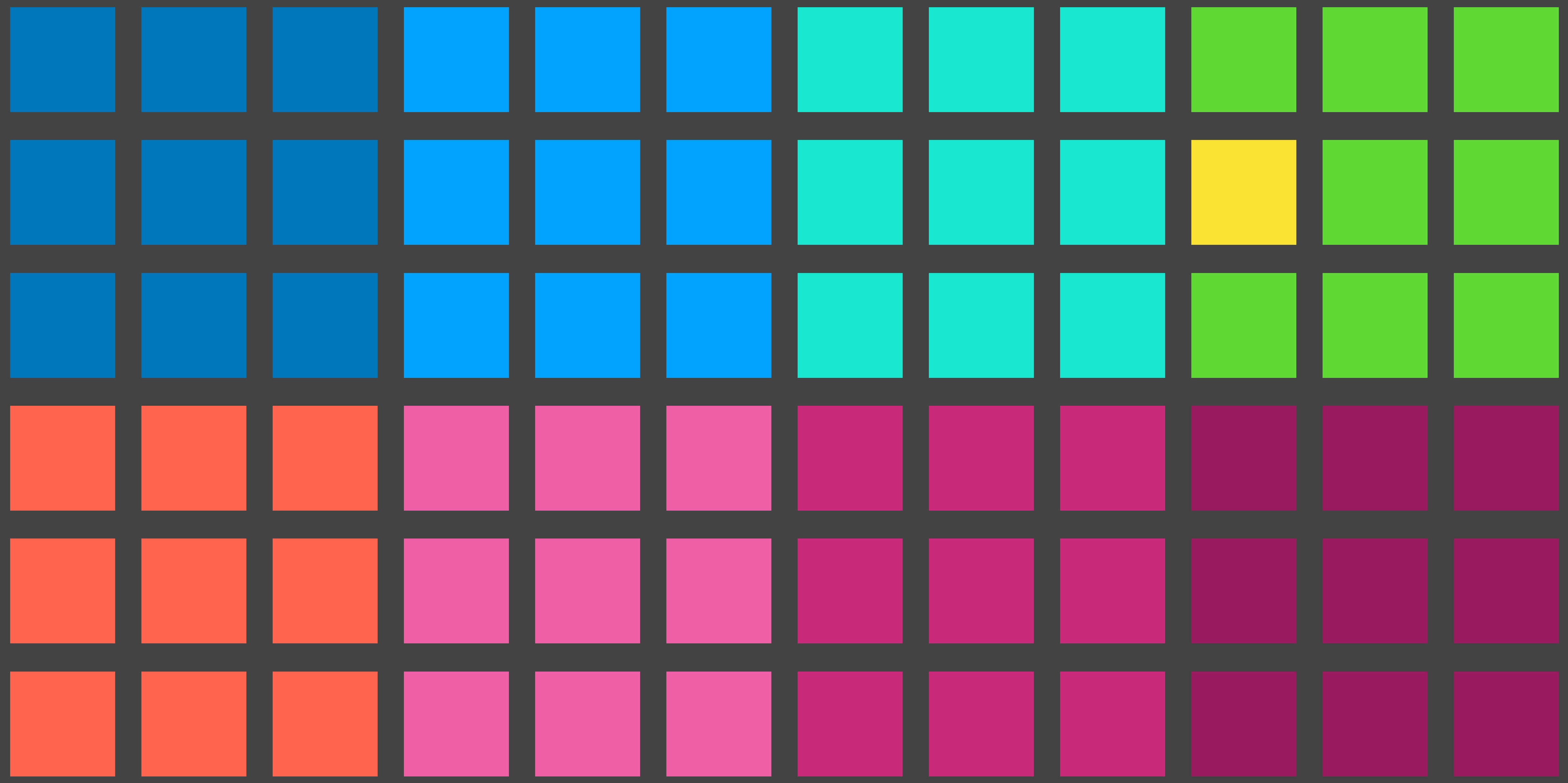

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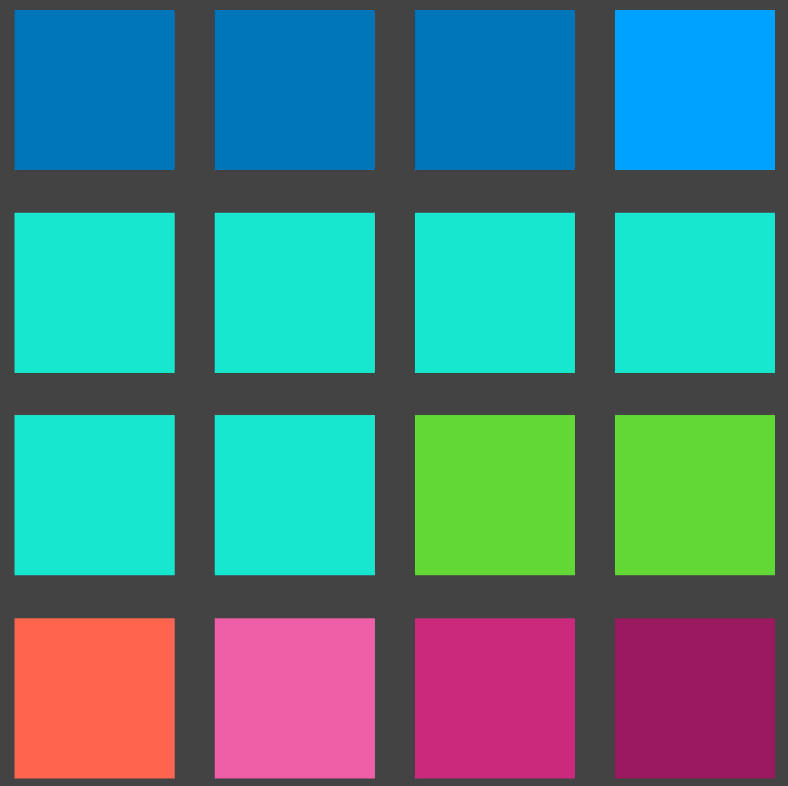

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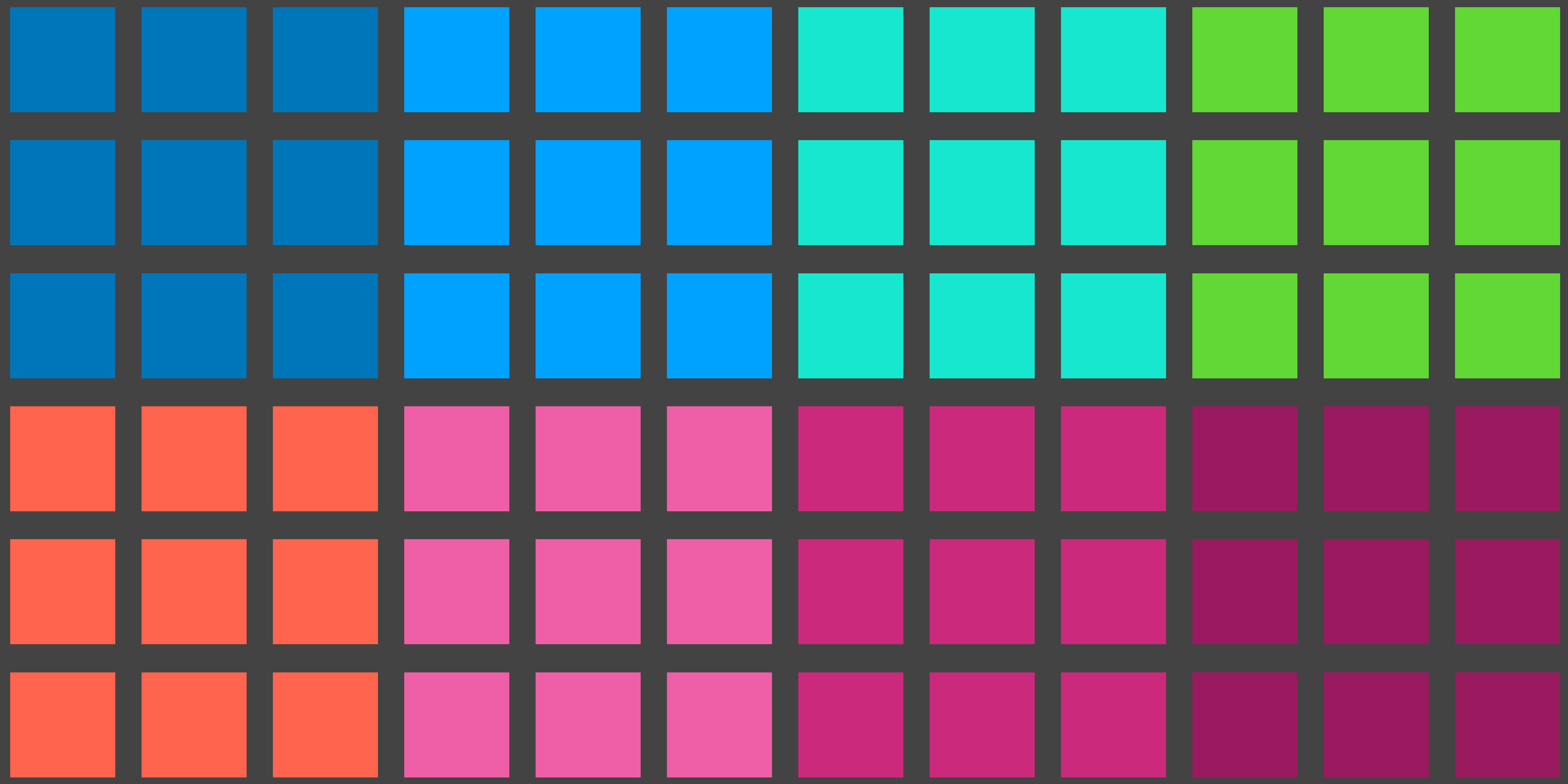

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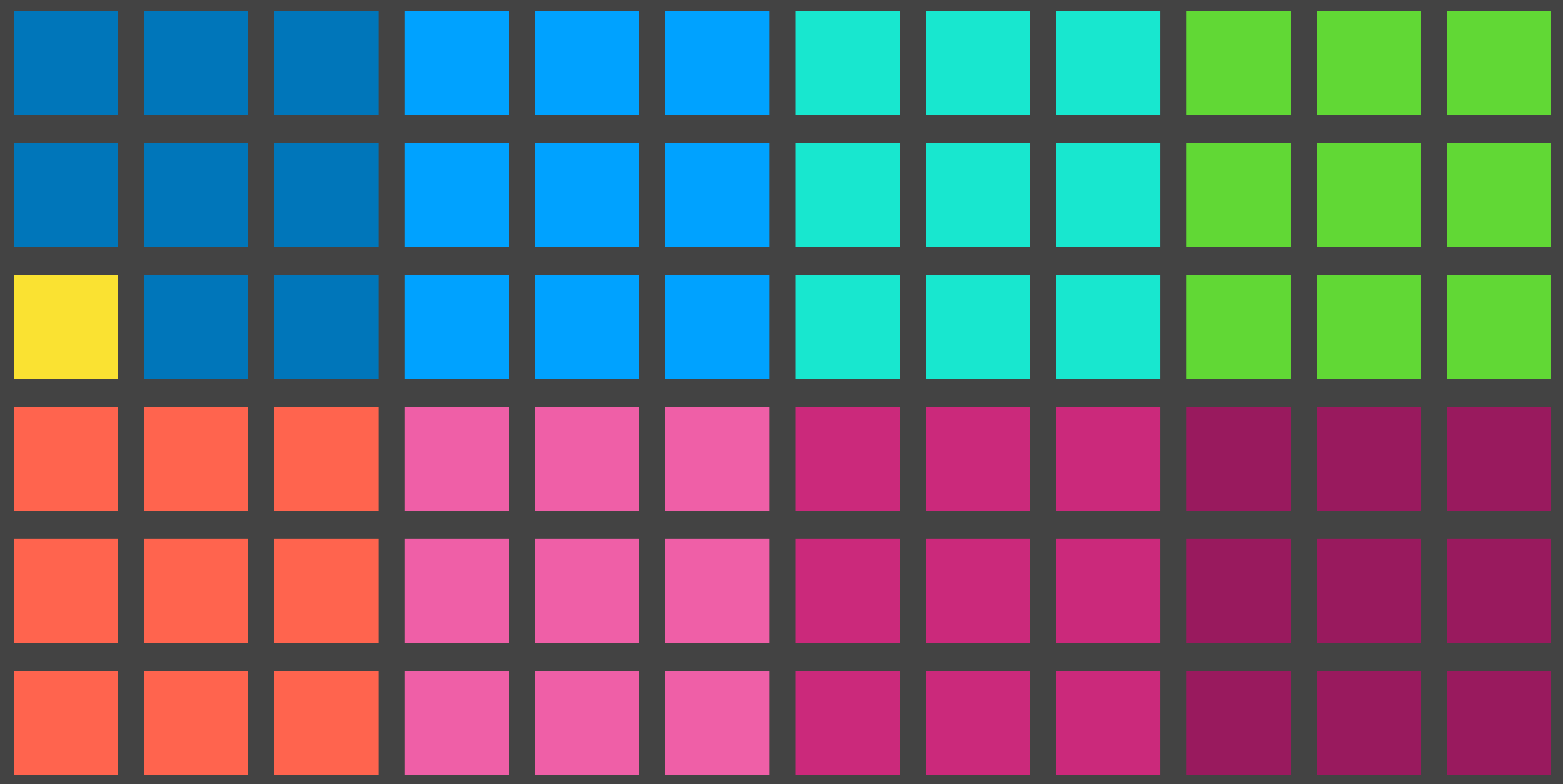

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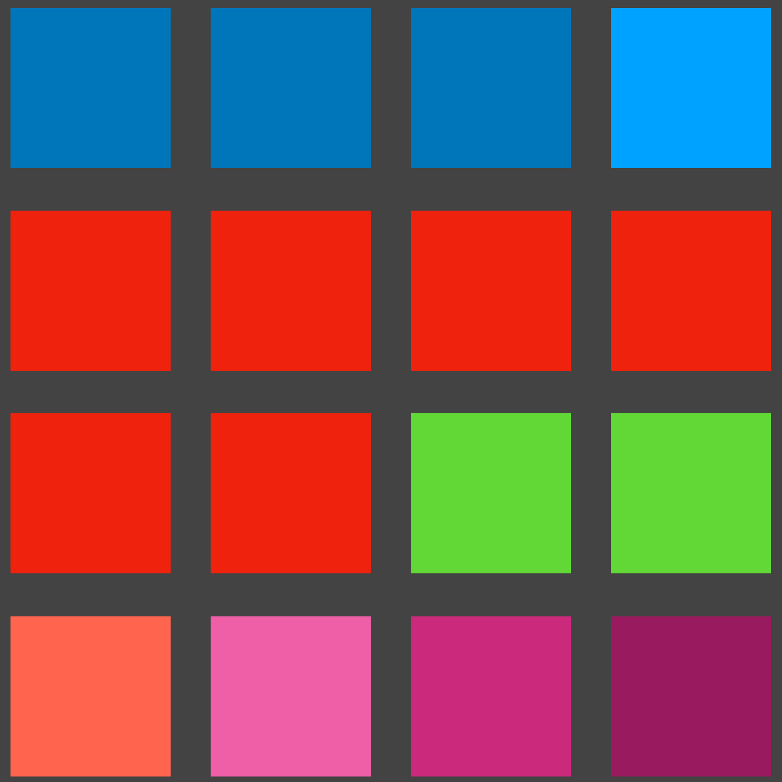

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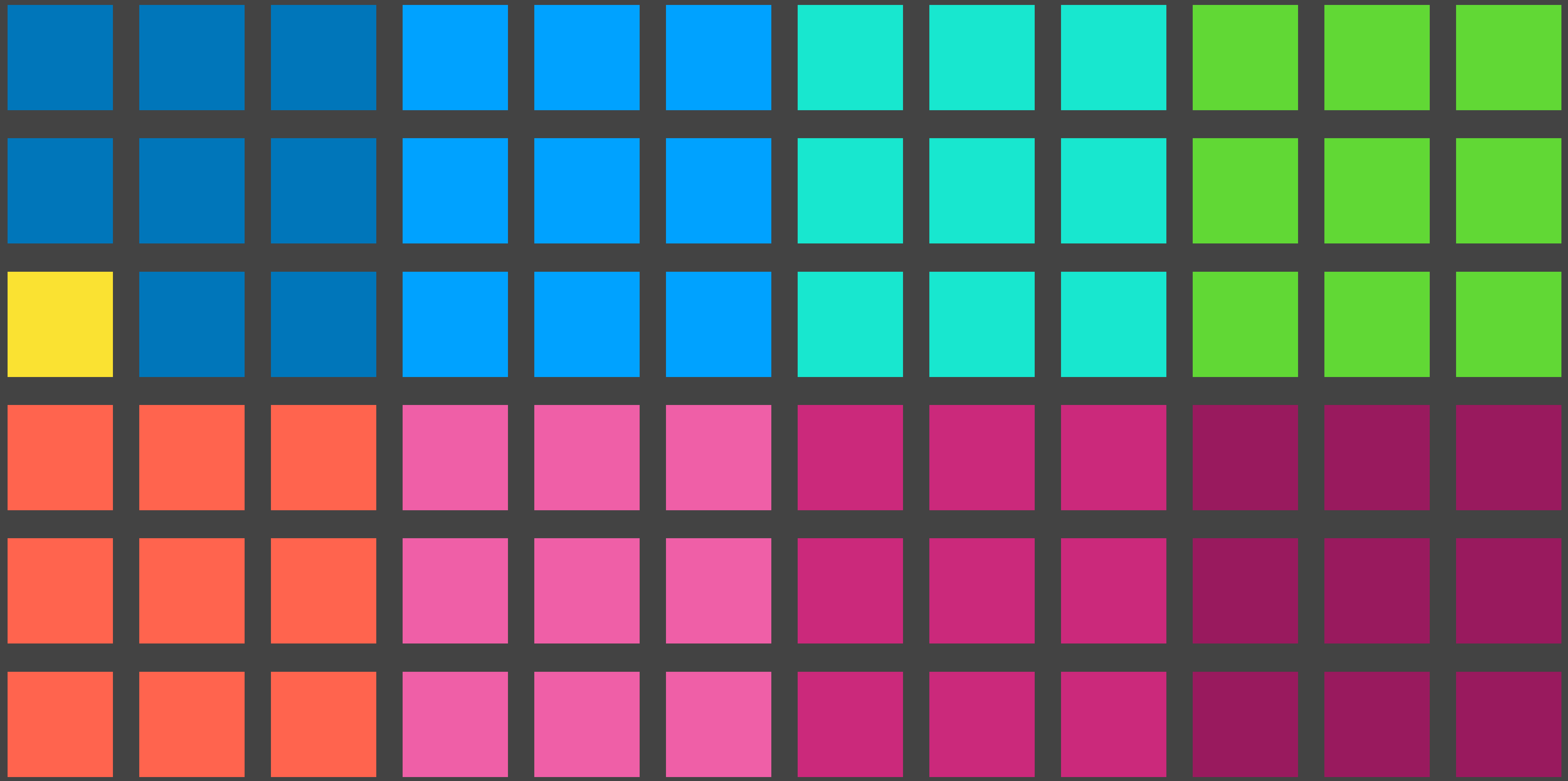

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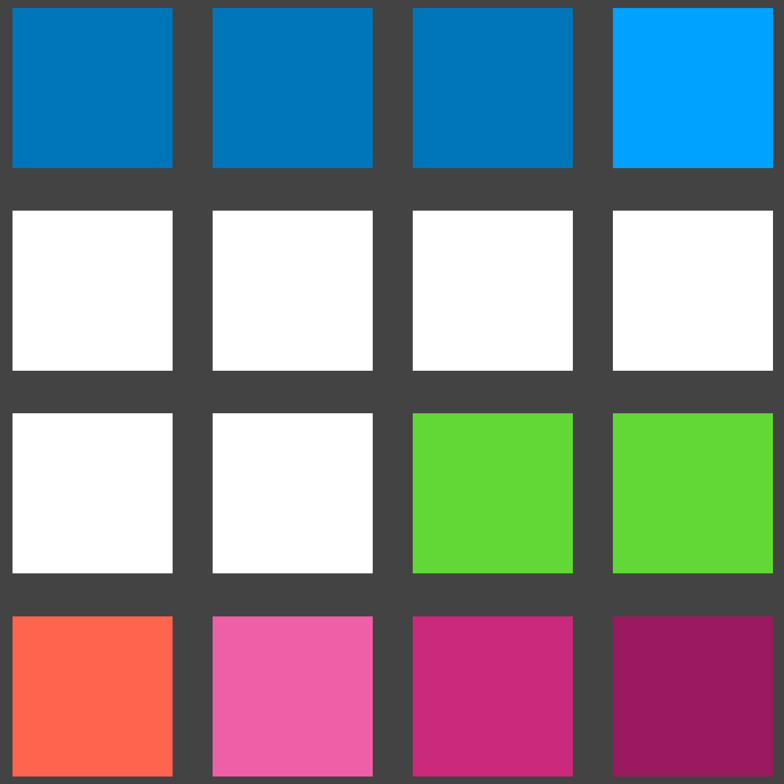

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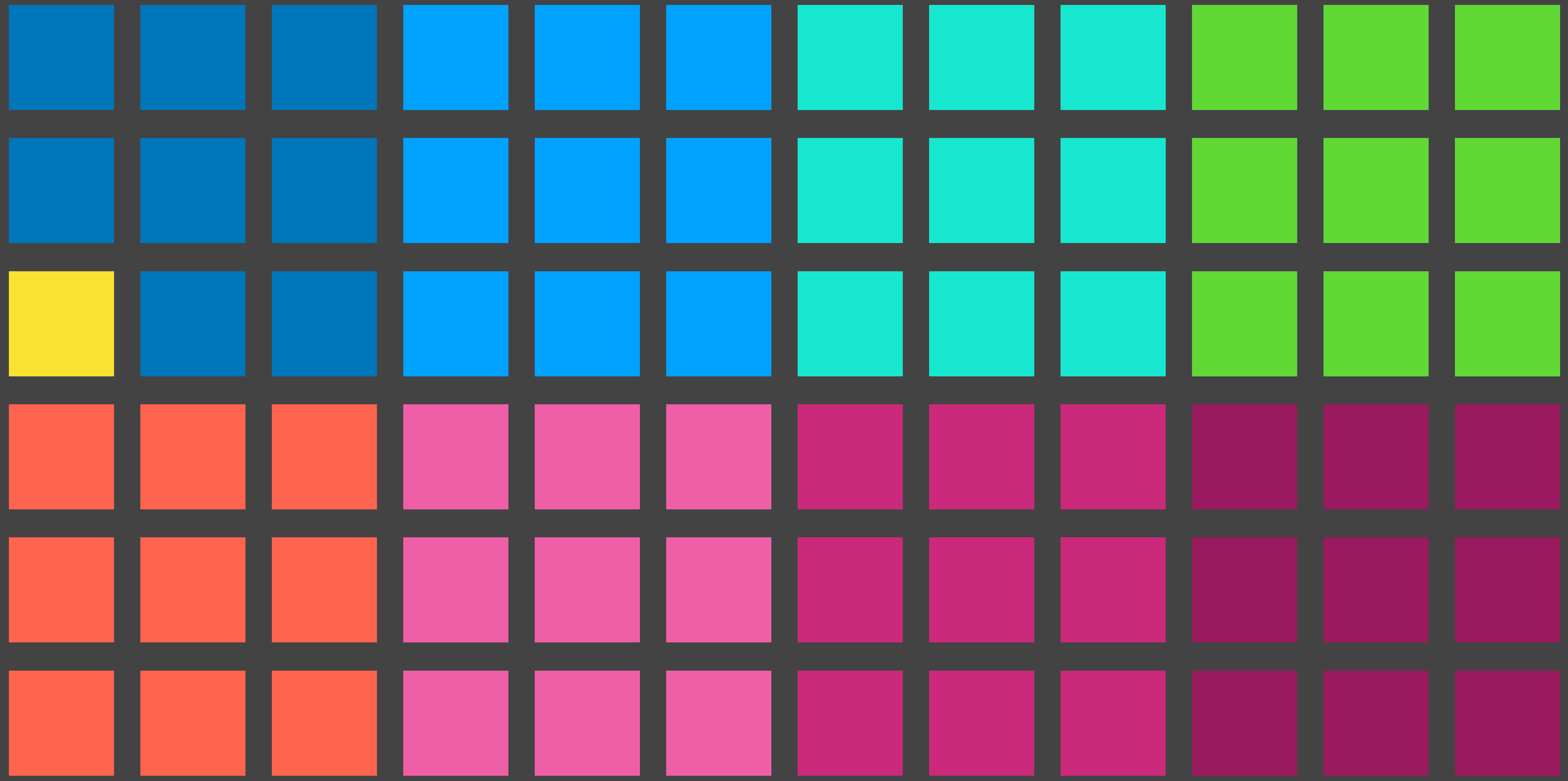

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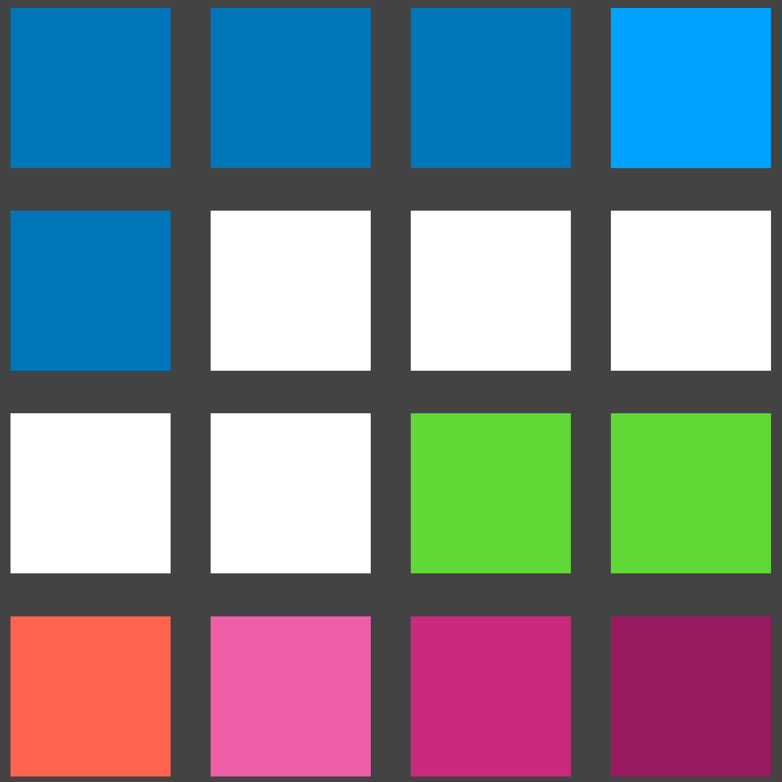

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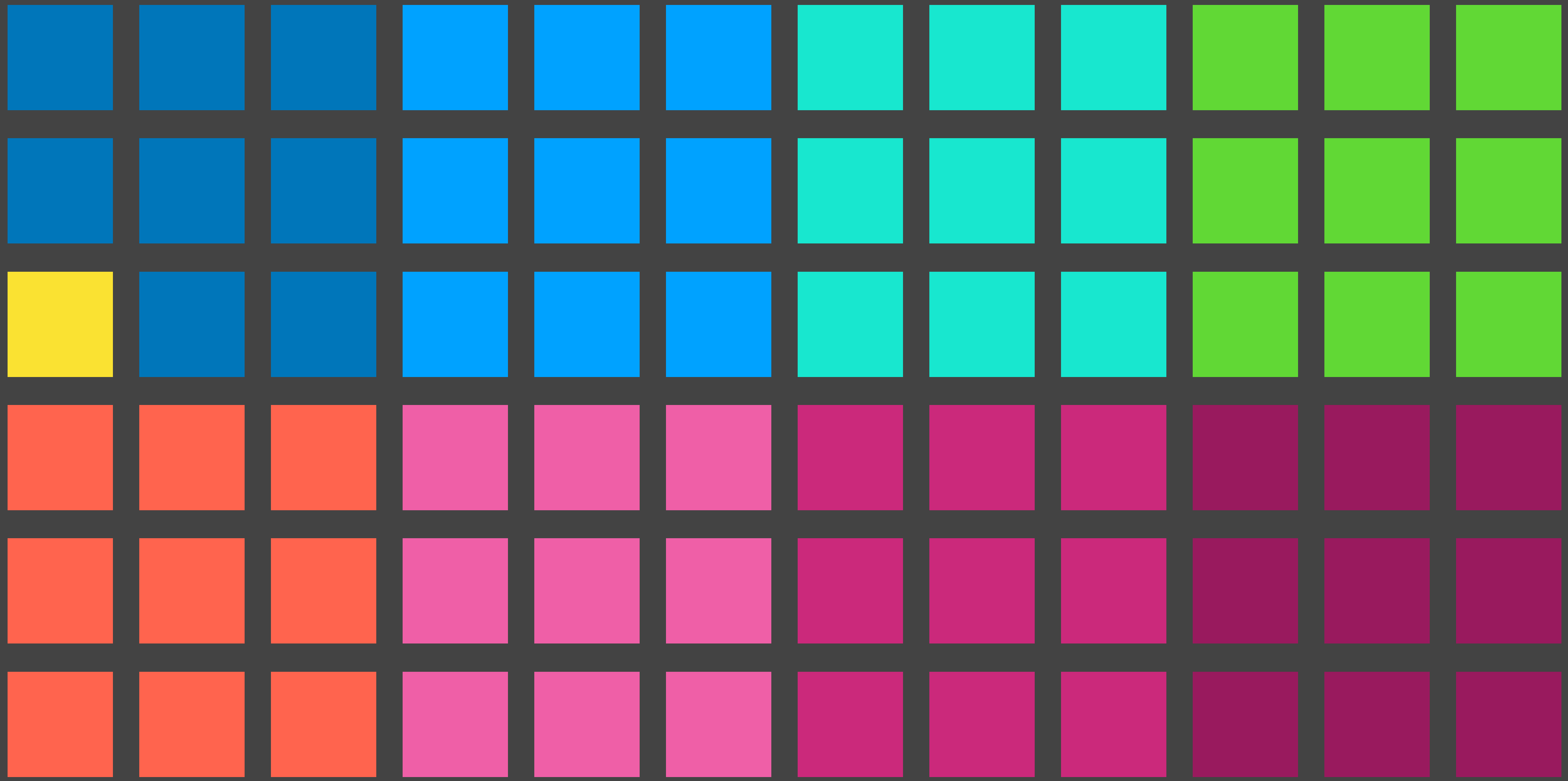

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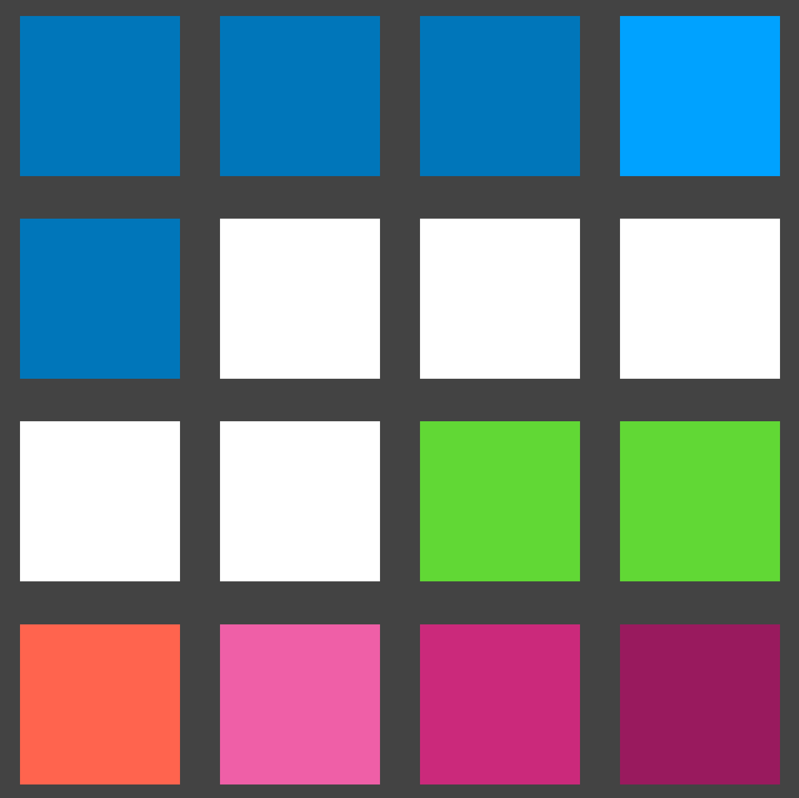

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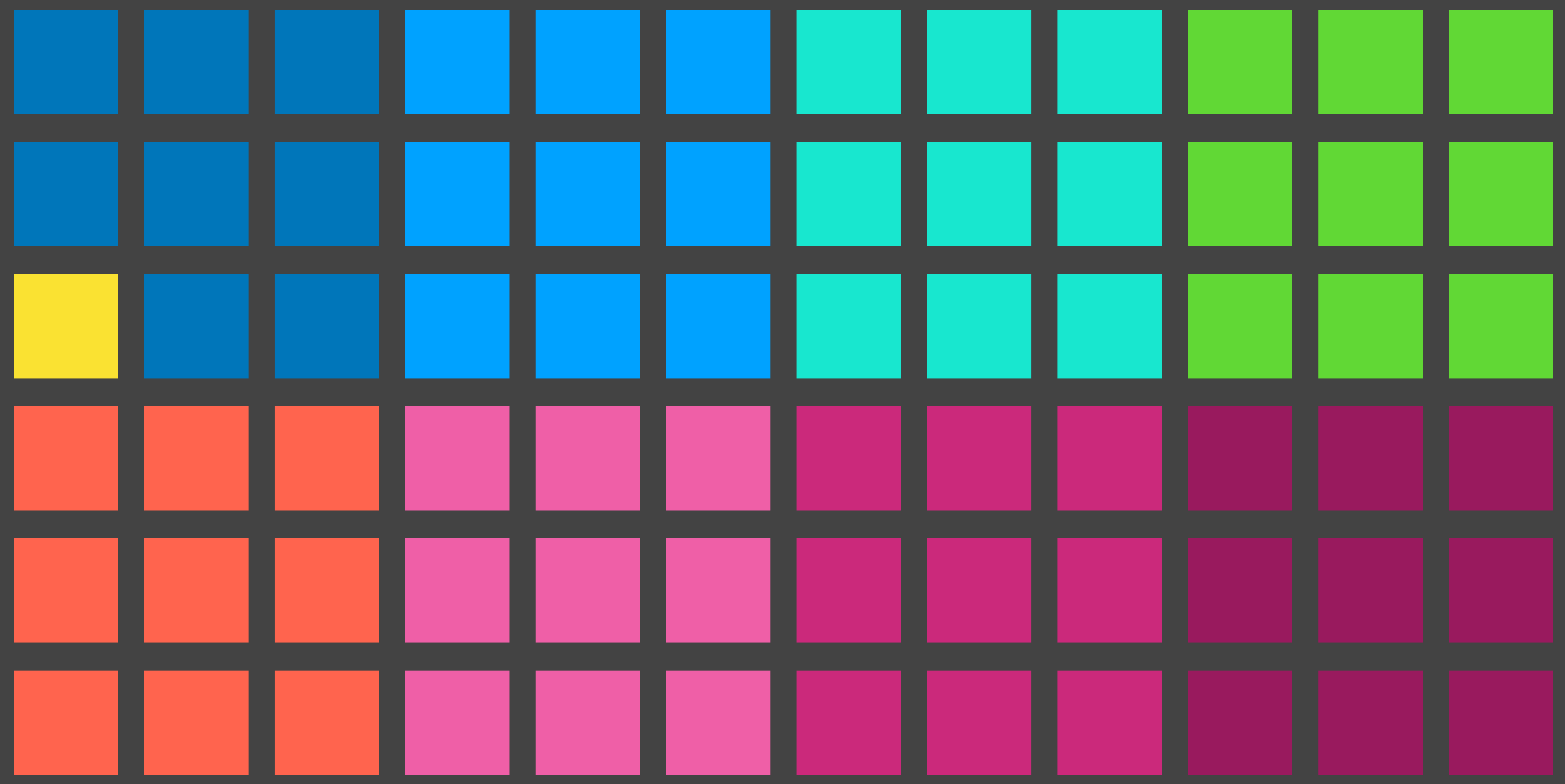


Goal is to minimize number of **blocks** evicted!

Results [Coester, Naor L., Talmon, SPAA 22]

New!

	Classic	Block-Aware
Offline	1	
Deterministic Online	k	
Randomized Online	O(log k)	

Results [Coester, Naor L., Talmon, SPAA 22]

New!

	Classic	Block-Aware
Offline	1	β
Deterministic Online	k	β k
Randomized Online	O(log k)	O(β log k)

Trivial!

Results [Coester, Naor L., Talmon, SPAA 22]

New!

	Classic	Block-Aware
Offline	1	O(log k)
Deterministic Online	k	k
Randomized Online	O(log k)	O($\log^2$ k)

Our Result

Results [Coester, Naor L., Talmon, SPAA 22]

New!

	Classic	Block-Aware
Offline	1	O(log k)
Deterministic Online	k	k
Randomized Online	O(log k)	O($\log^2$ k)

Our Result

Also show $\Omega(\beta)$ lower bound for randomized algorithms in **fetching cost model…**

Results [Coester, Naor L., Talmon, SPAA 22]

New!

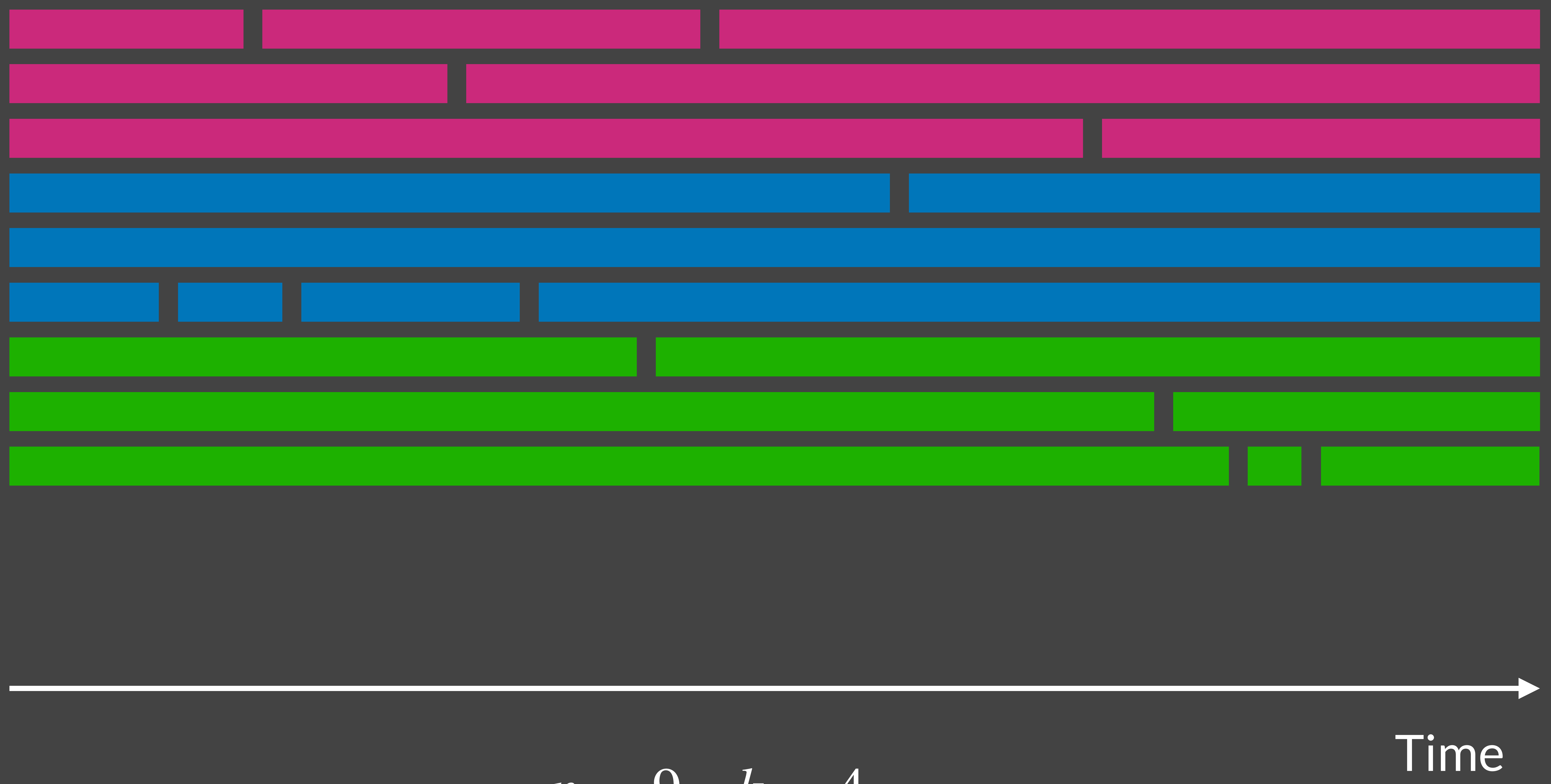
	Classic	Block-Aware
Offline	1	O(log k)
Deterministic Online	k	k
Randomized Online	O(log k)	O(log^2 k)

Our Result

Also show $\Omega(\beta)$ lower bound for randomized algorithms in **fetching cost model…**

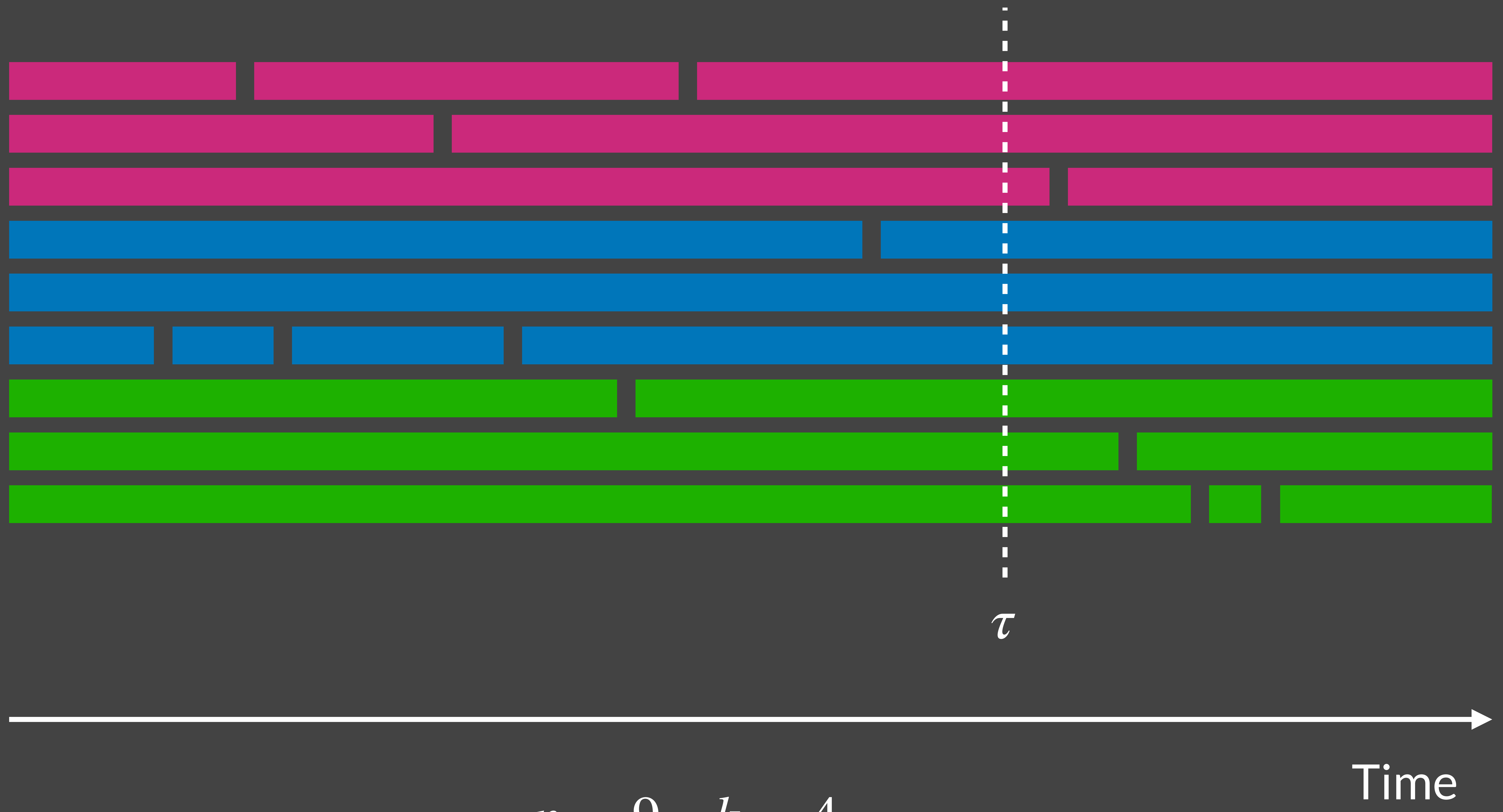
… separation of eviction/ fetching cost models!

What does this have to do with Submodular Cover?



$n = 9, \quad k = 4$

What does this have to do with Submodular Cover?



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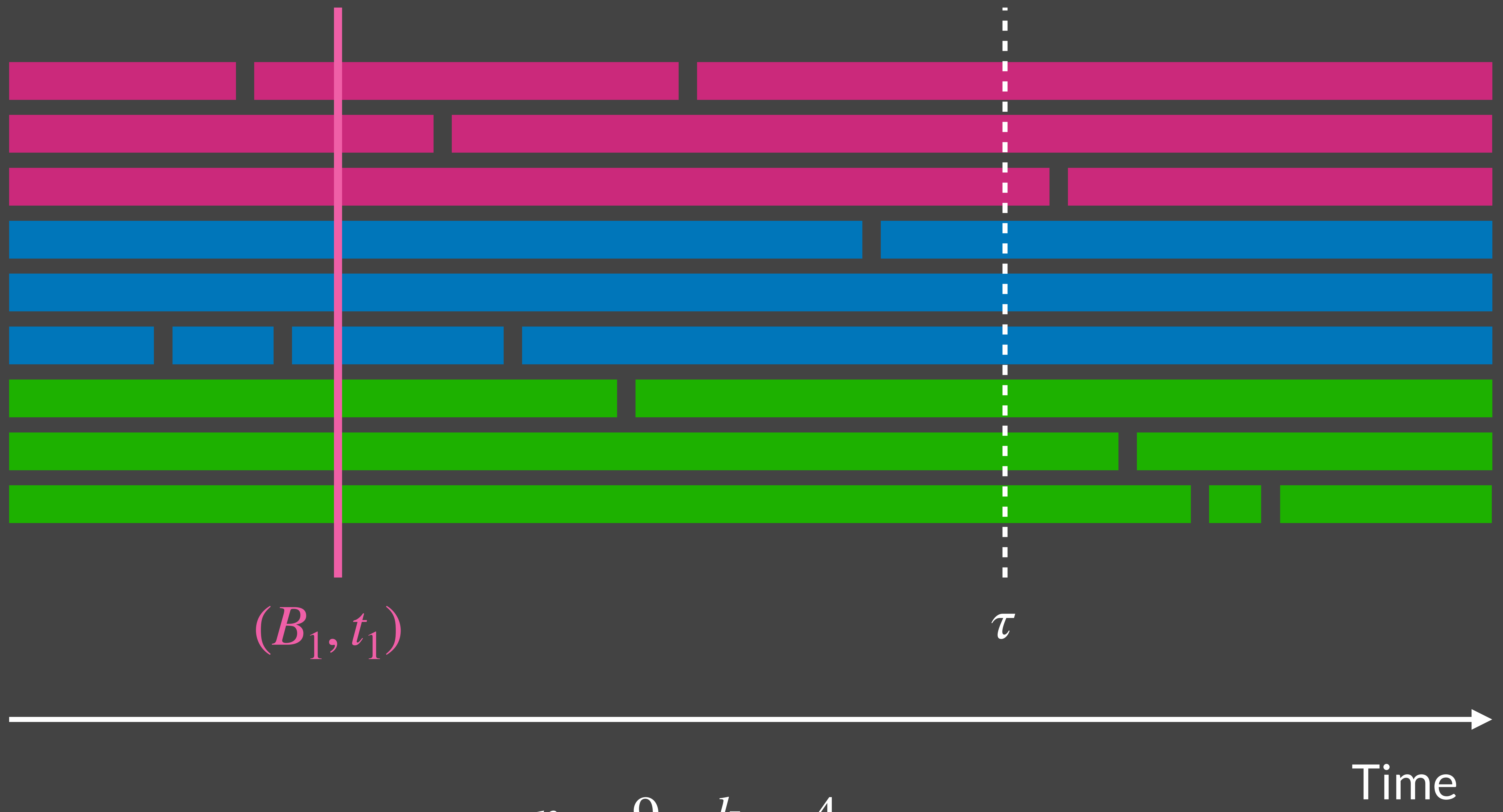
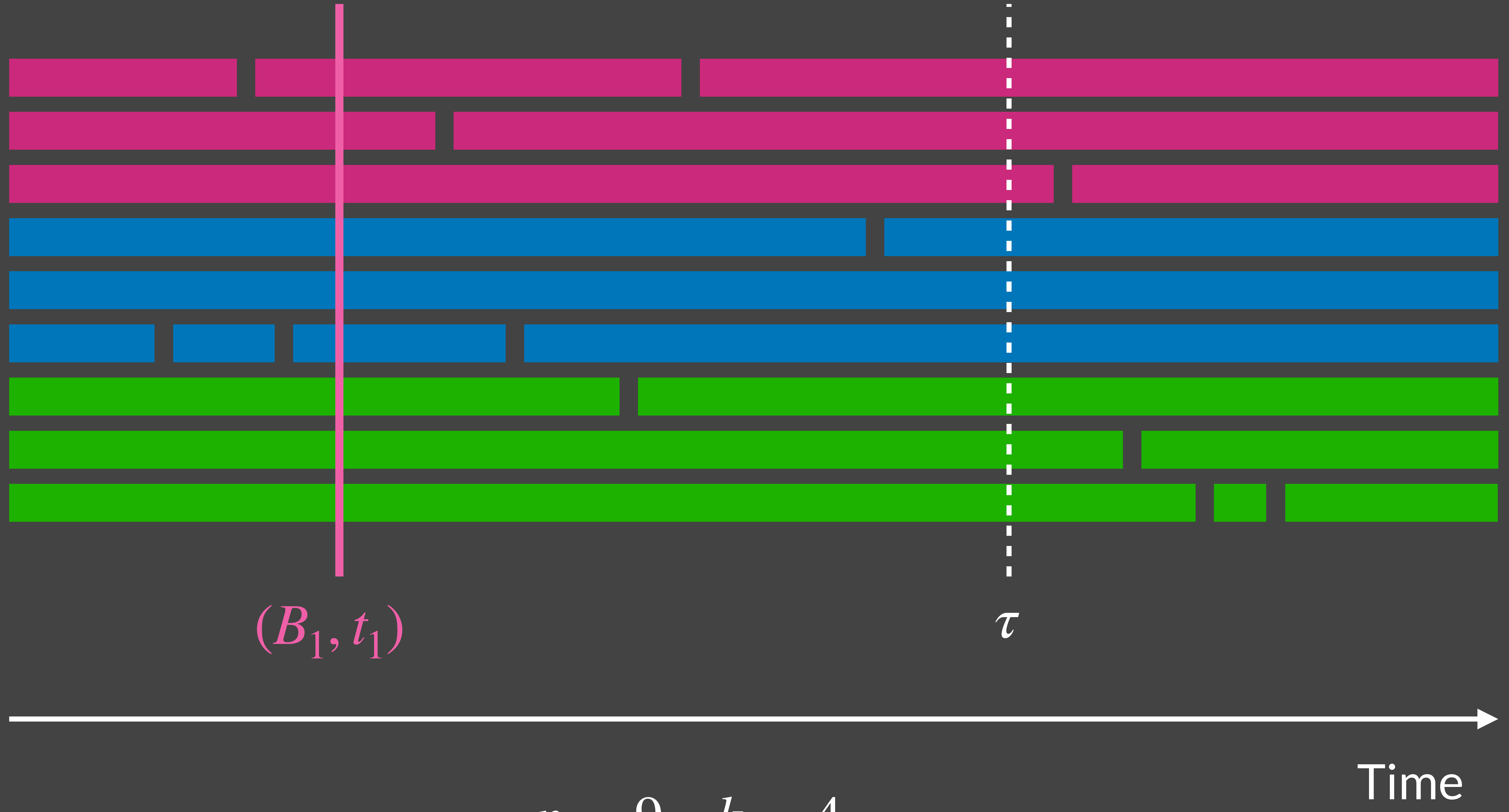


$n = 9, \quad k = 4$

What does this have to do with Submodular Cover?

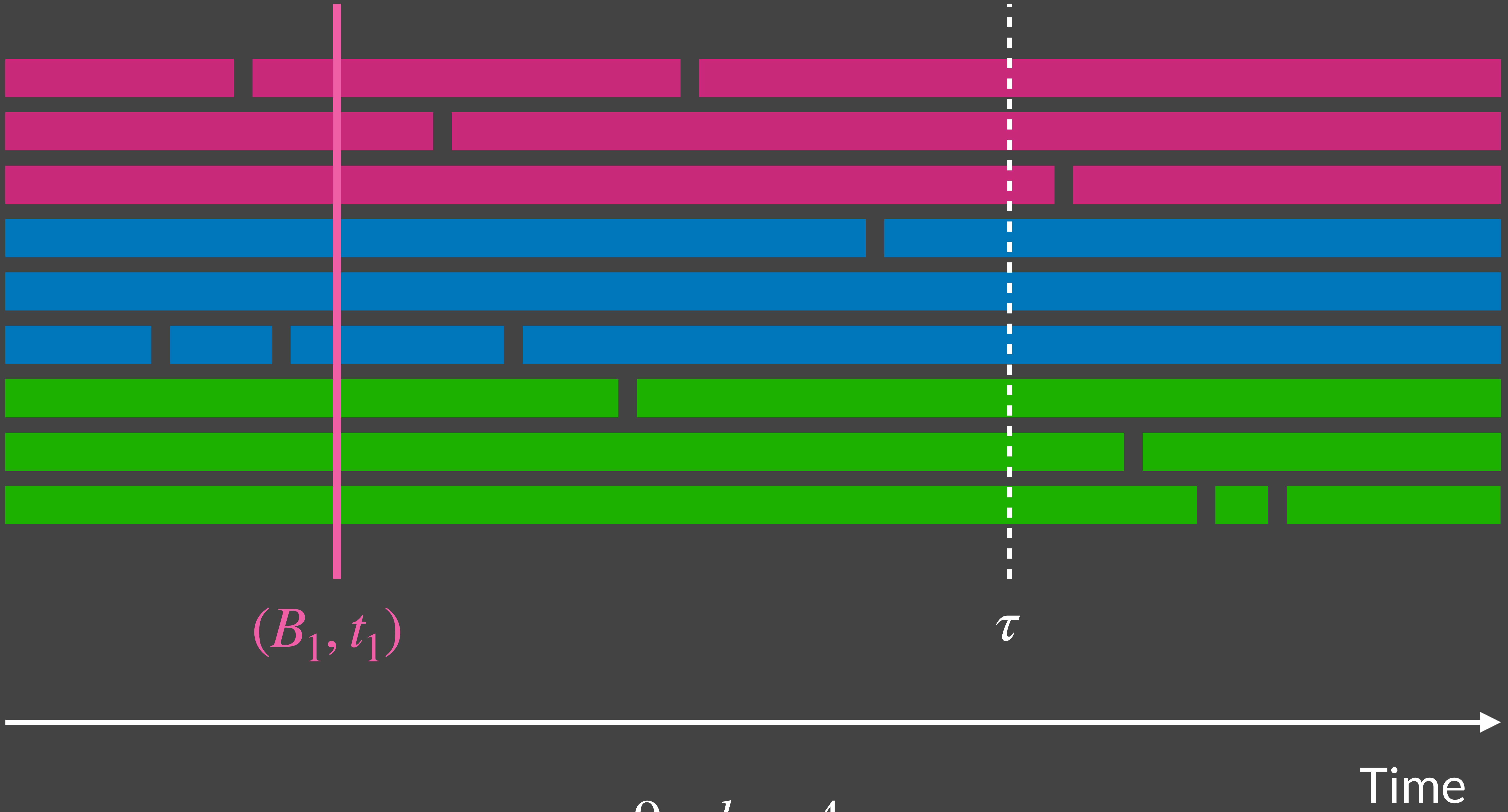
Reduces overflow at time τ by ____.



$$n = 9, \quad k = 4$$

What does this have to do with Submodular Cover?

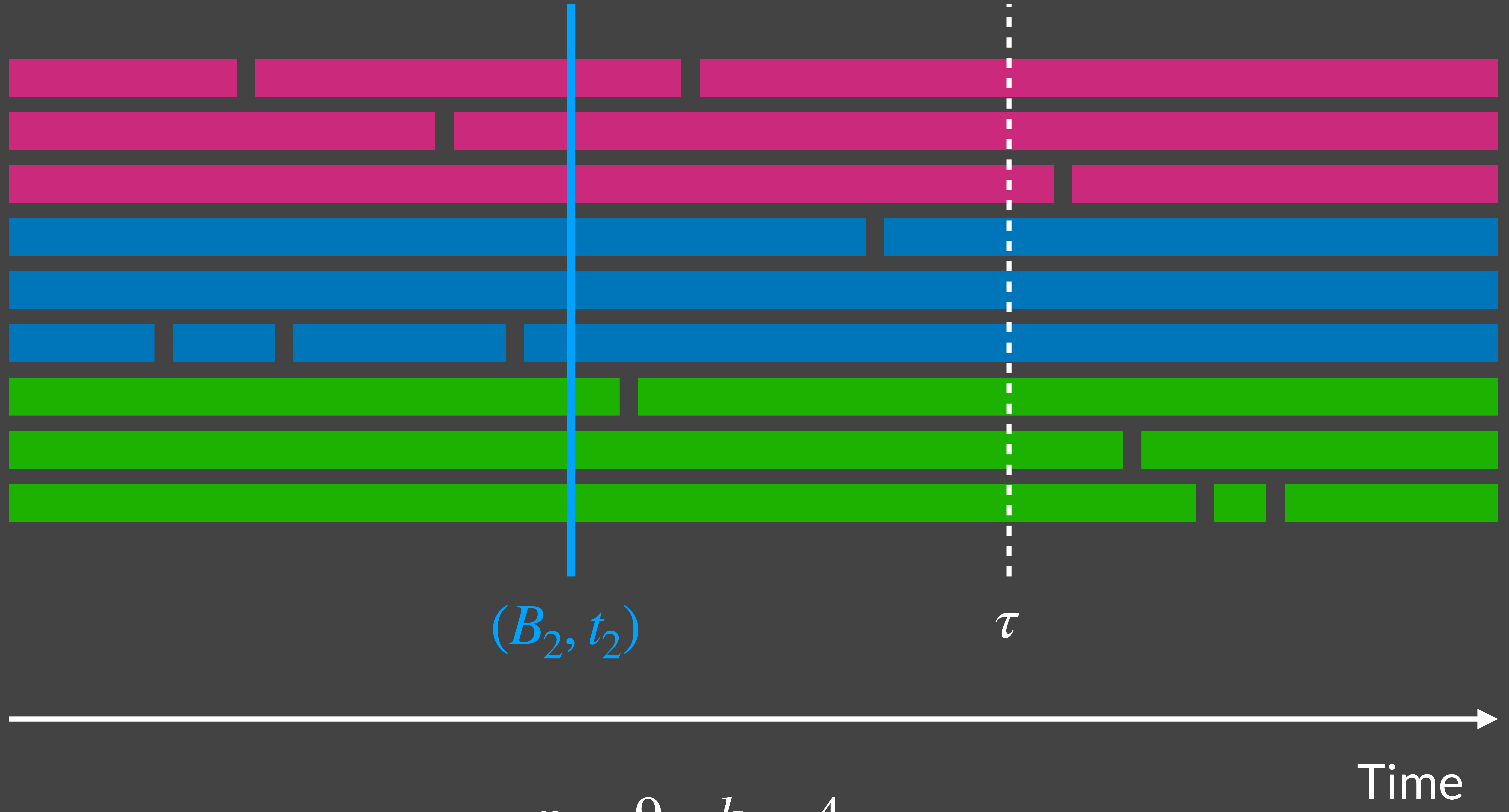
Reduces overflow at time τ by $\underline{\ 1\ }$.



$n = 9, \quad k = 4$

What does this have to do with Submodular Cover?

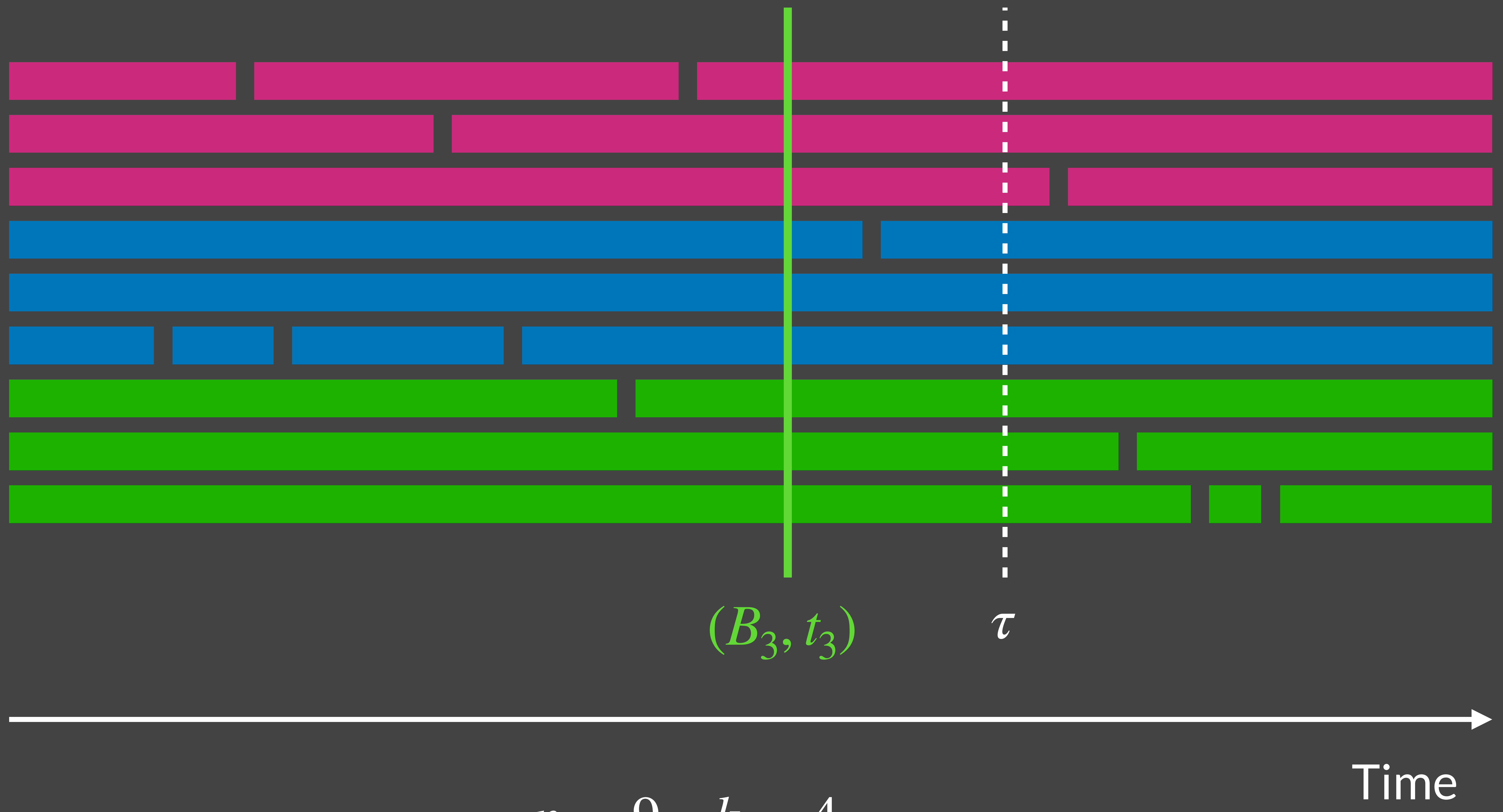
Reduces overflow at time τ by <u>2</u>.



$n = 9, \quad k = 4$

What does this have to do with Submodular Cover?

Reduces overflow at time τ by $\underline{\ 3\ }$.



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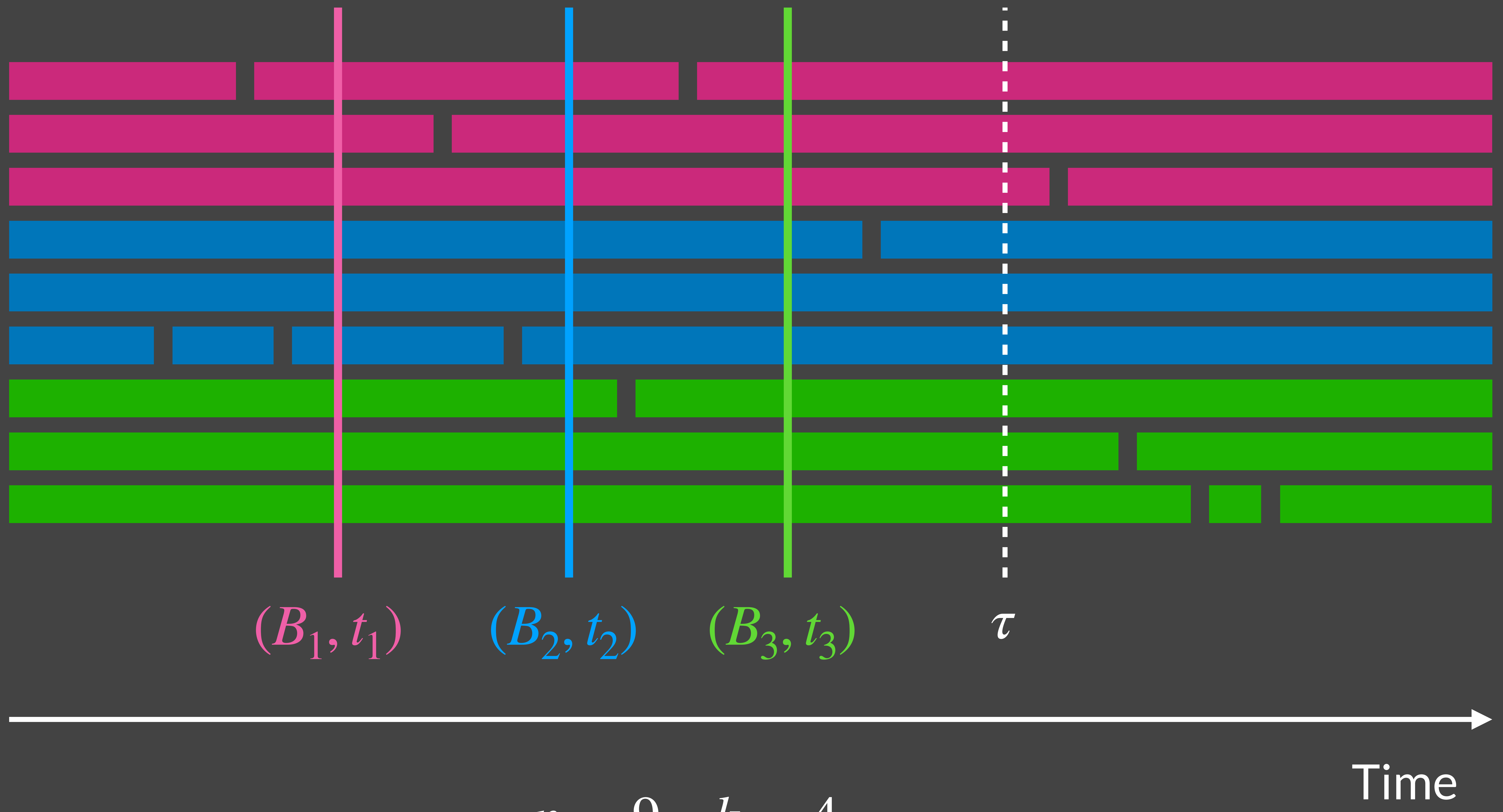
What does this have to do with Submodular Cover?

Reduces overflow at time τ by _5_.

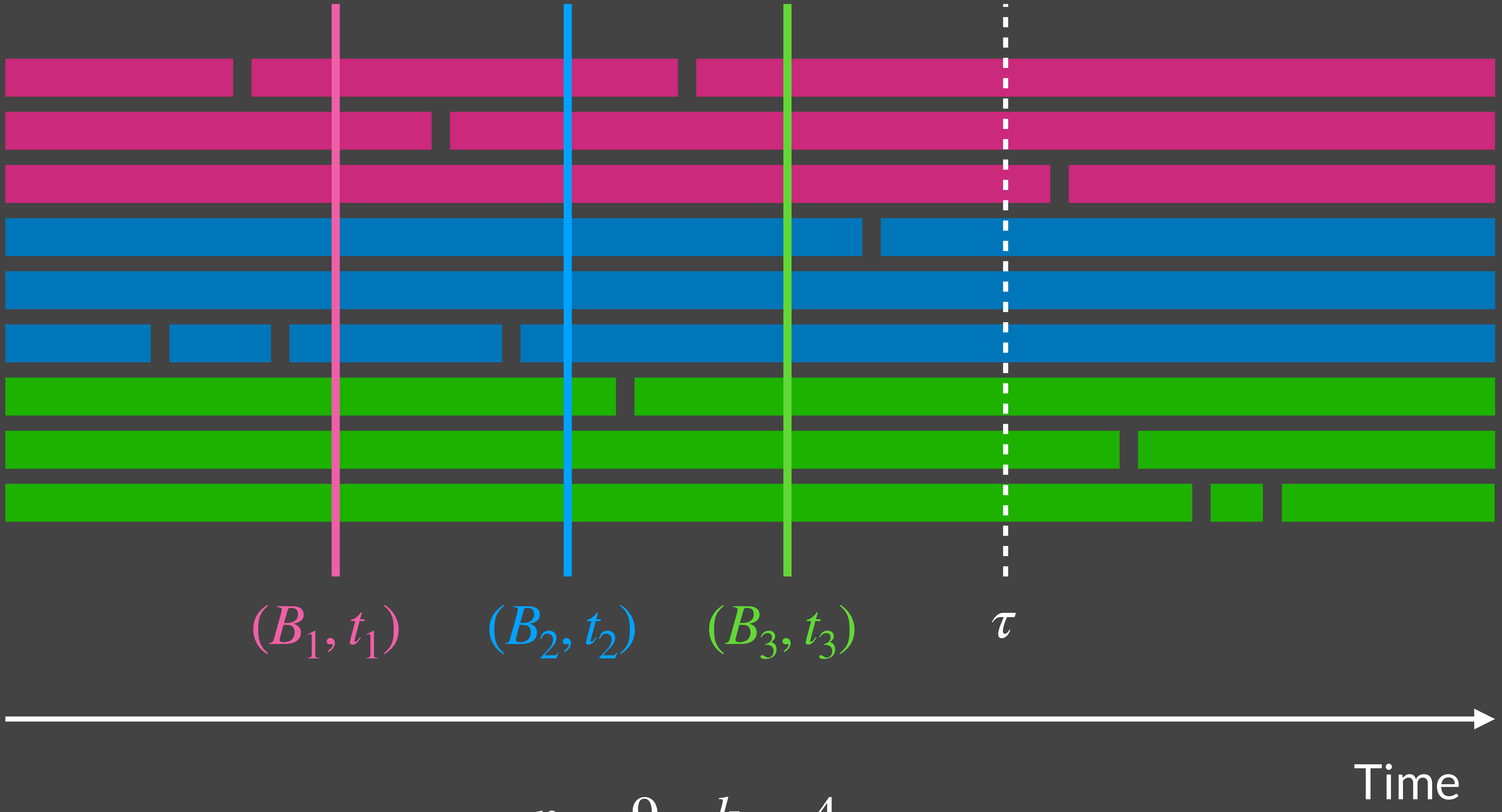


$n = 9, \quad k = 4$

What does this have to do with Submodular Cover?

Reduces overflow at time τ by <u>5</u>.

f^{τ} := "reduction in overflow at time τ" is submodular!



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Formulation as Submodular Cover

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$$\min_S |S|$$

$$\forall \tau : \quad f^\tau(S) \geq n - k$$

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This is an instance of Online Submodular Cover!

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Bounds from Part I <u>too weak</u>, depend on total time T.

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This is an instance of Online Submodular Cover!

Bounds from Part I too weak, depend on total time T.

We show our bounds via finer analysis… but reuse some ideas!

Talk Outline

Intro

Part I — Online/Dynamic Submodular Cover

Part II — Application: Block-Aware Caching

Part III — Random Order Online Set Cover

Conclusion

Talk Outline

Intro

Part I — Online/Dynamic Submodular Cover

Part II — Application: Block-Aware Caching

➡ Part III — Random Order Online Set Cover

Conclusion

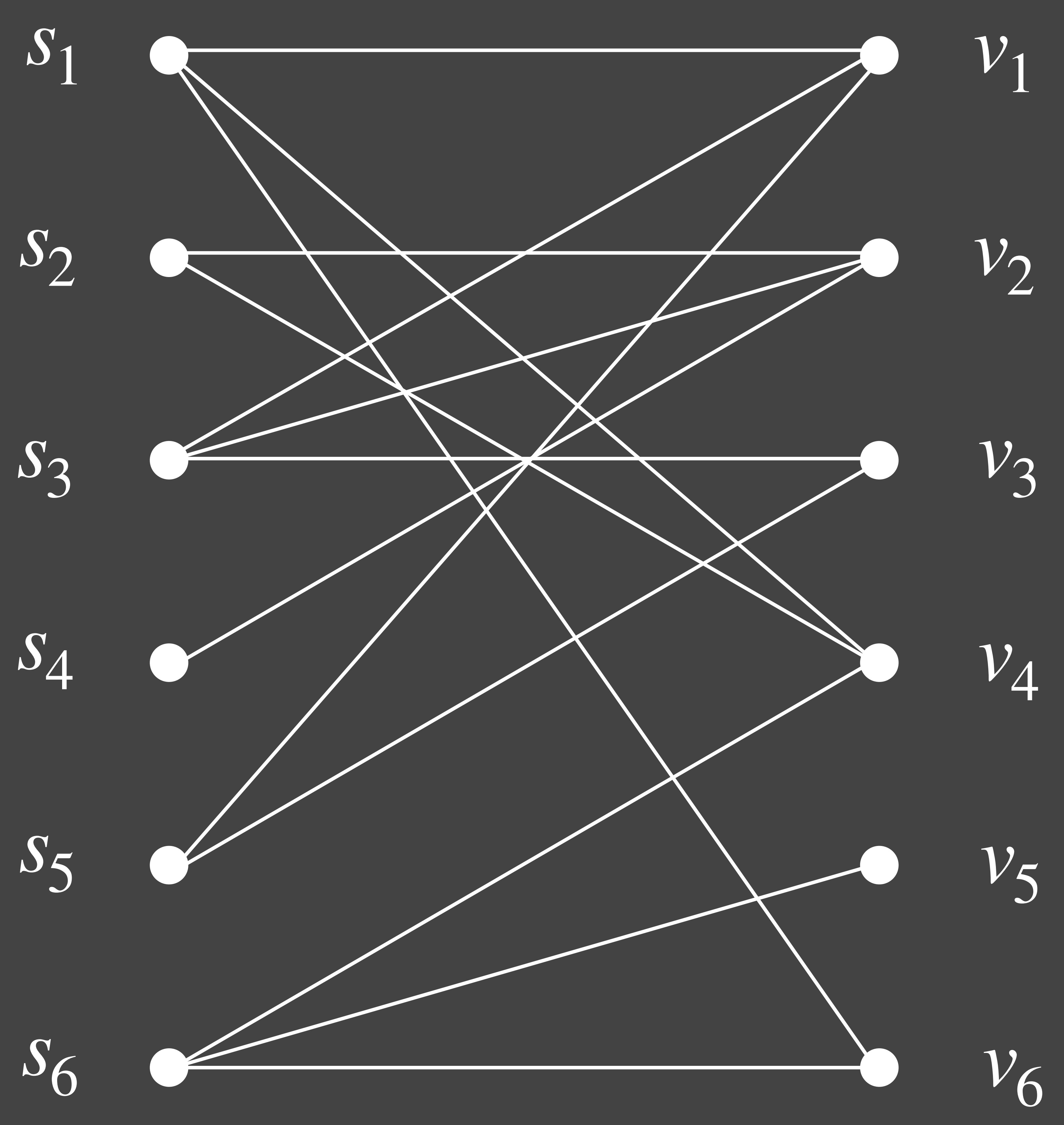
Part III — Random Order Online Set Cover

with Anupam Gupta and Gregory Kehne

Random Order (RO) Online Set Cover

$\mathcal{S}$

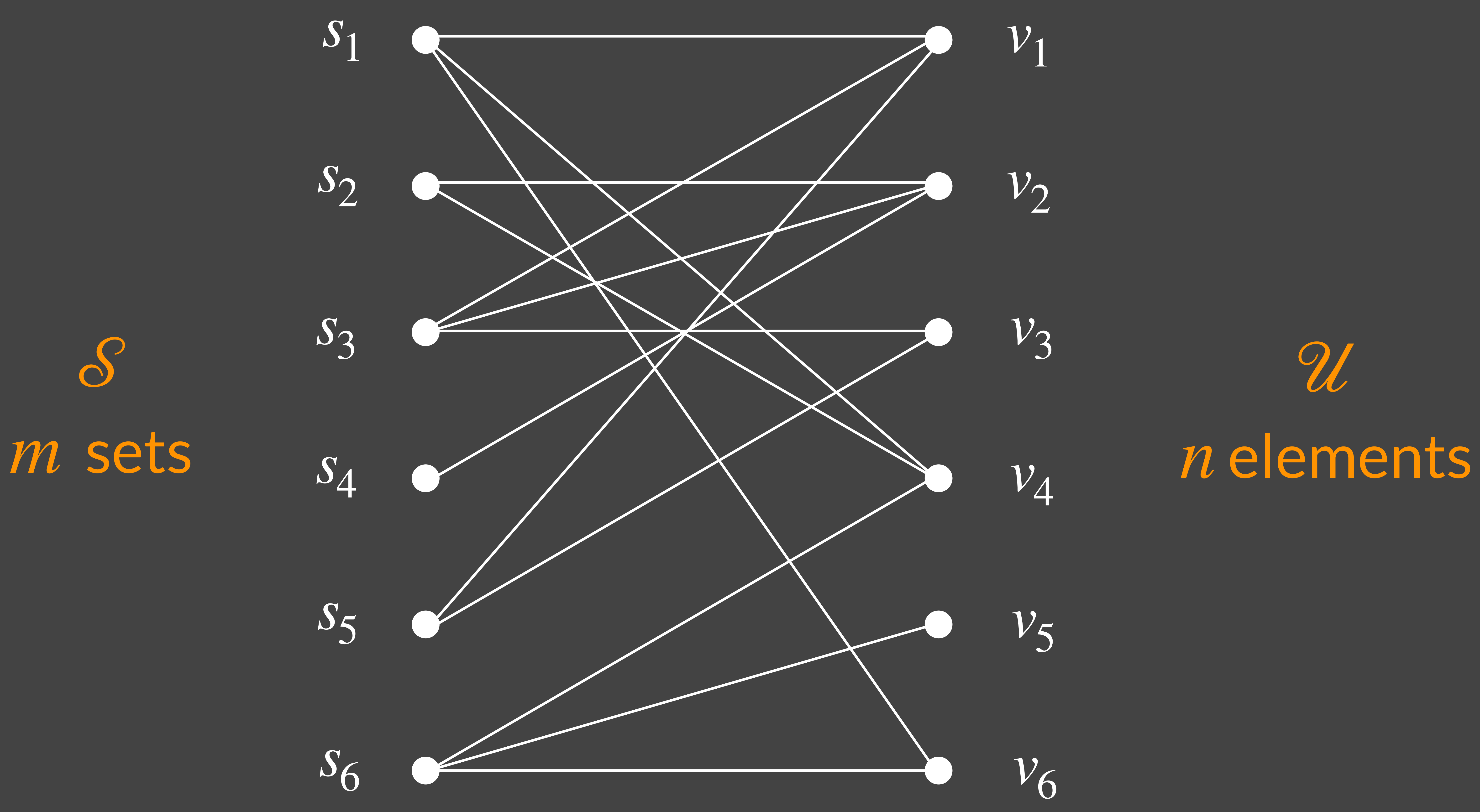
m sets



$\mathcal{U}$

n elements

Random Order (RO) Online Set Cover



v_1
v_2
v_3
v_4
v_5
v_6

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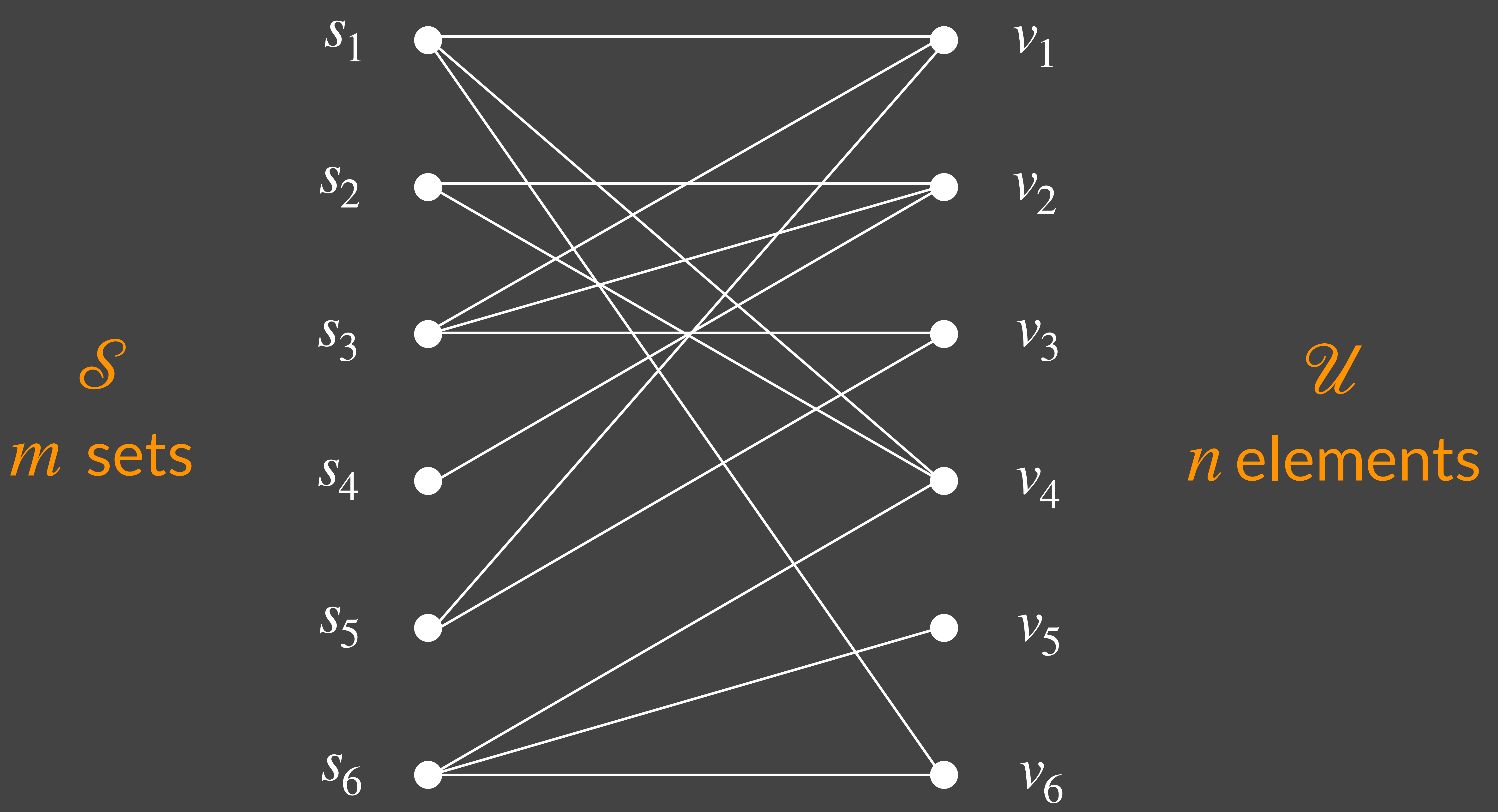

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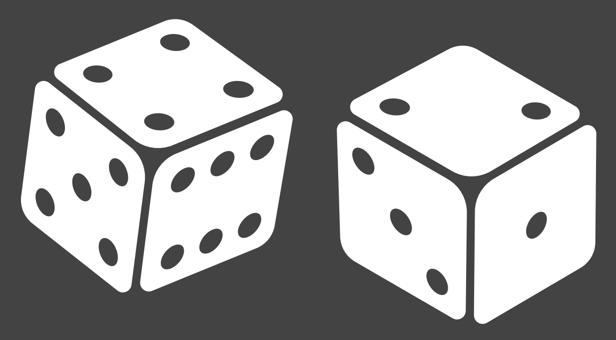

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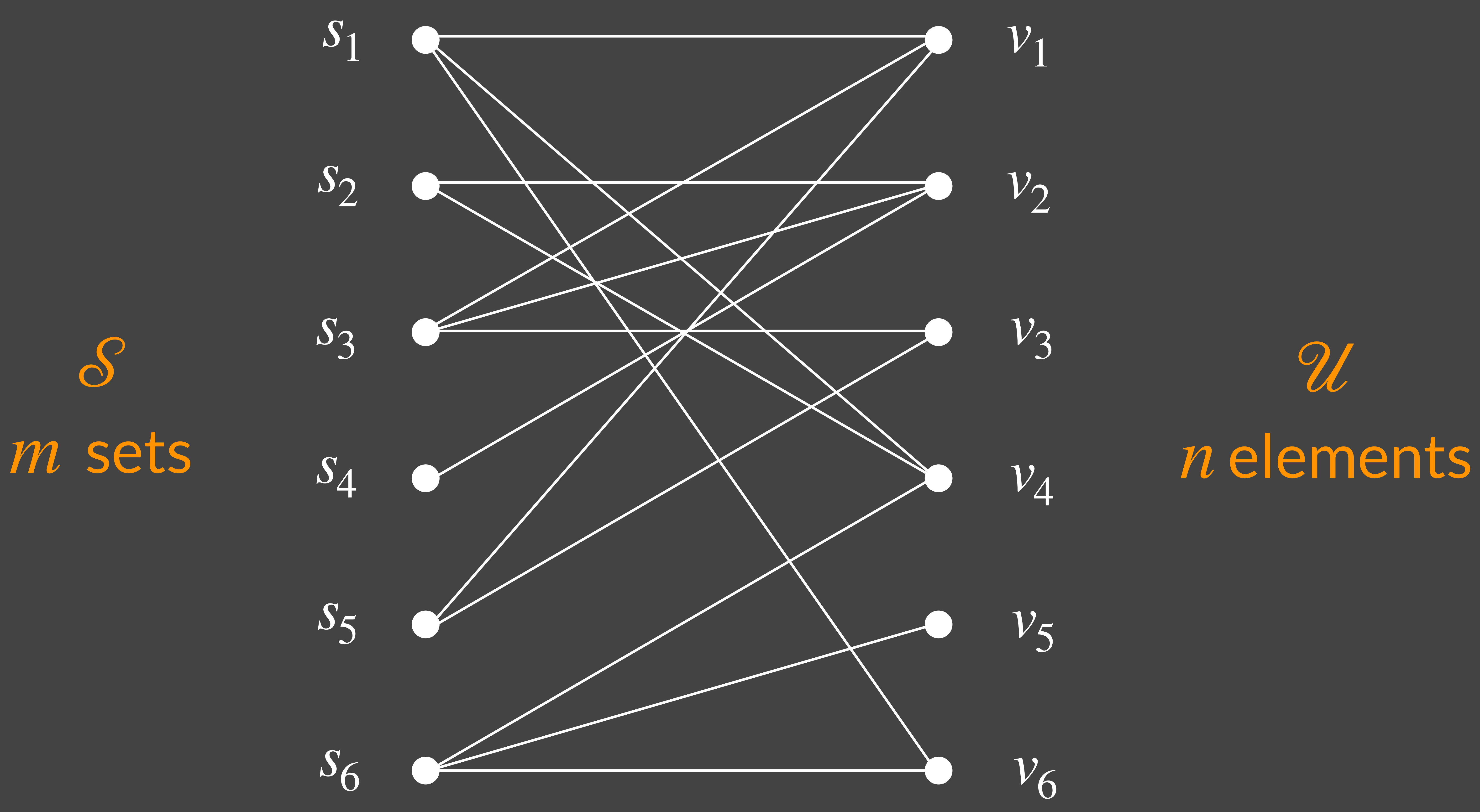

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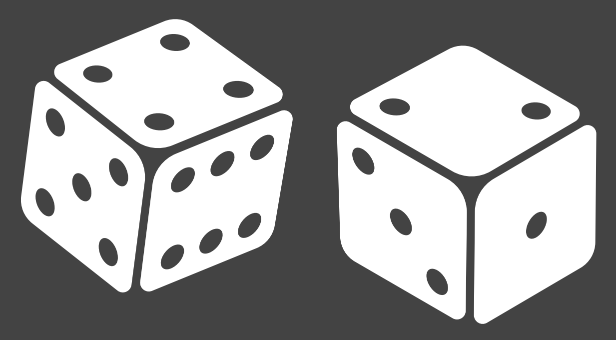

Random Order (RO) Online Set Cover

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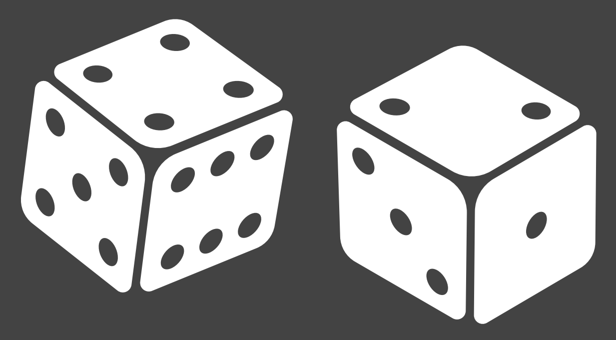
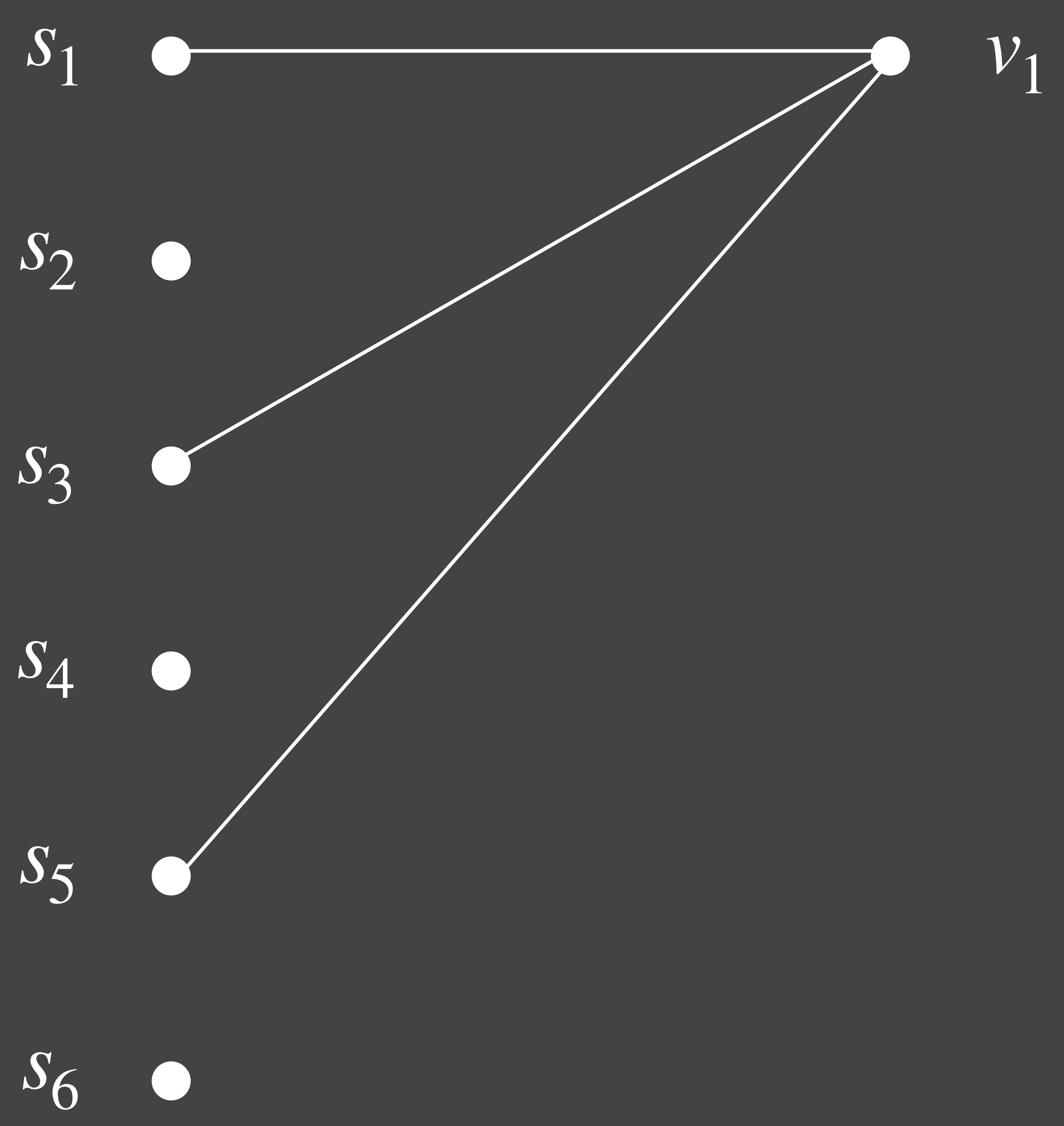


Random Order (RO) Online Set Cover

$\mathcal{S}$

m sets



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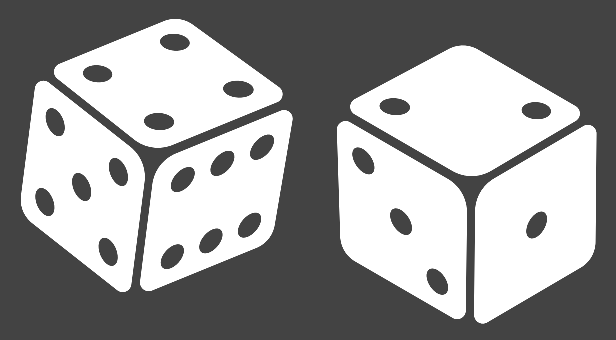

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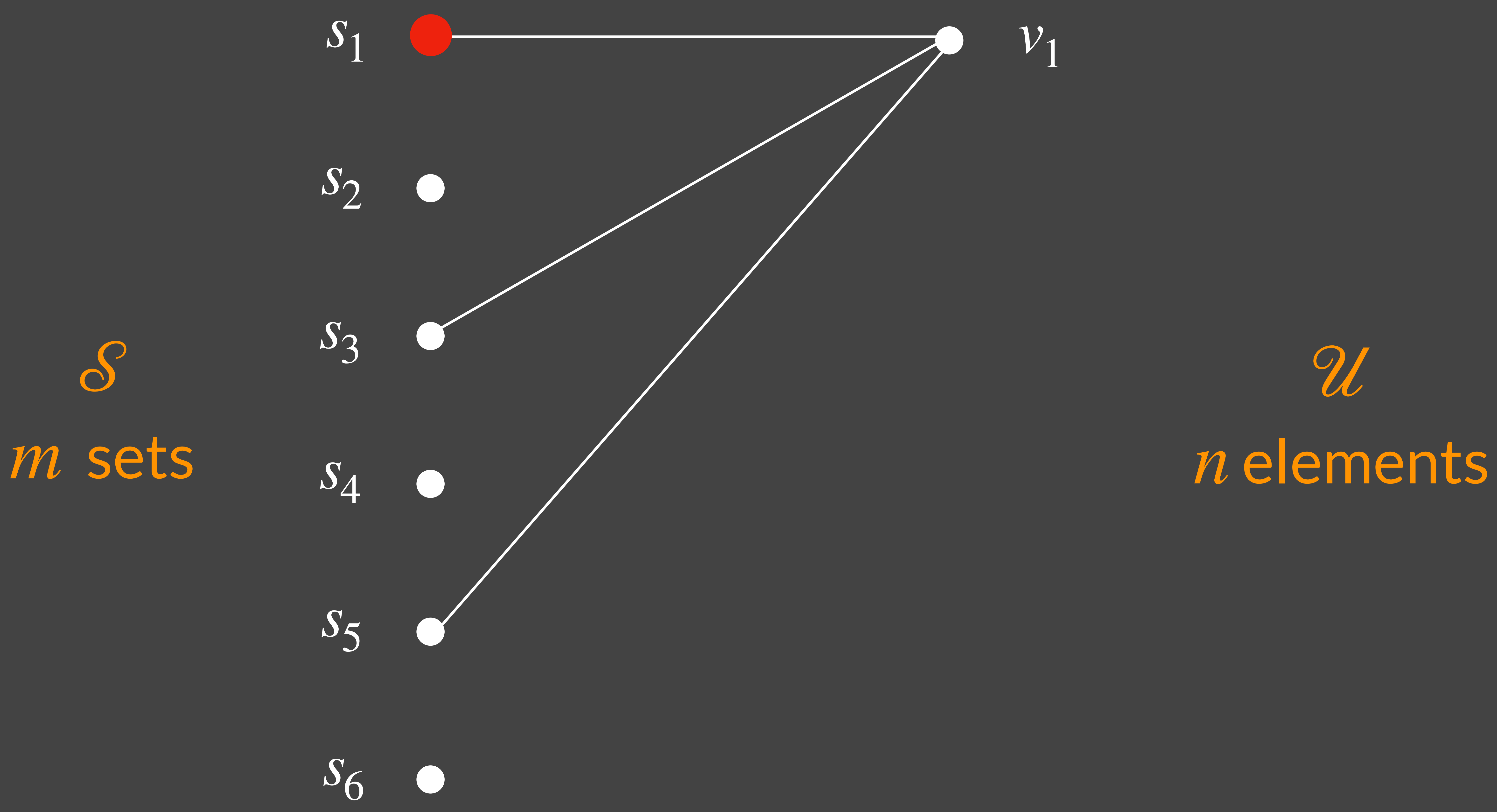

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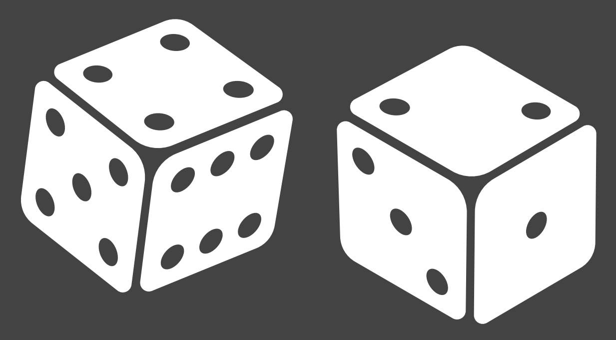

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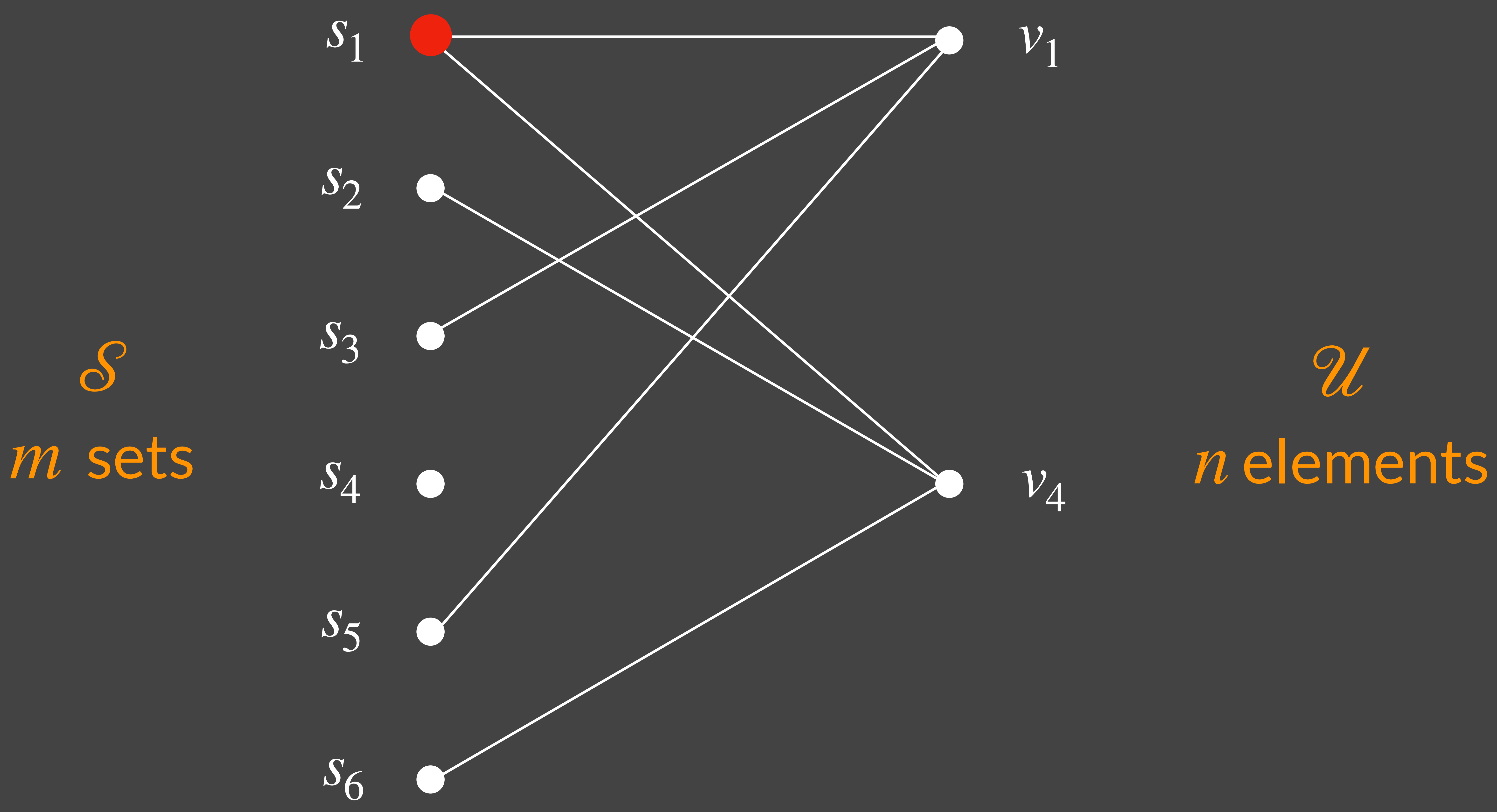

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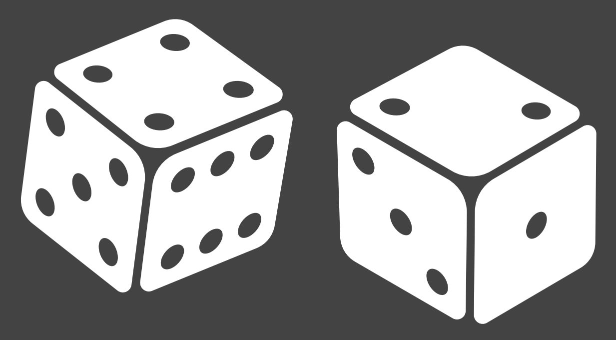

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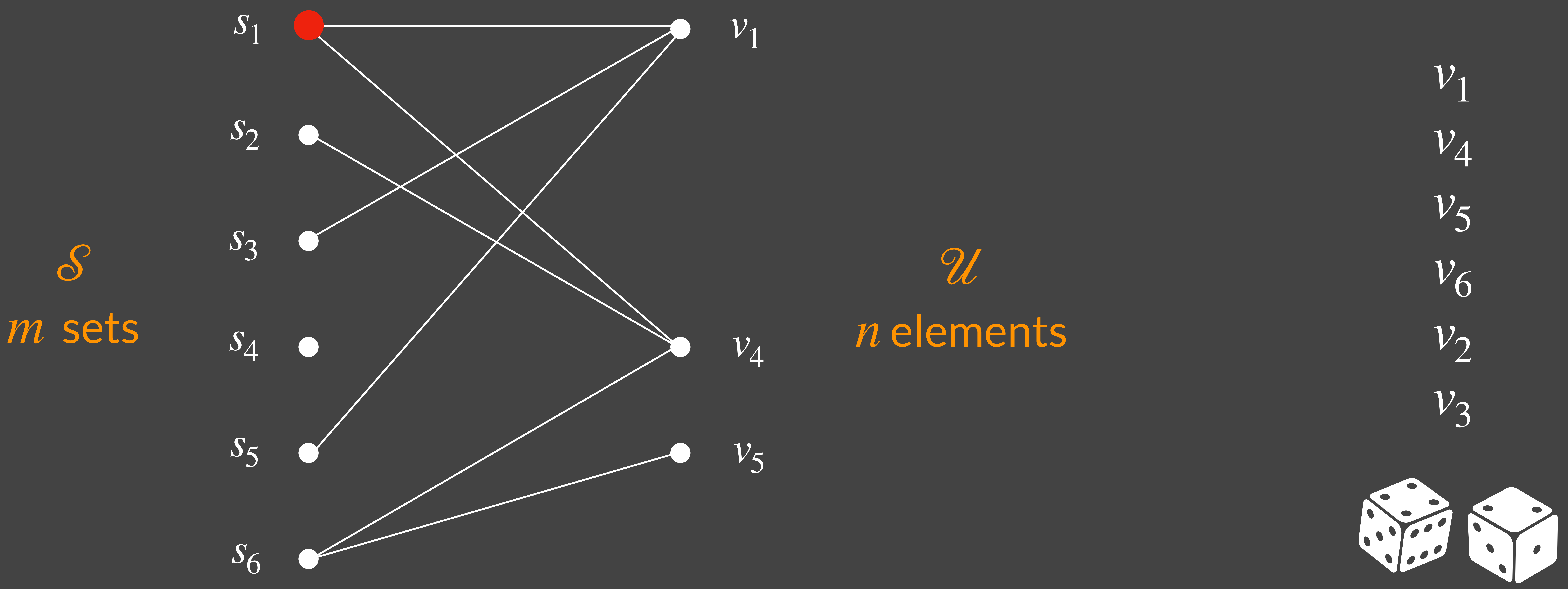

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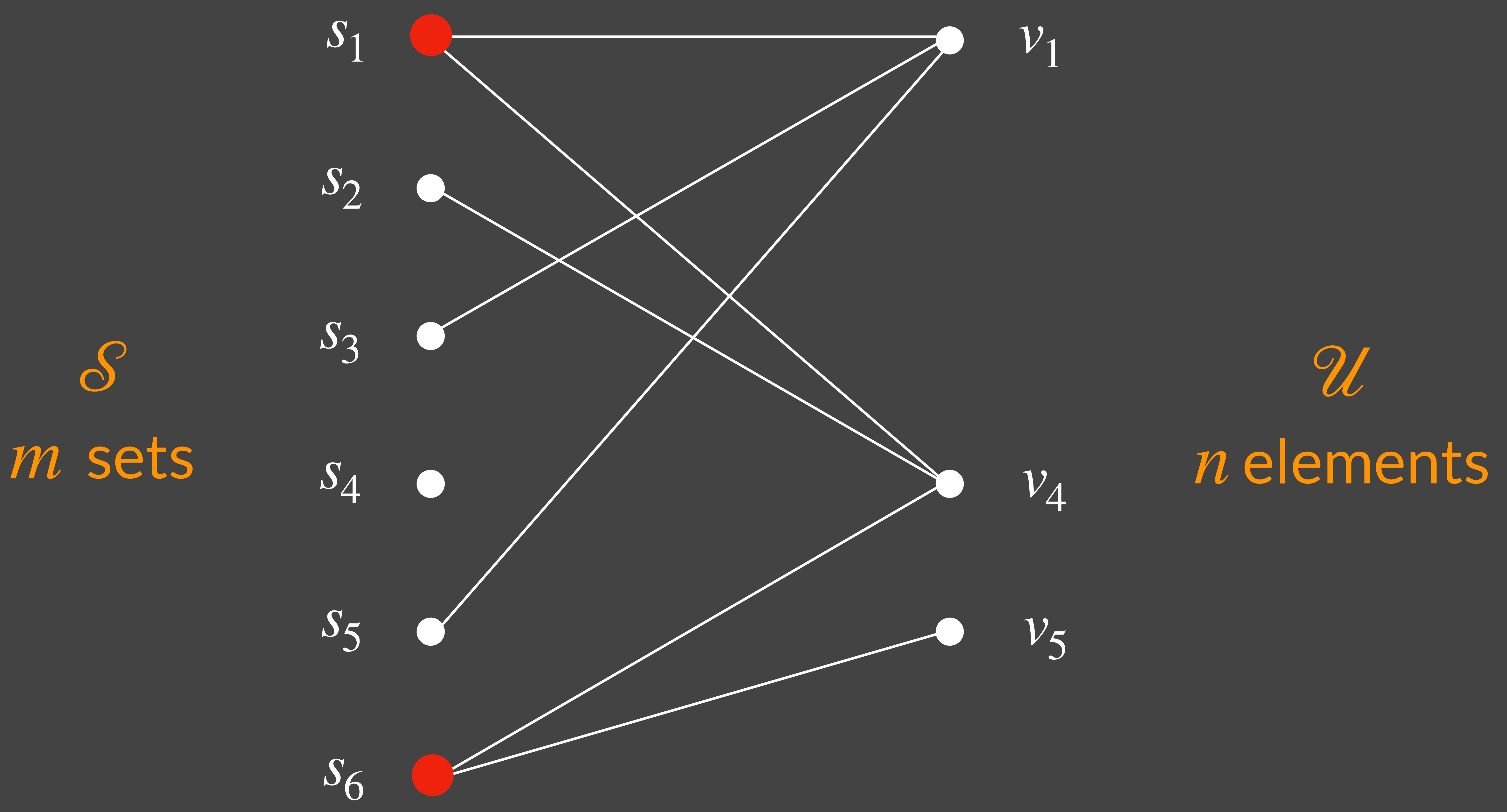

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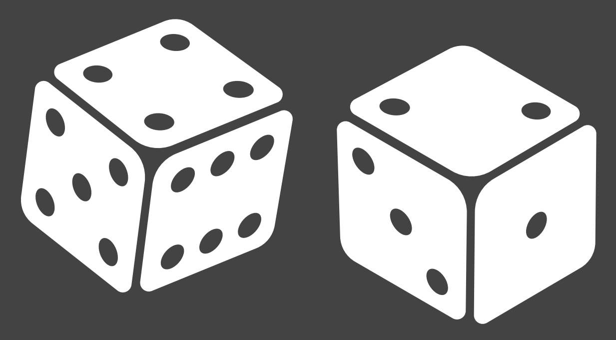

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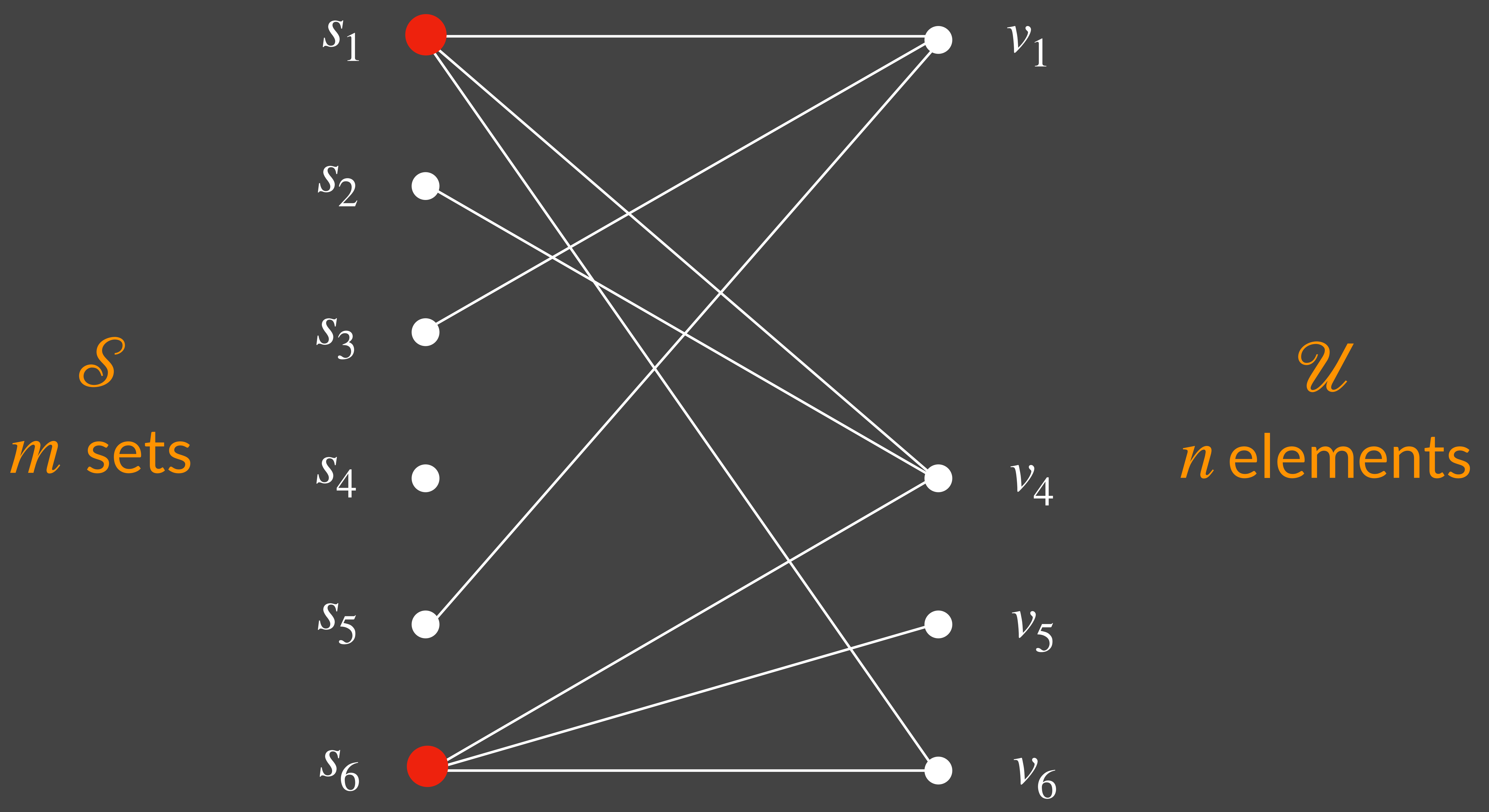

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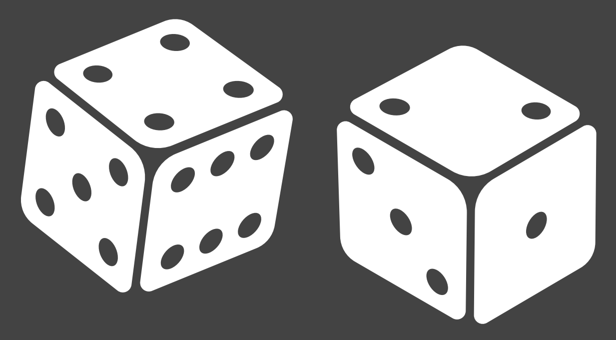

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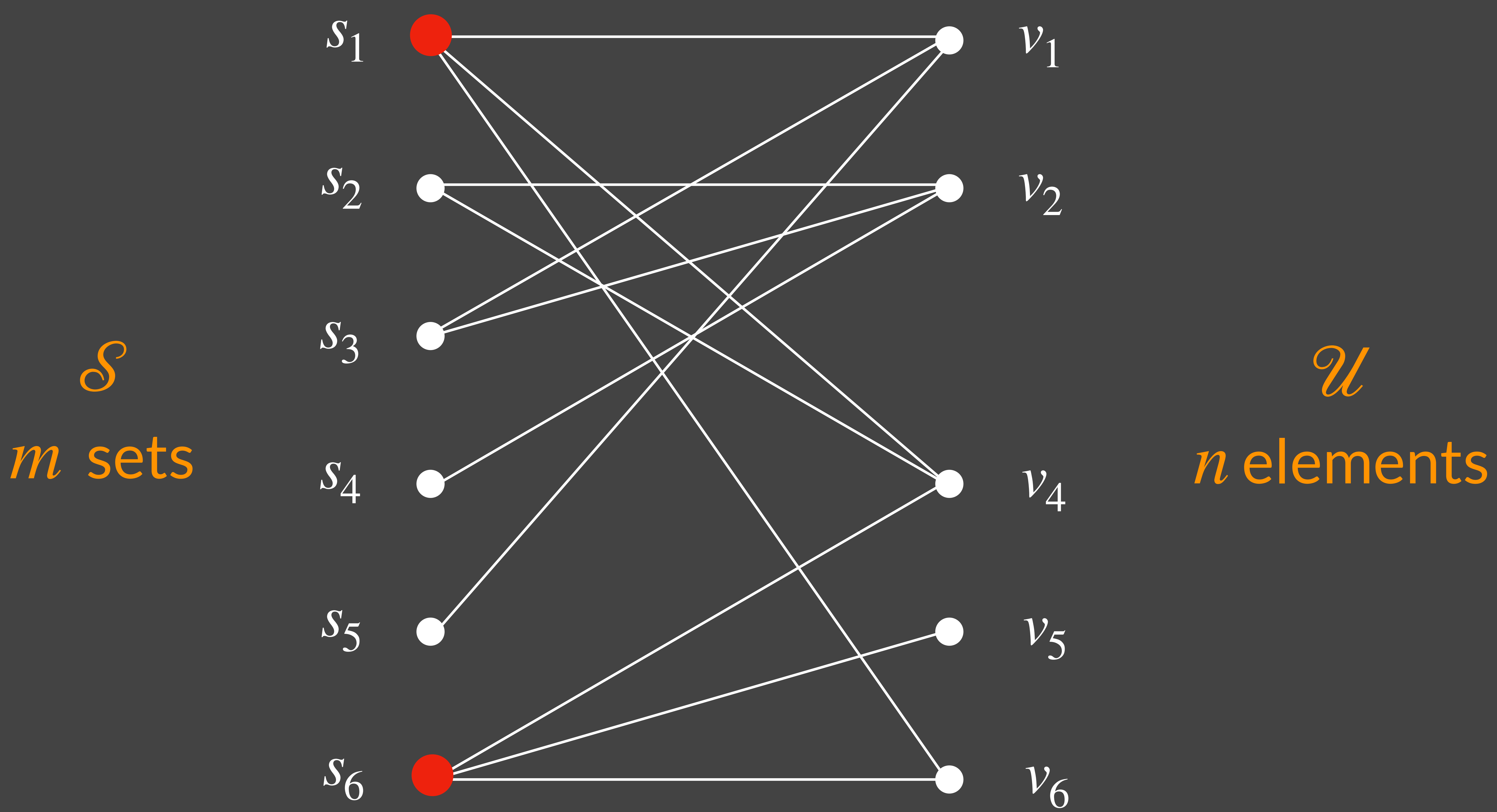

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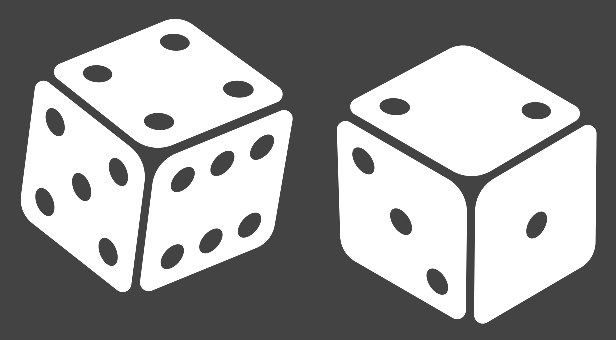

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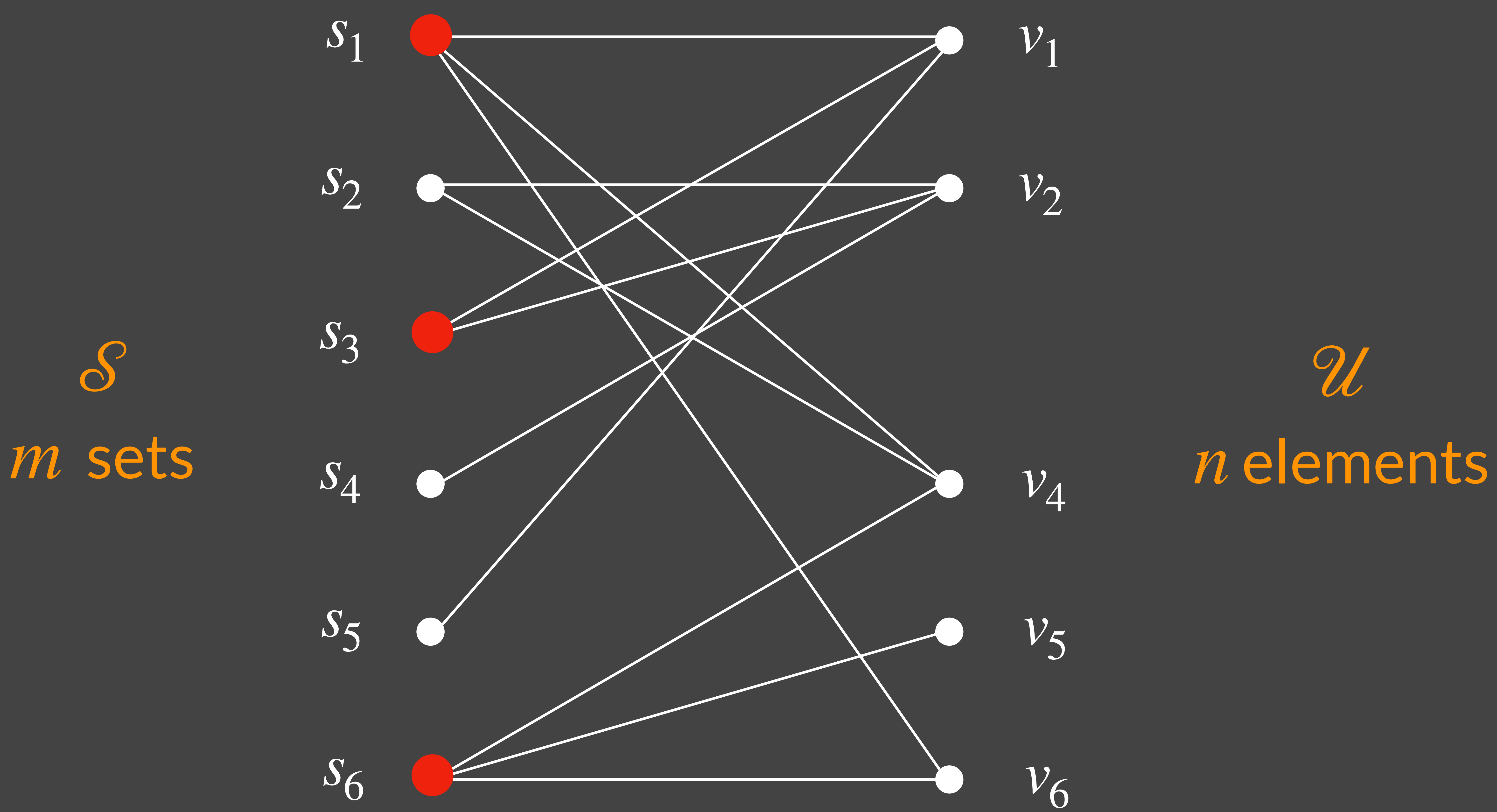

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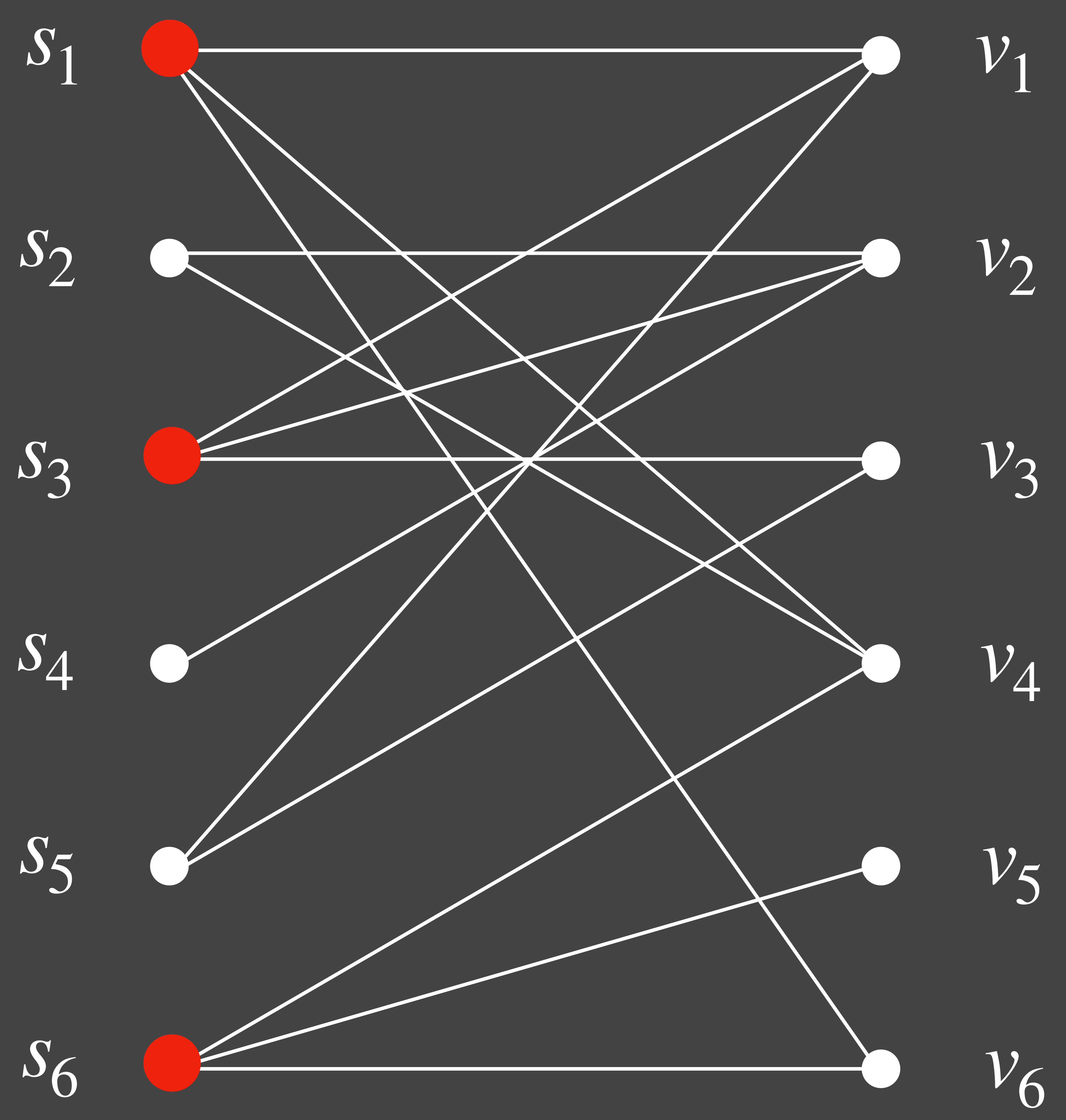
Random Order (RO) Online Set Cover

$\mathcal{S}$

m sets



$\mathcal{U}$

n elements

v_1
v_4
v_5
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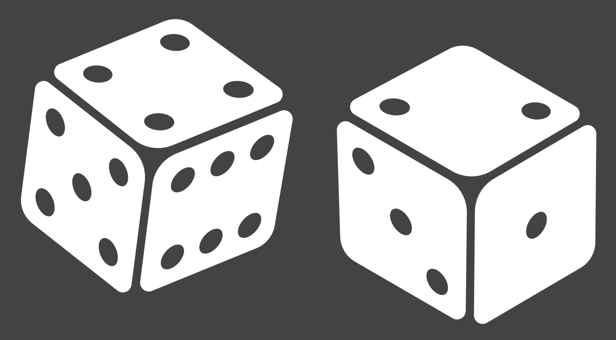

What is known?

Offline	log n + 1 [Johnson74],[Lovasz75], [Chvatal79]
Adversarial Online	O(log n log m) [Alon+03] [BuchbinderNaor09]
Stochastic Online	O(log mn) [Gupta Grandoni Leonardi Miettinen Sankowski Singh 08]
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Believable

$o(\log n \log m)$ not possible...

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Theorem [Gupta Kehne **L.** 21]:

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New algorithm! We show how to learn distribution & solve at same time.

RO Set Cover
(Exponential Time Warmup)

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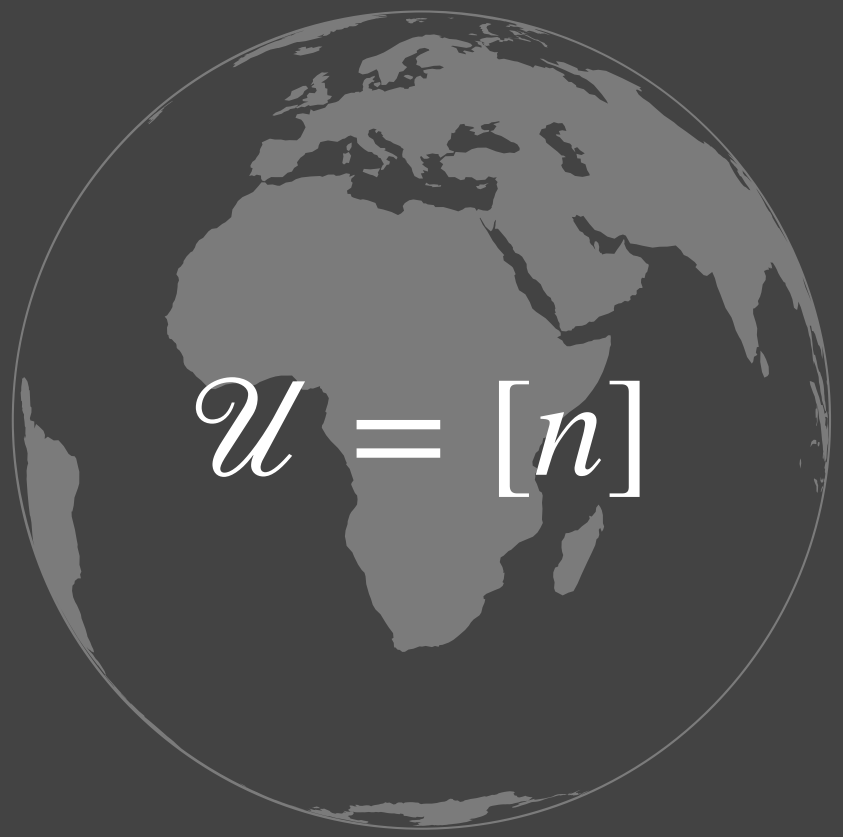
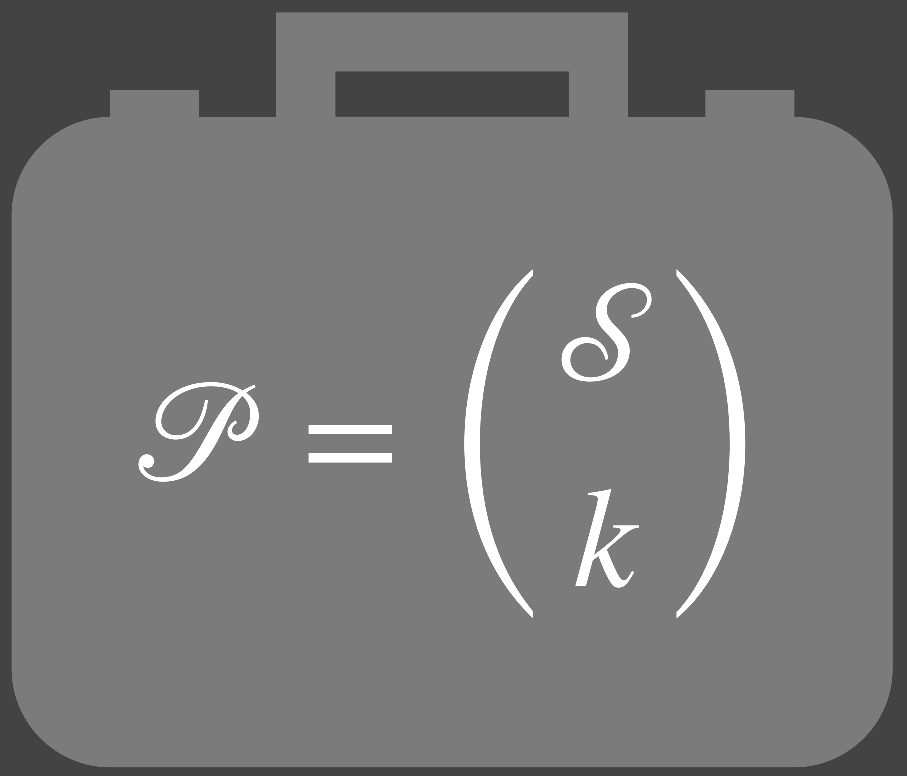


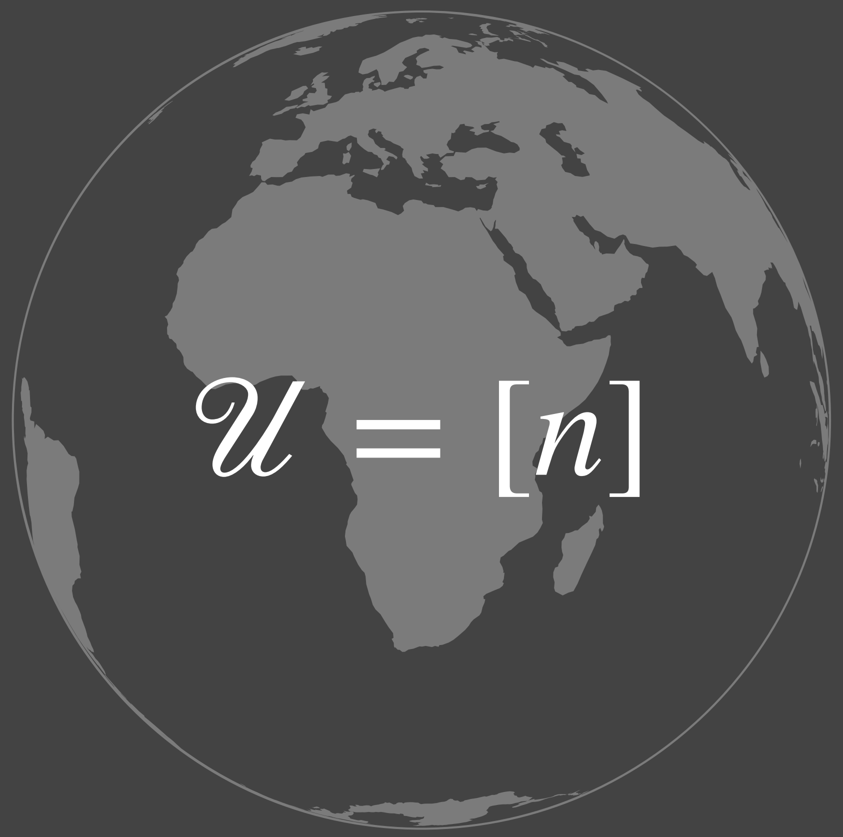
$$\mathscr{U} = [n]$$



$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$k \approx |OPT|$

RO Set Cover
(Exponential Time Warmup)



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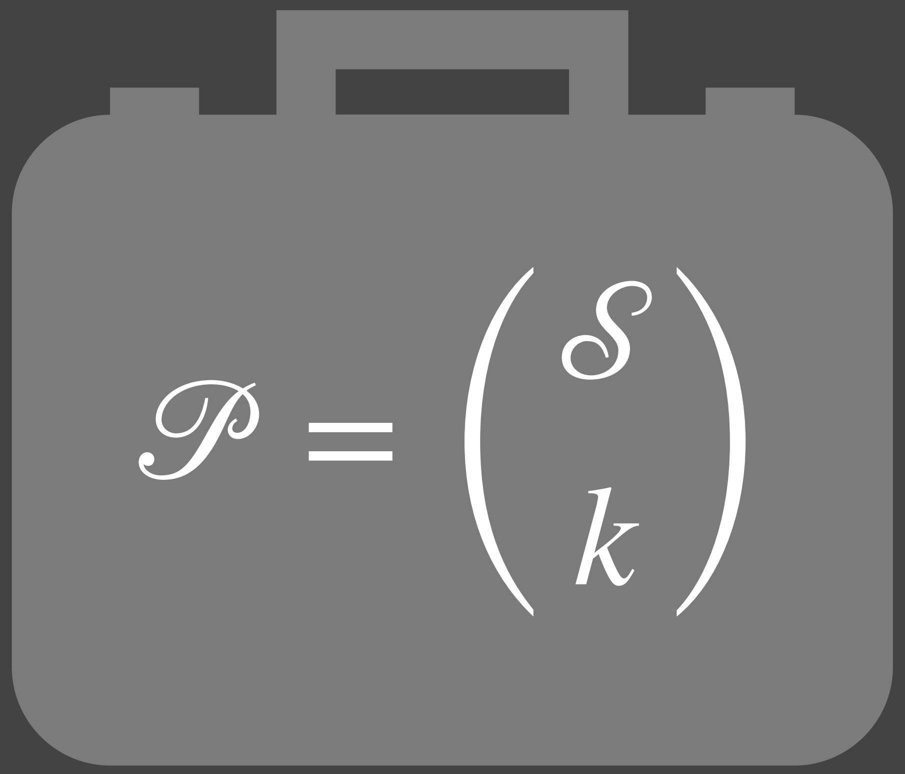

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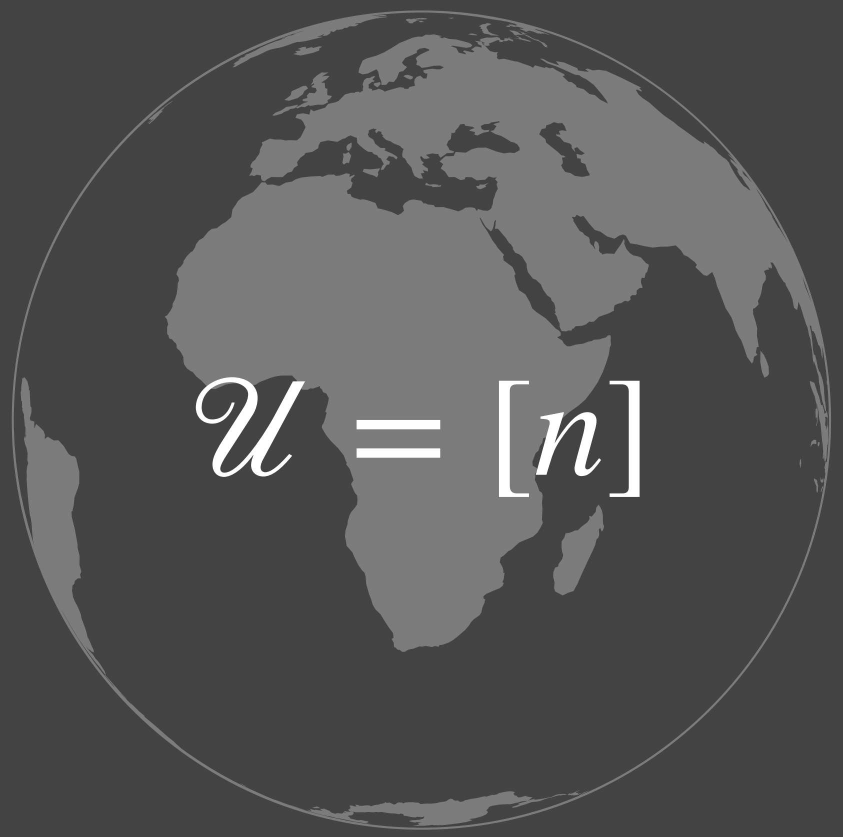
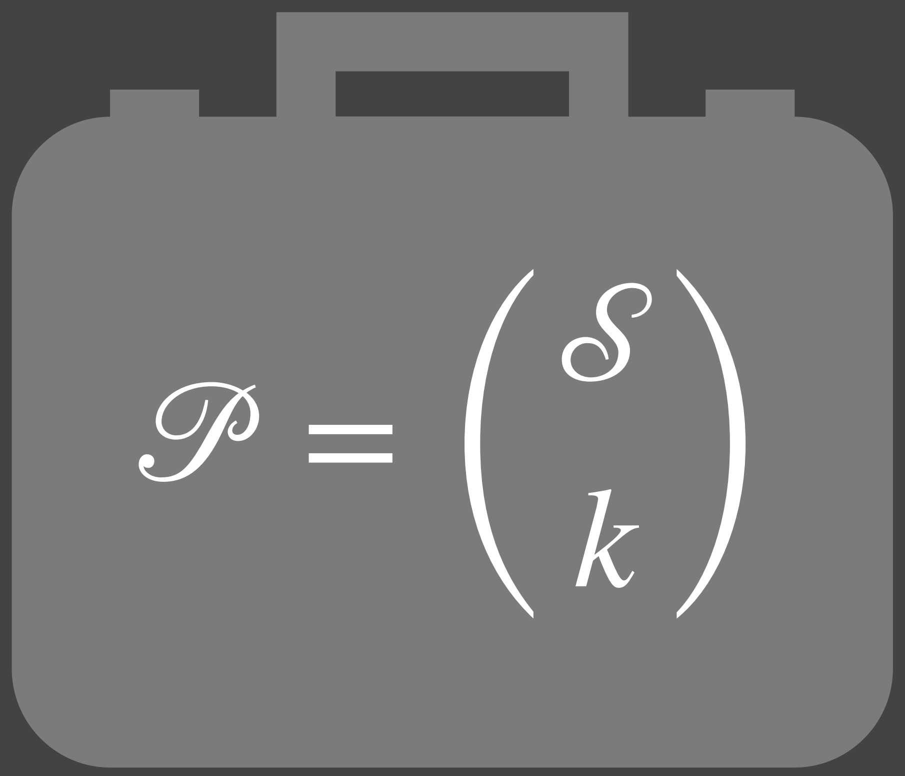
@ time t:

RO Set Cover
(Exponential Time Warmup)





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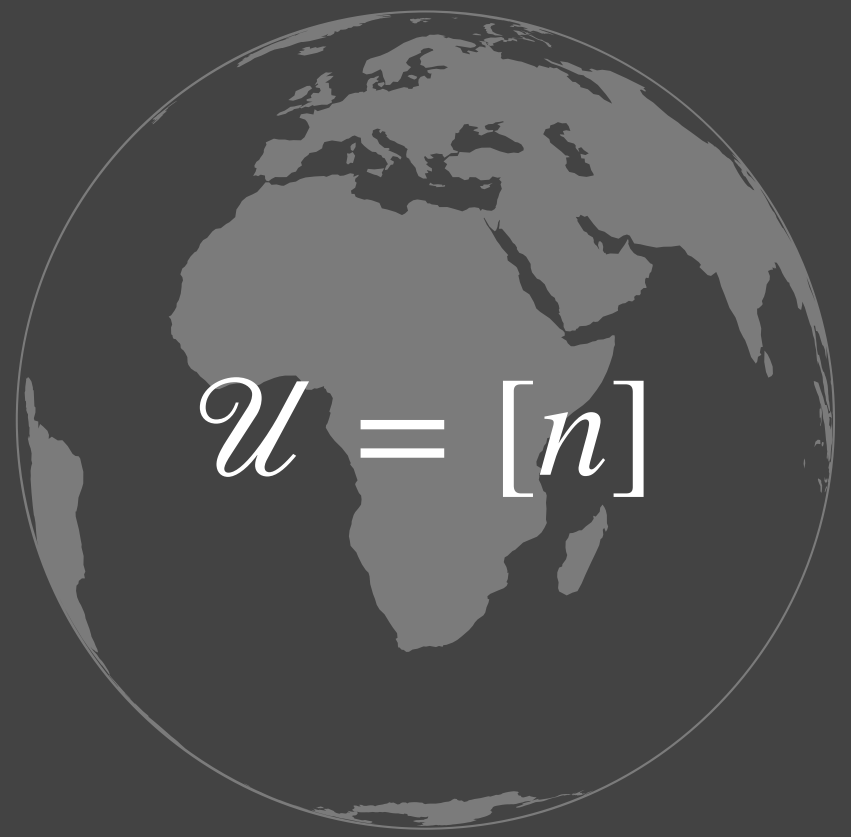

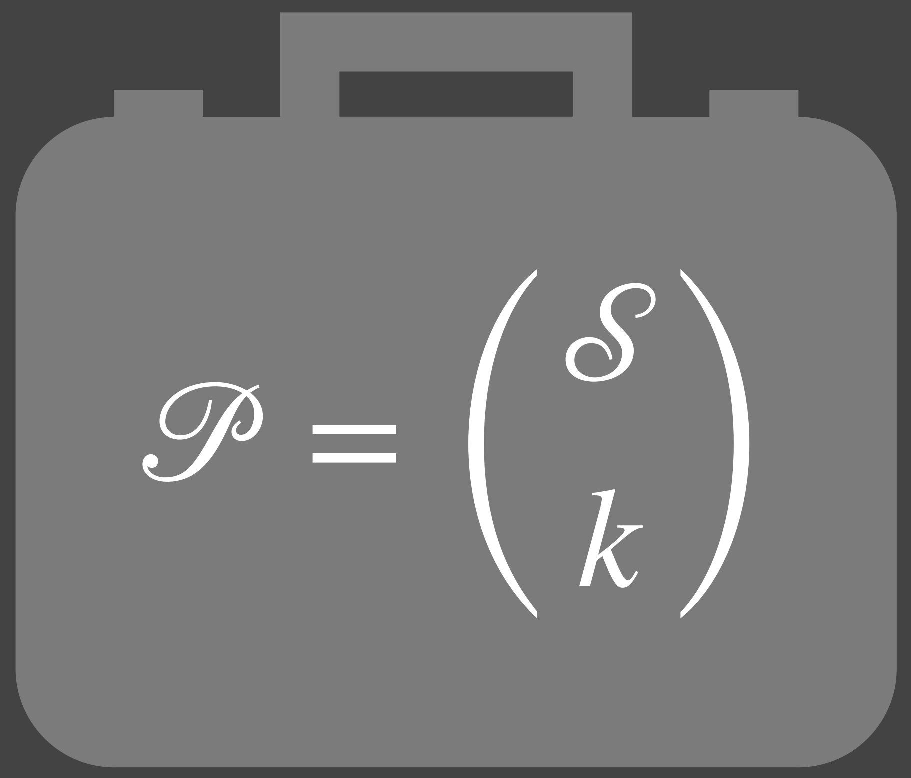

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RO Set Cover
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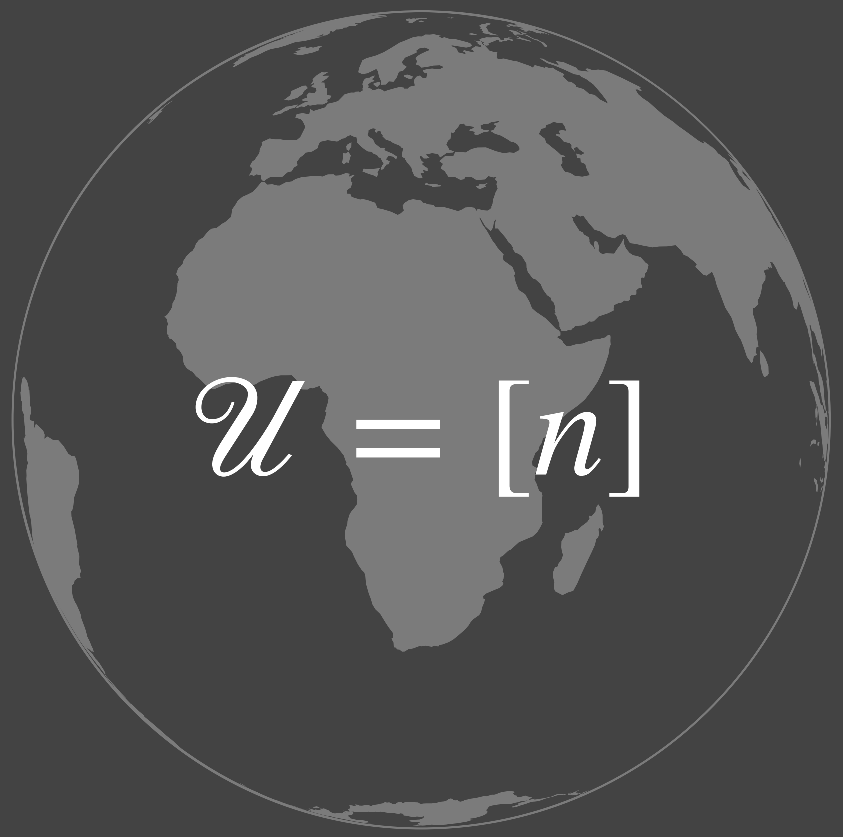

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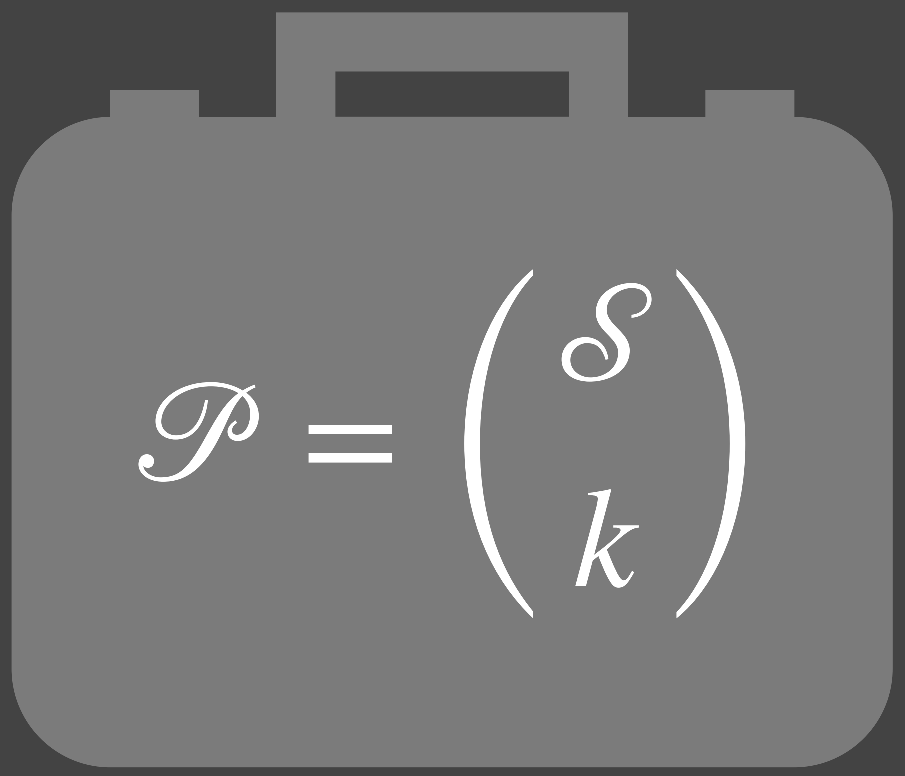

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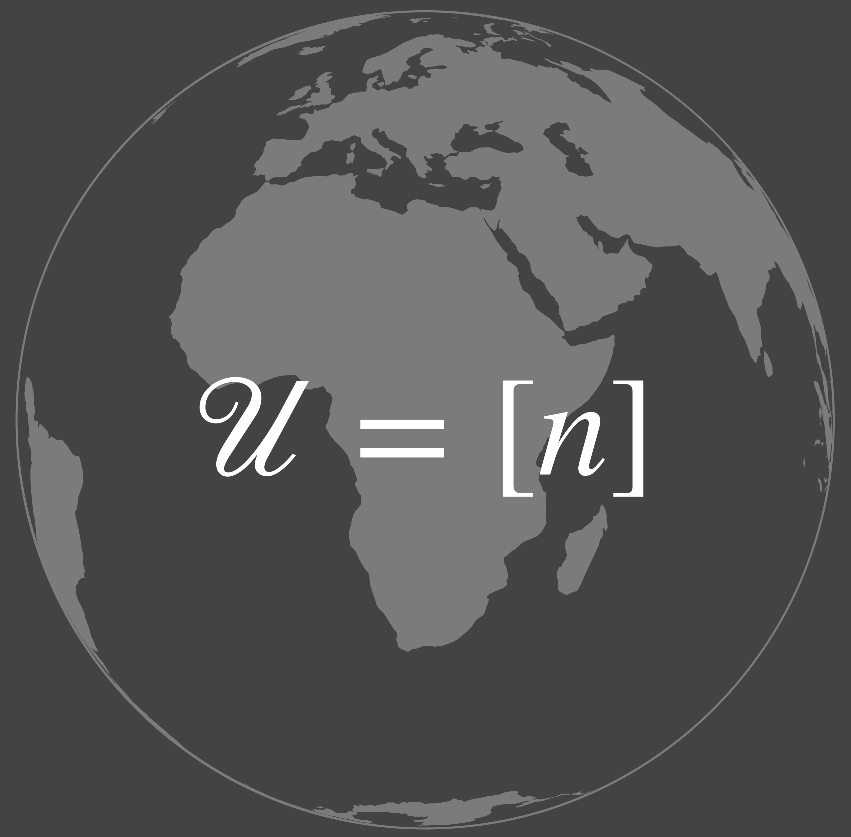

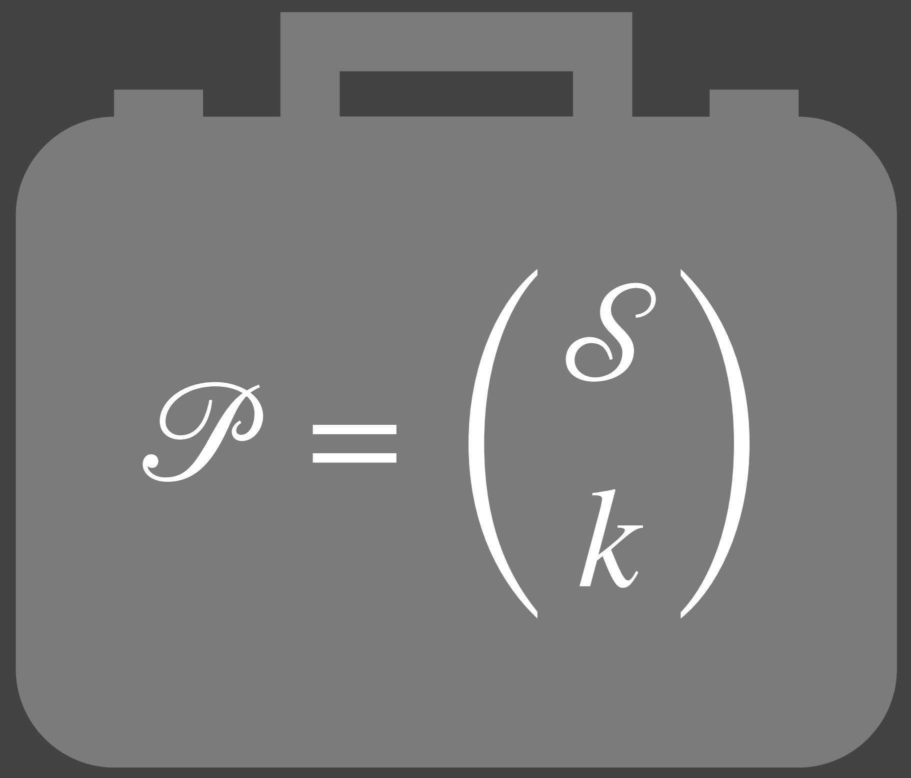

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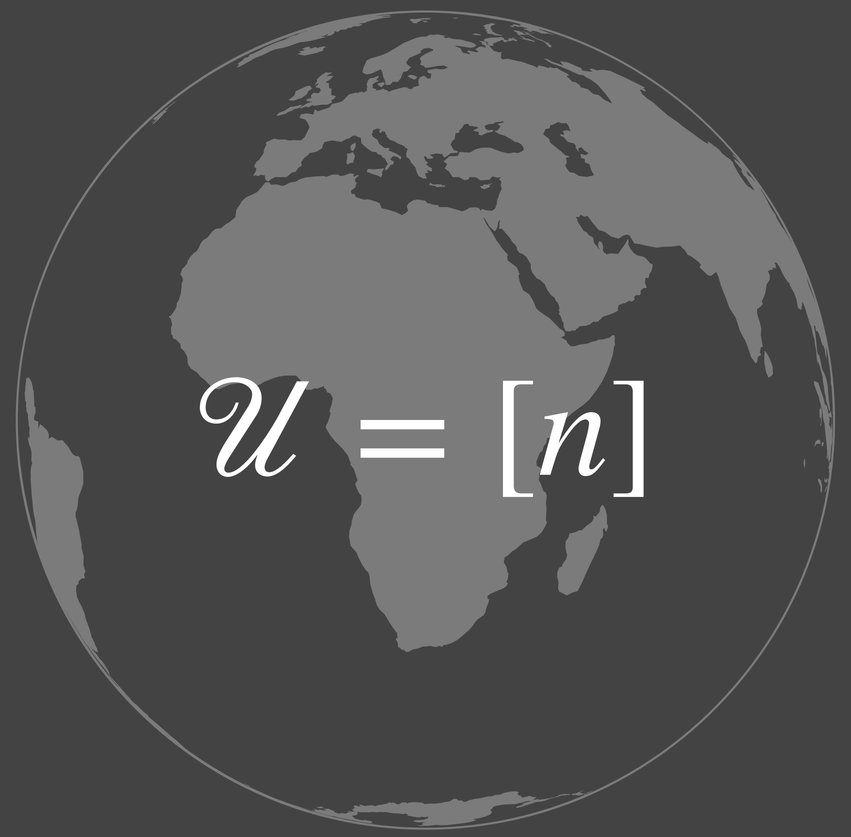
RO Set Cover
(Exponential Time Warmup)



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$\Rightarrow R$ covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

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RO Set Cover

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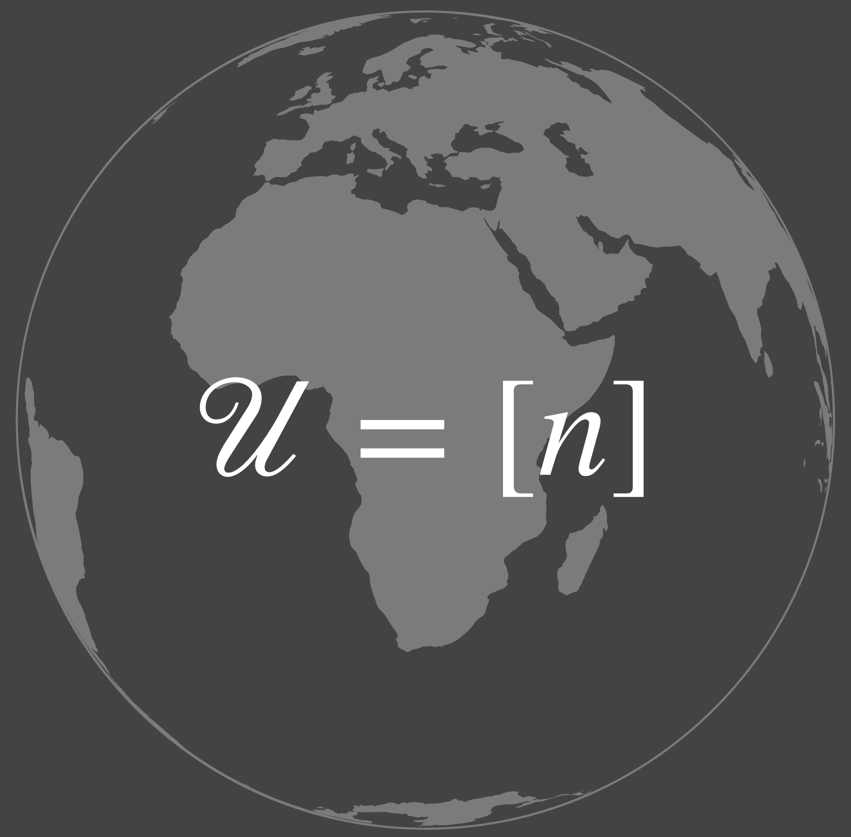

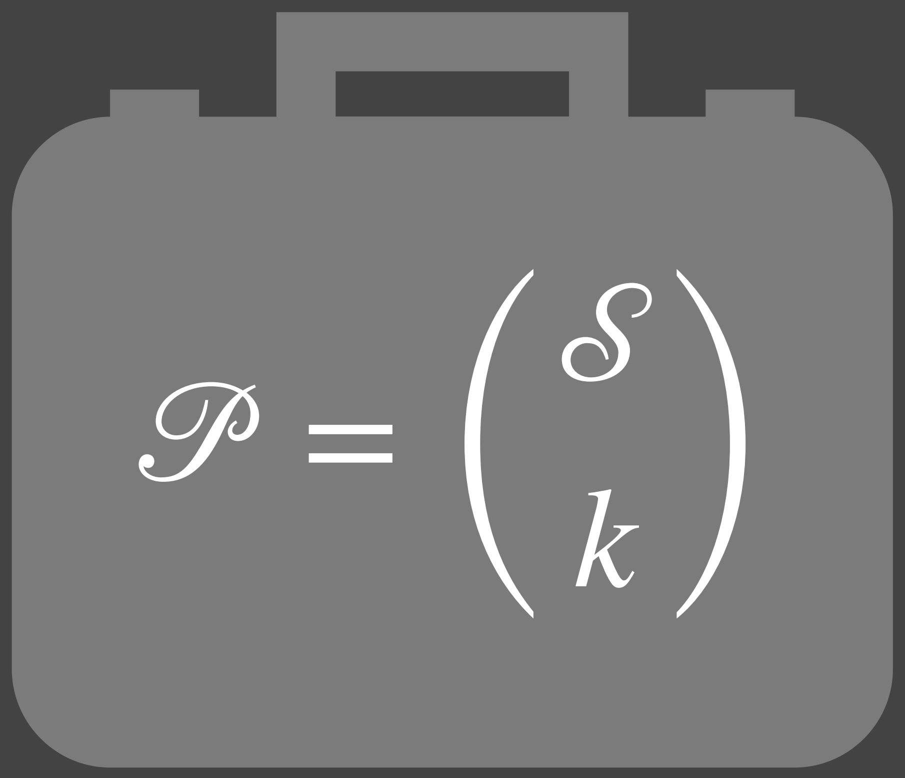

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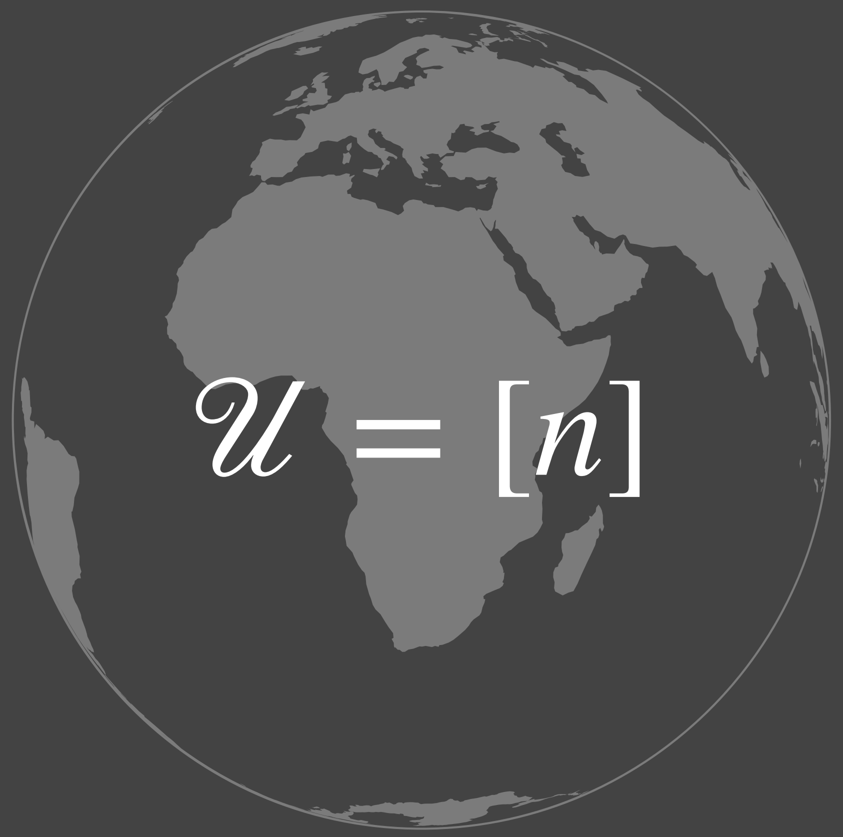

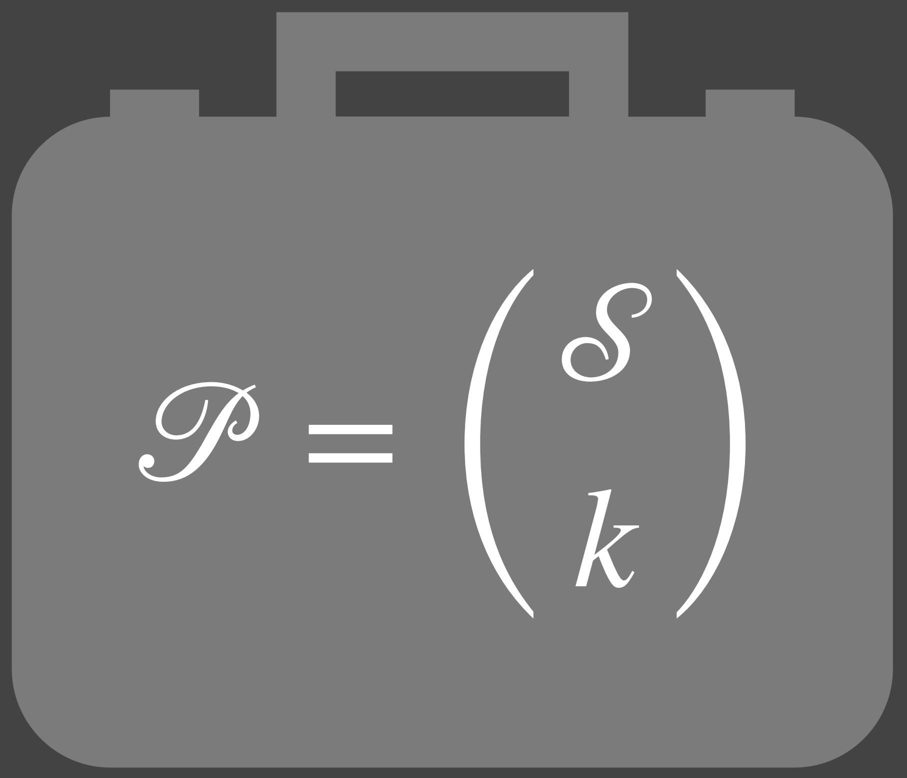

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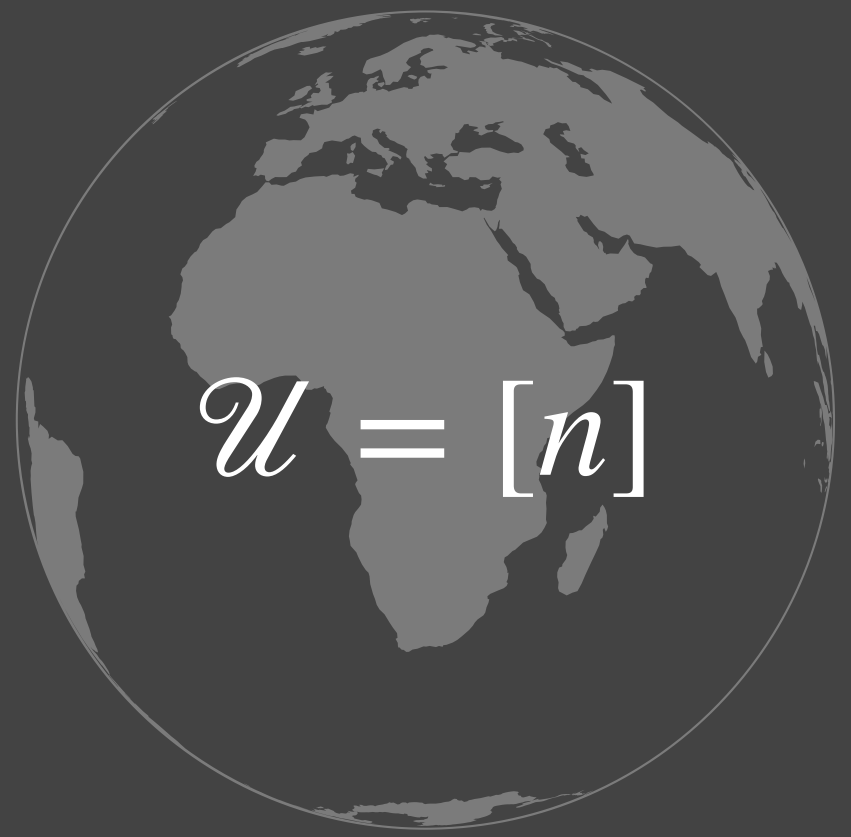
$\Rightarrow$ $\geq 1/2$ of $T \in \mathscr{P}$ pruned w.p. $1/2$.

RO Set Cover
(Exponential Time Warmup)



$$\mathscr{P} = \binom{\mathscr{S}}{k}$$

$k \approx |OPT|$

@ time t:

If v^t covered, do nothing.

Else:

(I) choose $T \sim \mathscr{P}$, buy random $R \sim T$.

(II) "Prune" $T \not\ni v^t$ from $\mathscr{P}$.

Buy arbitrary set to cover v^t.

Case 1: $\geq 1/2$ **of** $T \in \mathscr{P}$ **cover** $\geq 1/2$ **of** $\mathscr{U}$.

$\Rightarrow$ R covers $\dfrac{|\mathscr{U}|}{4k}$ in expectation.

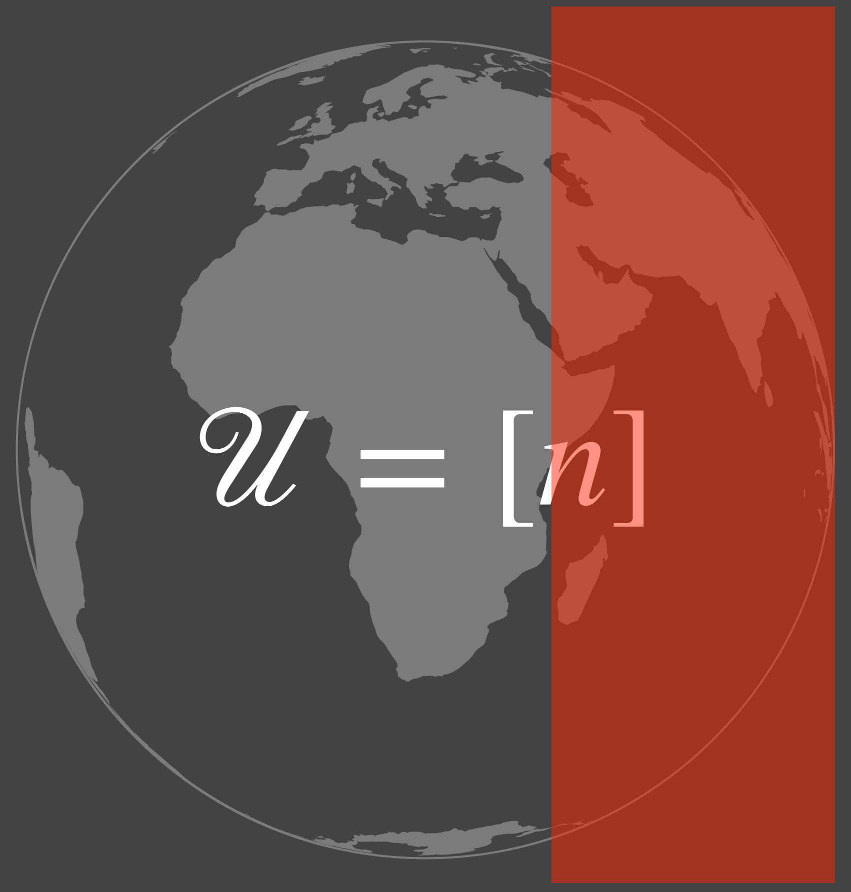
$\Rightarrow$ $\mathscr{U}$ shrinks by $\left(1 - \dfrac{1}{4k}\right)$ in expectation.

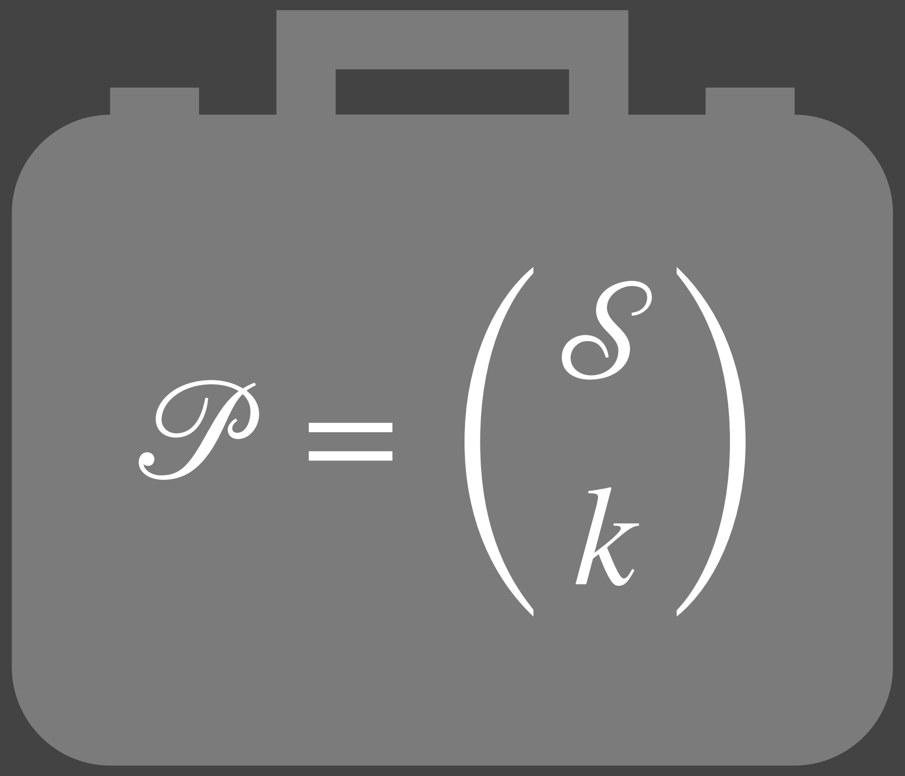
Case 2: $\geq 1/2$ **of** $T \in \mathscr{P}$ **cover** $< 1/2$ **of** $\mathscr{U}$.

$\Rightarrow$ $\geq 1/2$ of $T \in \mathscr{P}$ pruned w.p. $1/2$.

$\Rightarrow$ $\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover
(Exponential Time Warmup)





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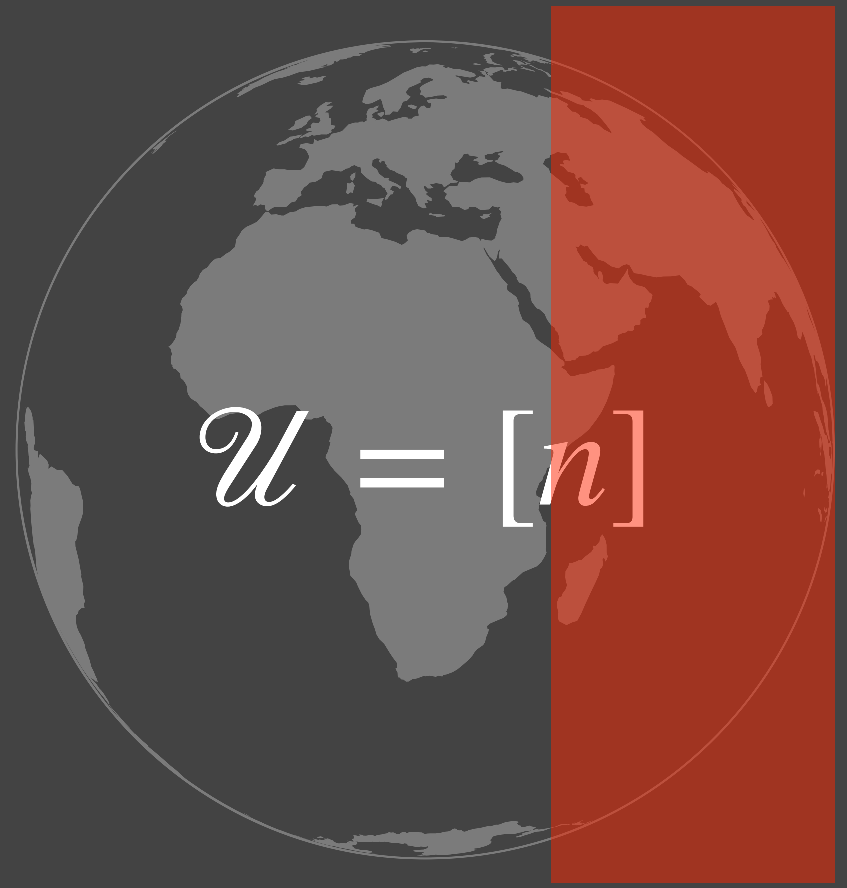

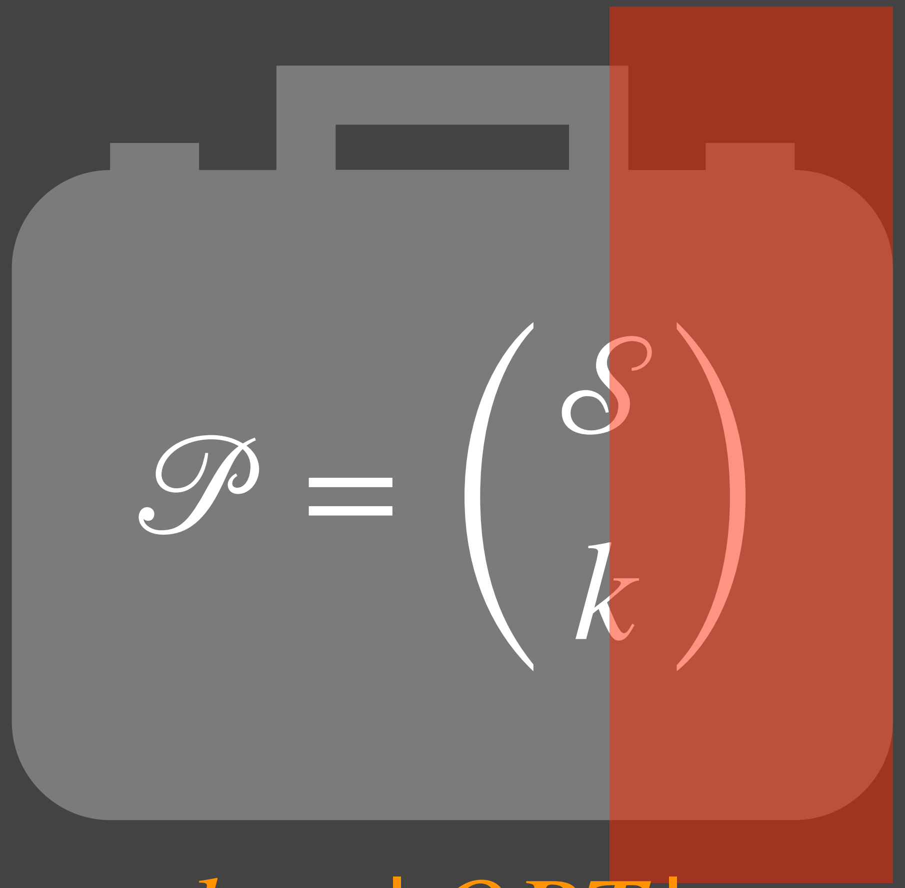

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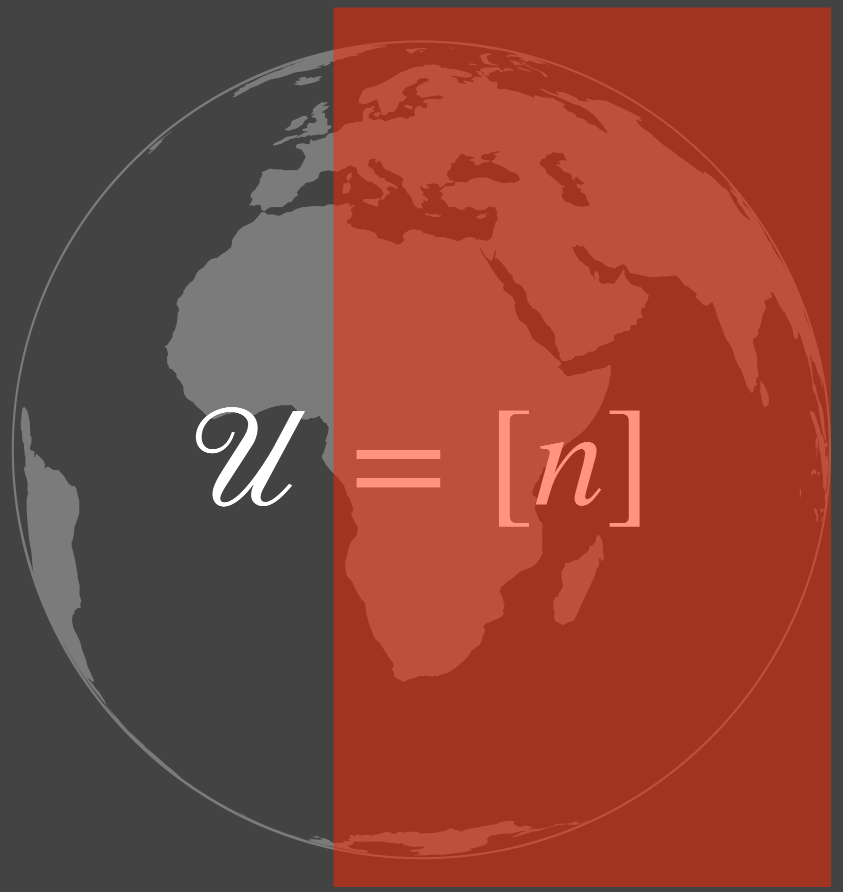

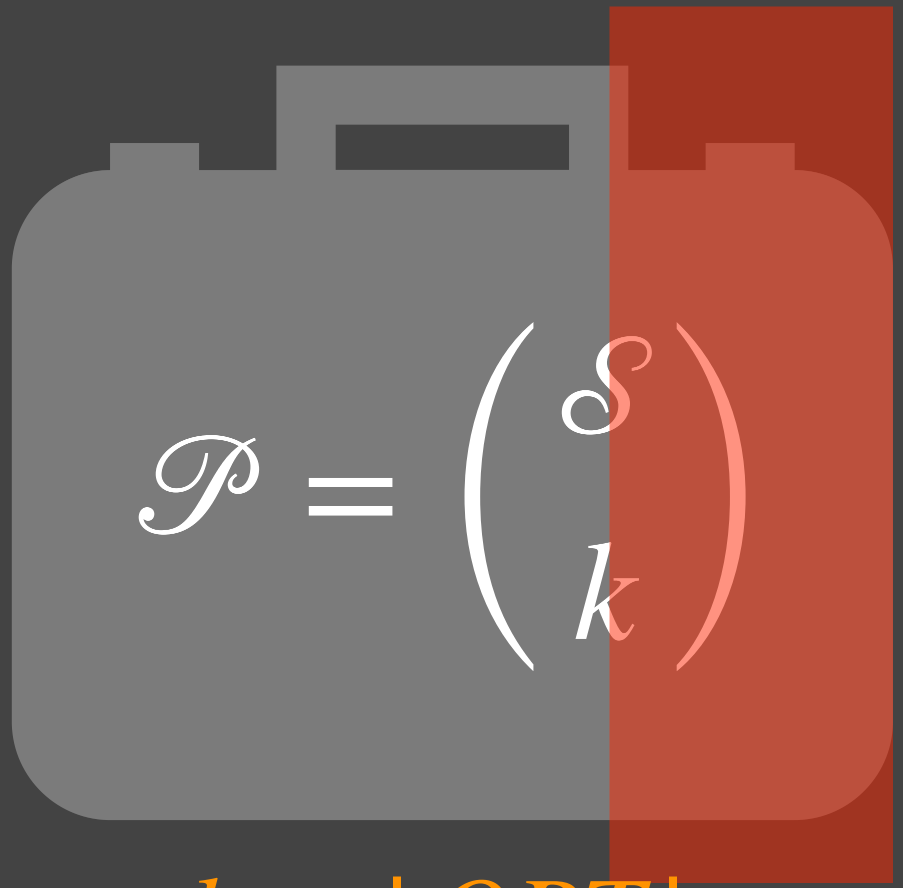

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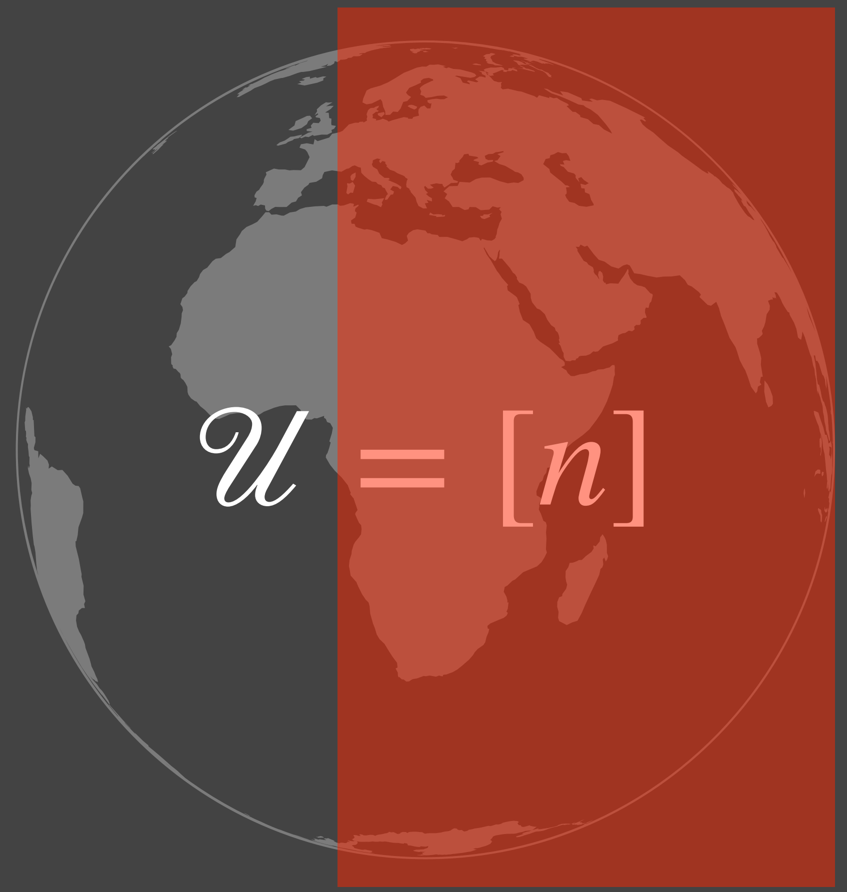

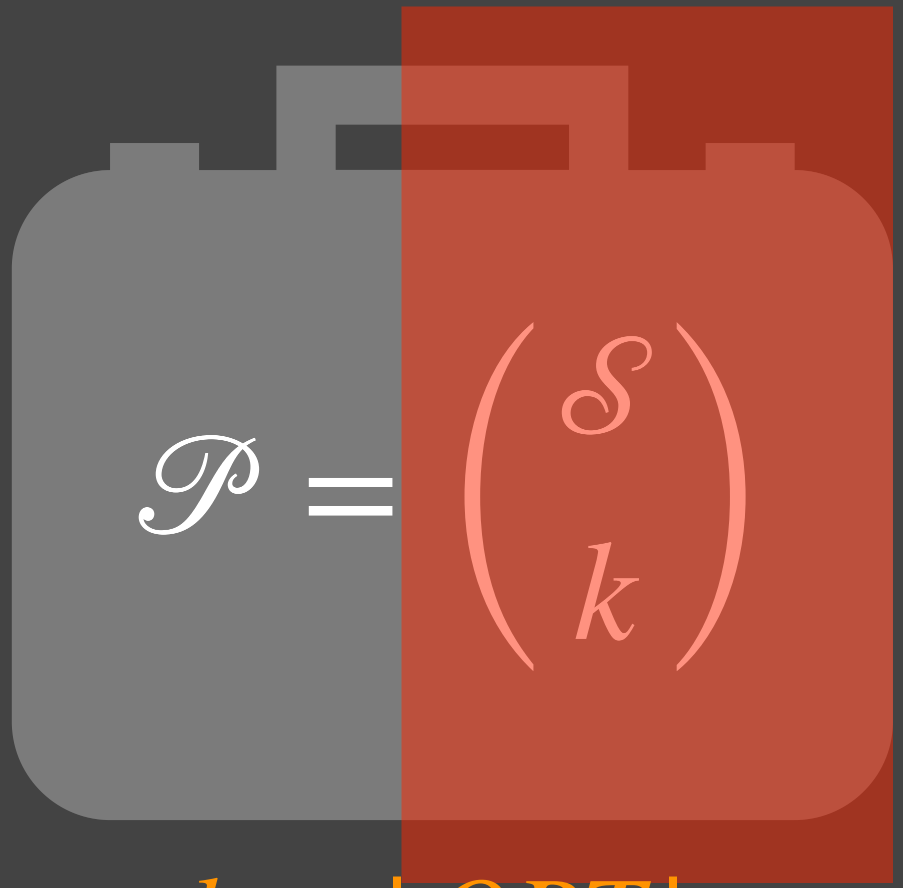

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RO Set Cover
(Exponential Time Warmup)

Case 1: (COVER)

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: (LEARN)

$\mathscr{P}$ shrinks by $3/4$ in expectation.

RO Set Cover
(Exponential Time Warmup)

Case 1: (COVER)

$\mathscr{U}$ shrinks by $\left(1 - \frac{1}{4k}\right)$ in expectation.

Case 2: (LEARN)

$\mathscr{P}$ shrinks by $3/4$ in expectation.

$|\mathscr{U}|$ initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

RO Set Cover
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Case 1: (COVER)

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Case 2: (LEARN)

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$|\mathscr{U}|$ initially n, $\Rightarrow$ $O(k \log n)$ COVER steps suffice.

$|\mathscr{P}|$ initially $\binom{m}{k}$, $\Rightarrow$ $O(k \log m)$ LEARN steps suffice.

RO Set Cover
(Exponential Time Warmup)

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$\Rightarrow O(k \log mn)$ steps suffice.

But how to make polytime?

Can we reuse LEARN/ COVER intuition?

LearnOrCover
(Unit cost)

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Init. $x \leftarrow 1/m$.

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@ time t, element v arrives:

LearnOrCover
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If v covered, do nothing.

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(Unit cost)

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(I) Buy random $R \sim x$.

(II) $\forall S \ni v$, set $x_S \leftarrow e \cdot x_S$.

Renormalize $x \leftarrow x/\|x\|_1$.

LearnOrCover
(Unit cost)

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Buy arbitrary set to cover v.

LearnOrCover
(Unit cost)

Idea! Measure convergence with potential function:

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Idea! Measure convergence with potential function:

$$\boxed{\Phi(t) = c_1 \, \mathrm{KL}(x^* \,||\, x^t) + c_2 \log |\mathcal{U}^t|}$$

$\mathcal{U}^t$:= uncovered elements @ time t

x^* := uniform distribution on OPT

LearnOrCover
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$\sum_S x^*_S \log \frac{x^*_S}{x^t_S}$

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Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

LearnOrCover
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Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

Claim 2: If v uncovered, then $E[\Delta\Phi] \leq -\frac{1}{k}$.

(Recall $k = |OPT|$)

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(Unit cost)

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(Recall $k = |OPT|$)

Bound $E_R[\Delta \log |\mathcal{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v.

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(Unit cost)

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$\sum_S x_S^* \log \frac{x_S^*}{x_S^t}$

Idea! Measure convergence with potential function:

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(Recall $k = |OPT|$)

Bound $E_R[\Delta \log |\mathcal{U}^t|]$ over randomness of R.

Bound $E_v[\Delta KL]$ over randomness of v. $\longleftarrow$ This is where we use RO!

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathscr{U}^t|] \leq -\frac{1}{k}$$

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$$E[\Delta\Phi] = E_v[\Delta \mathrm{KL}] + (e-1) \cdot E_R[\Delta \log |\mathcal{U}^t|] \leq -\frac{1}{k}$$

Since $\Phi(0) = O(\log(mn))$, expected total cost is $k \log(mn)$.

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathscr{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \le (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\mathbf{KL}(x^* \,||\, x^t) - \mathbf{KL}(x^* \,||\, x^{t-1})$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \le -E_v\left[\sum_{S \ni v} x_S\right].$$

Proof:

Claim 2a: If v^t uncovered,

$$E_v[\Delta \mathrm{KL}] \leq (e-1) \cdot E_v\left[\sum_{S \ni v} x_S\right] - \frac{1}{k}.$$

Proof:

$$\sum_S x_S^* \log\left(\frac{x_S^*}{x_S^t}\right) - \sum_S x_S^* \log\left(\frac{x_S^*}{x_S^{t-1}}\right)$$

Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

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Claim 2a: If v^t uncovered,

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$$E_R[\Delta \log|\mathscr{U}^t|] \le -E_v\left[\sum_{S\ni v} x_S\right].$$

Proof:

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Proof:

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Claim 2b: If v^t uncovered,

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Proof:

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$$= \underbrace{\cancel{\sum_S x_S^*}}_{=1} \log \|x\|_1 - \sum_{S\ni v} x_S^* \underbrace{\cancel{\log e}}_{=1}$$

$$= \log\left(\sum_S x_S^{t-1} + \sum_{S\ni v}(e-1)\cdot x_S^{t-1}\right) - \sum_{S\ni v} x_S^*$$

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Claim 2b: If v^t uncovered,

$$E_R[\Delta \log |\mathcal{U}^t|] \leq -E_v\left[\sum_{S \ni v} x_S\right].$$

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Extensions & Lower bounds

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Theorem: $O(\log mn)$ for pure covering IPs in random order.

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Recall, in Part I [Gupta L. 20], we show $O(\log m \log(n \cdot f(\mathcal{N})))$ for adversarial order.

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Online set cover, but random 1/2 of elements known upfront (see [Kaplan Naori Raz 21]).

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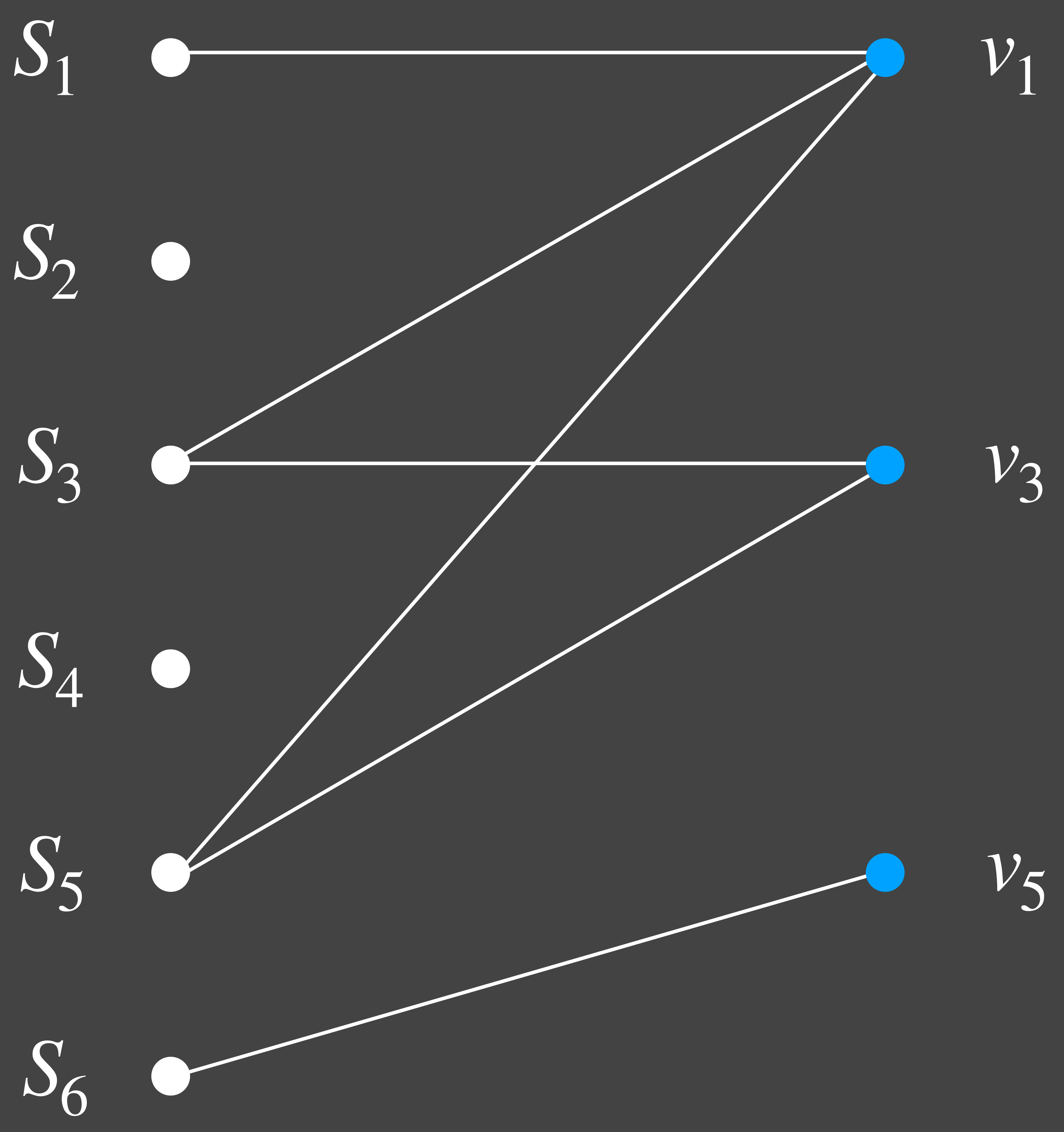

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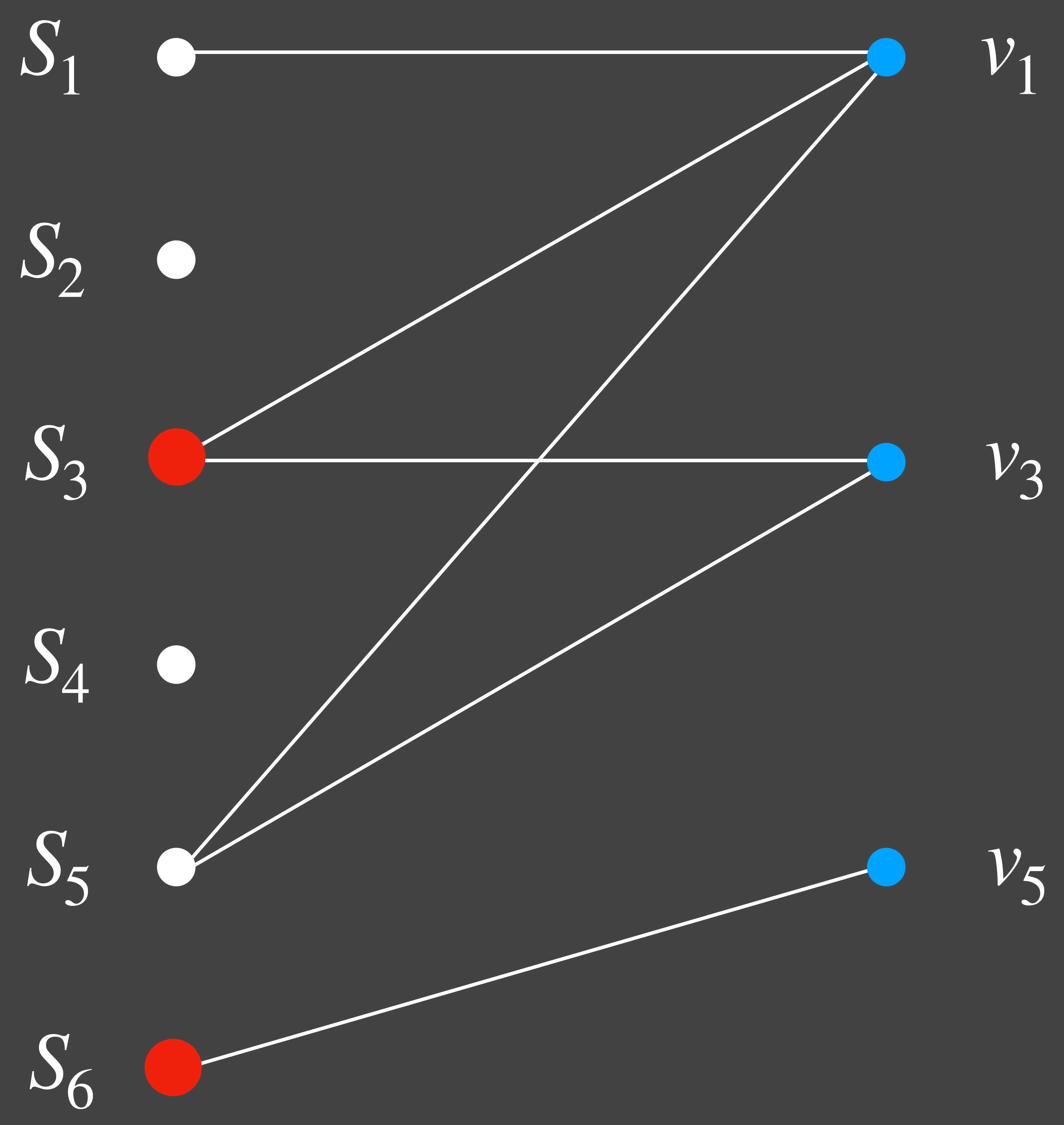

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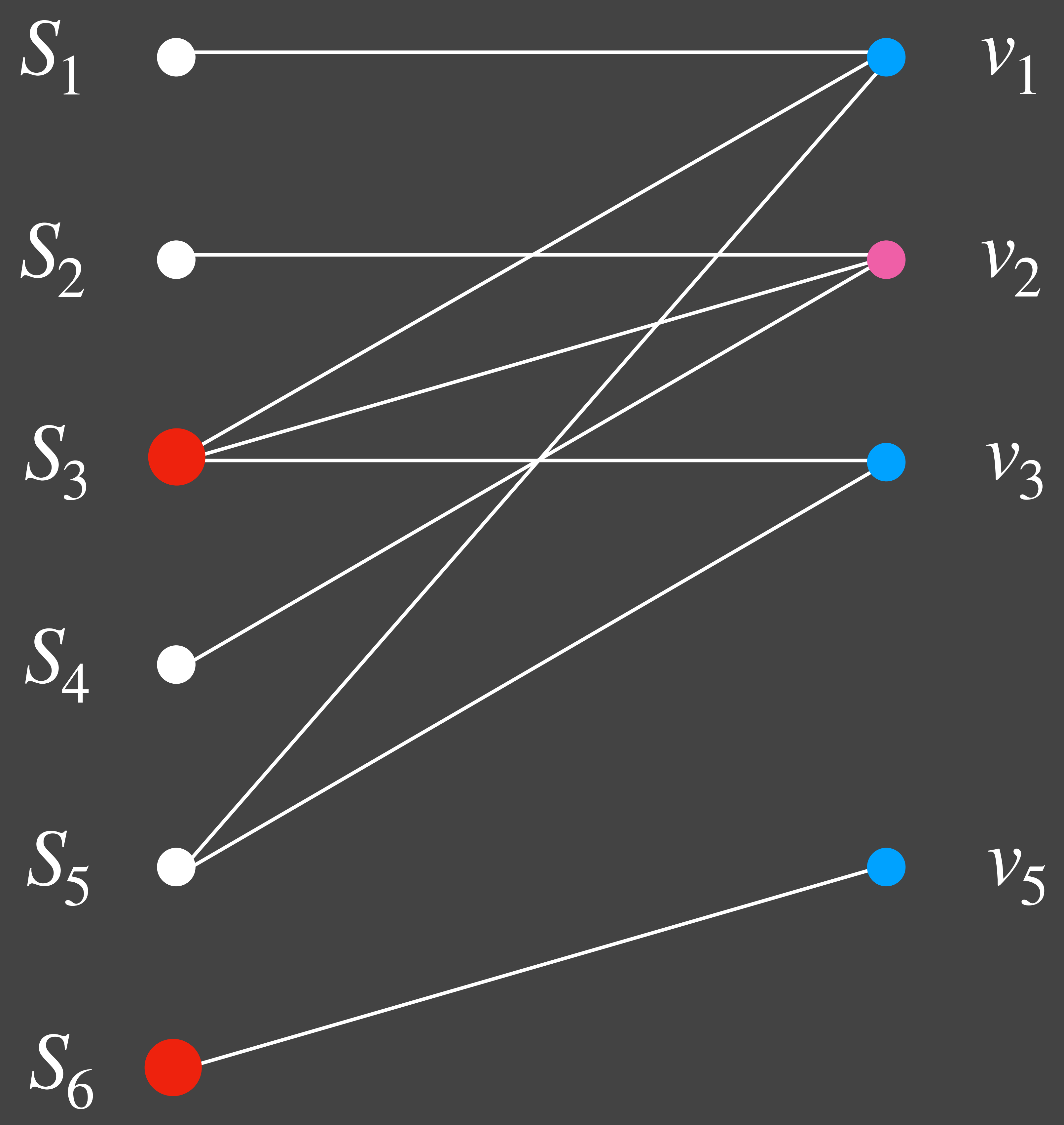

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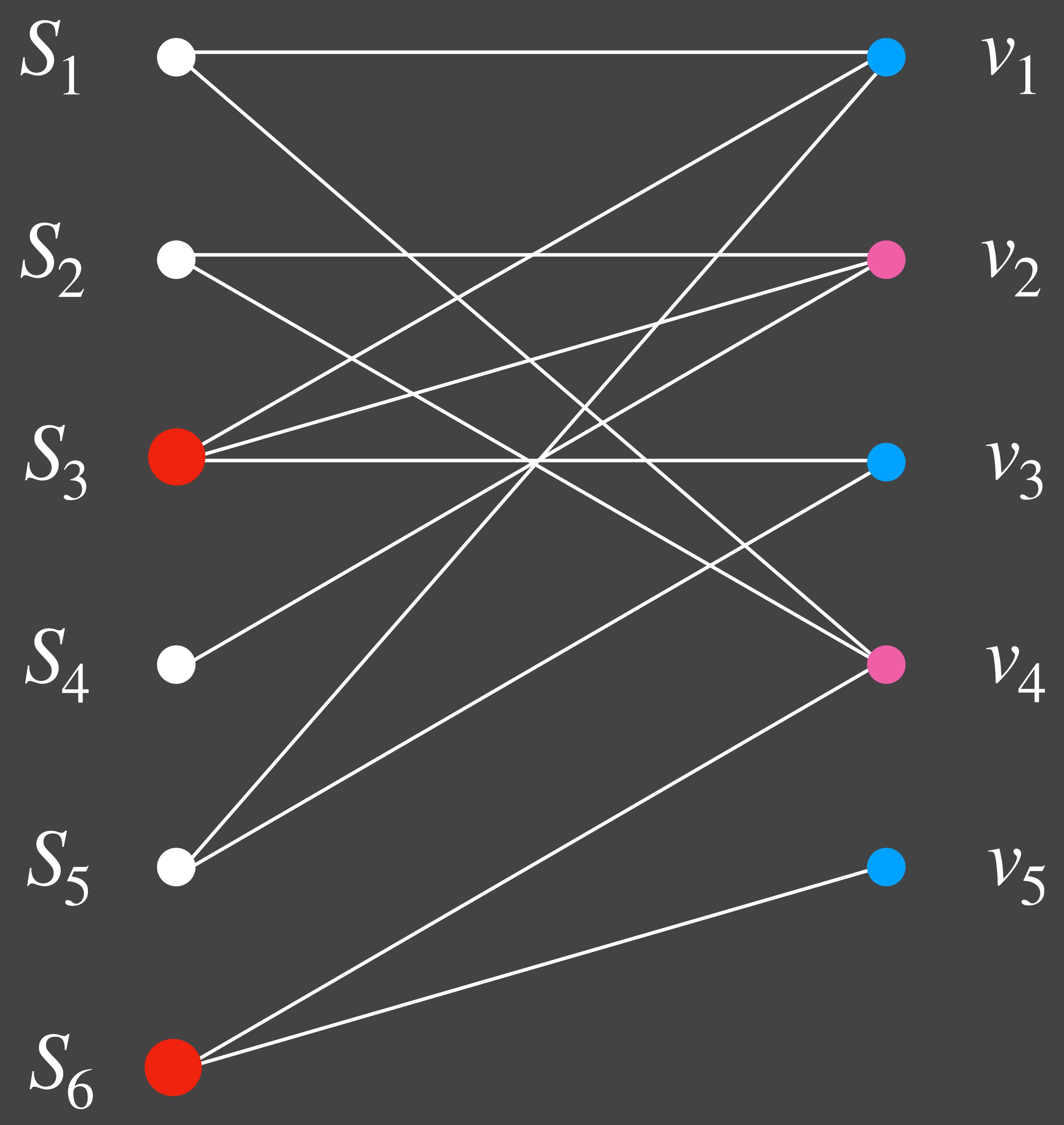

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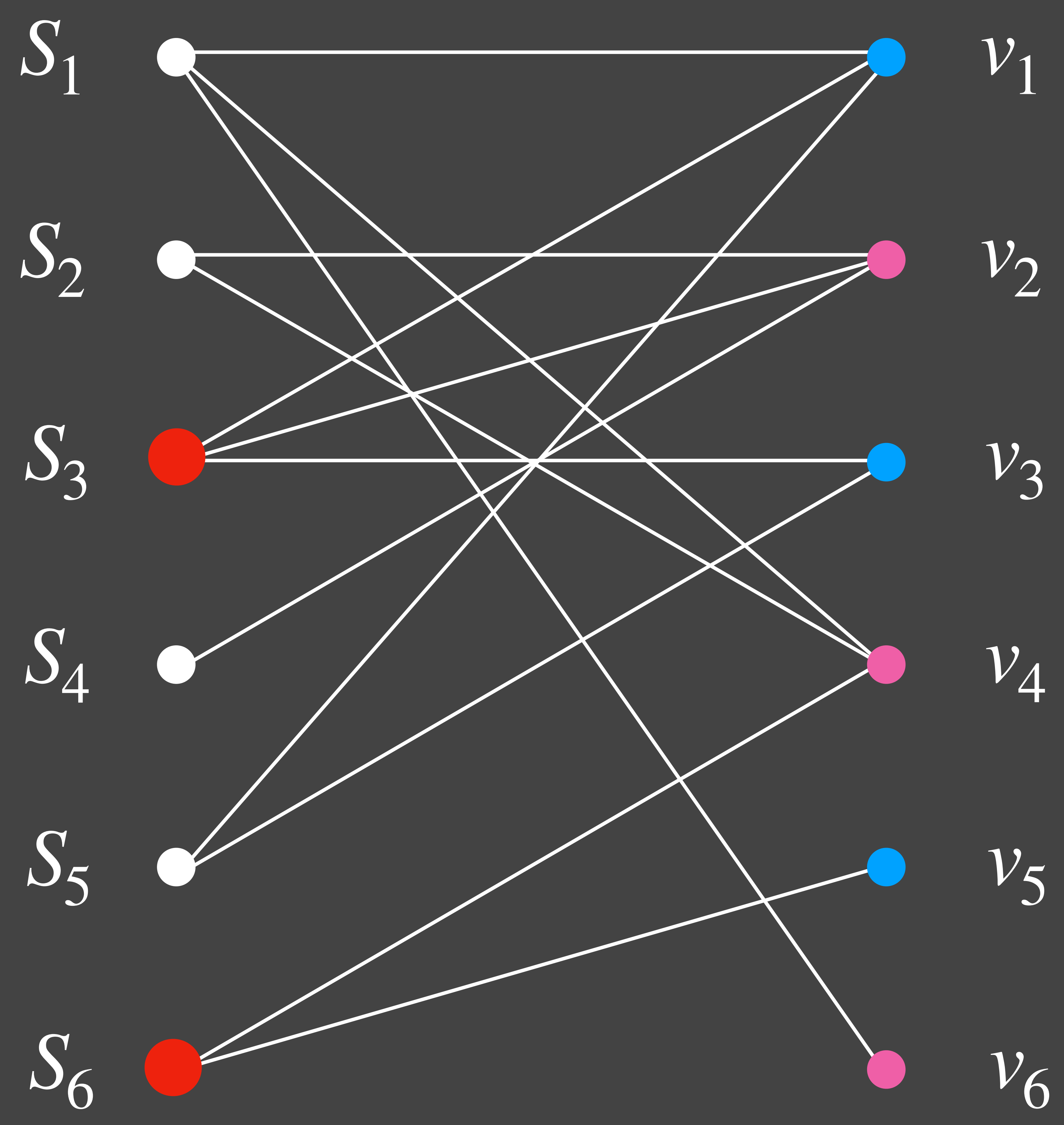

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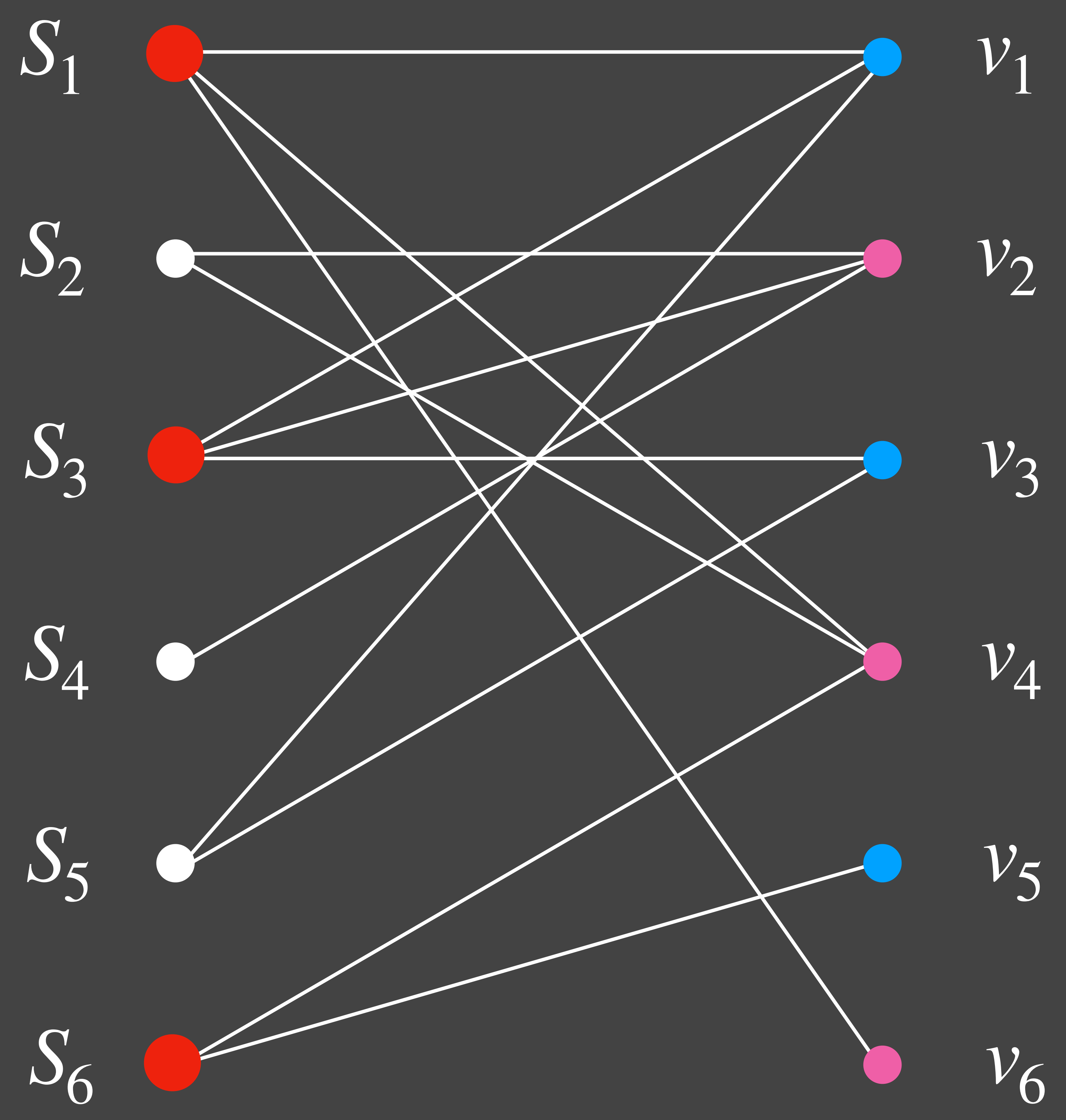

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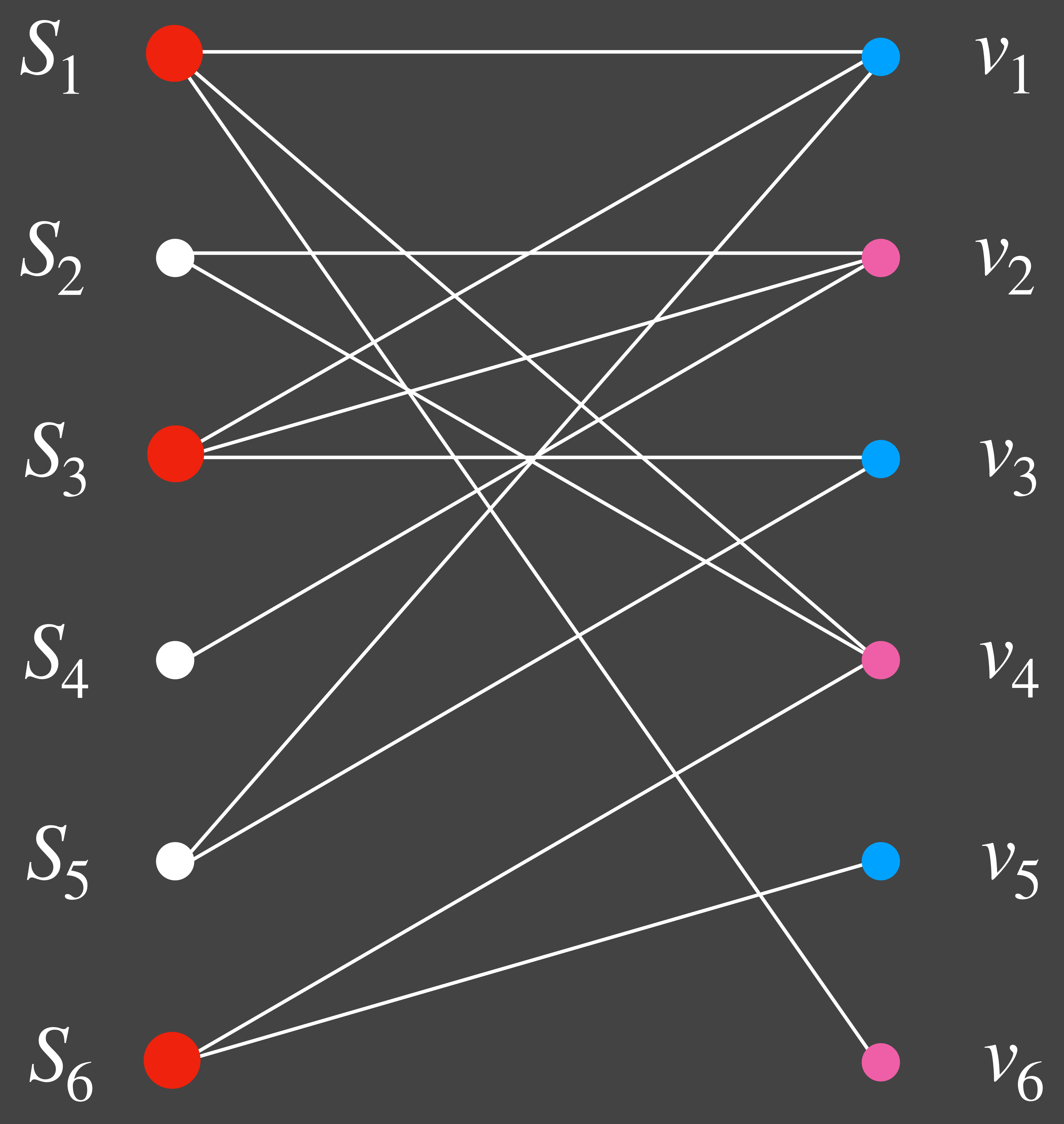

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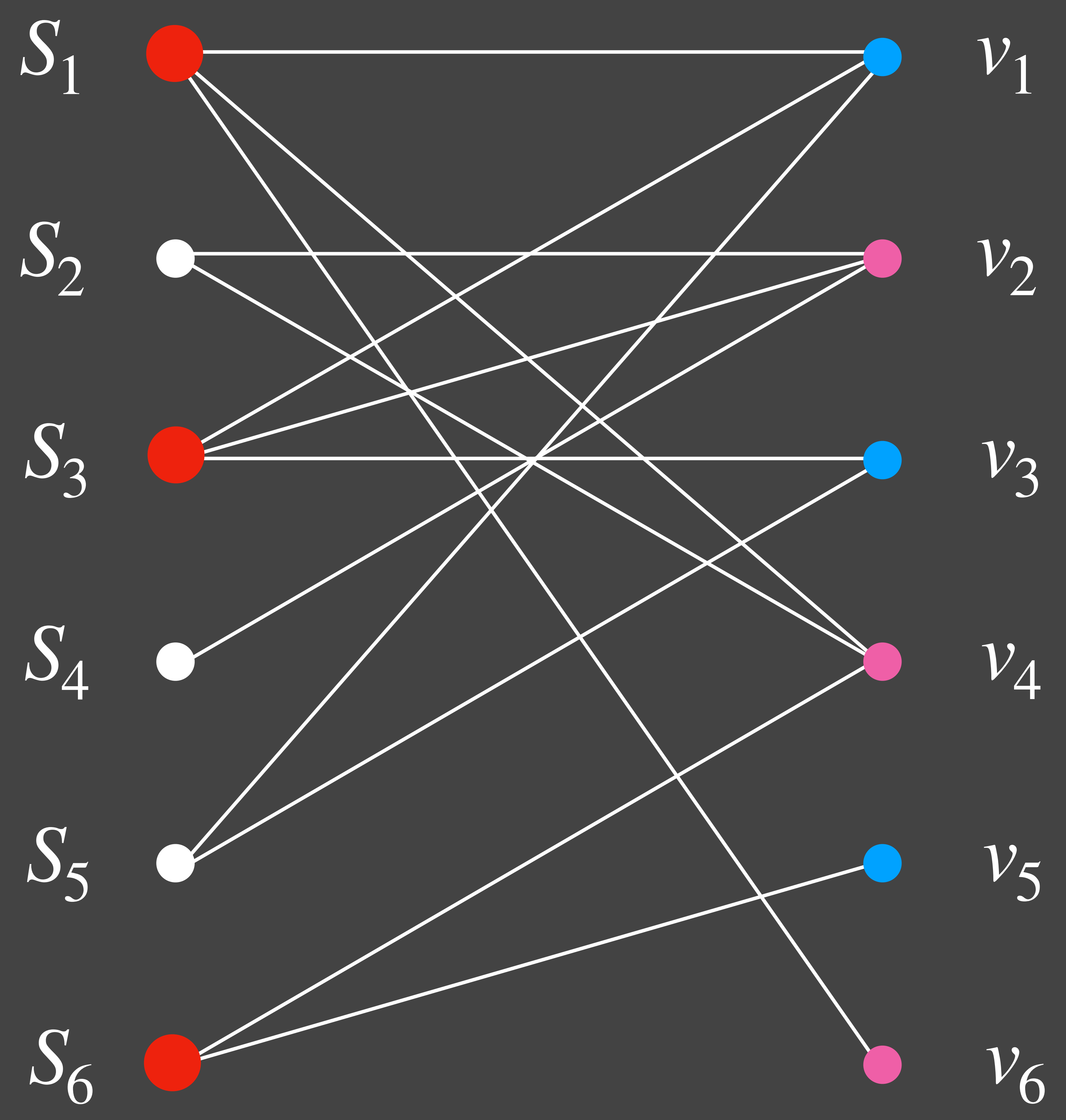

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Theorem:

*There is a **randomized poly time** algorithm for **Online Set** Cover With-a-Sample with competitive ratio* $O(\log(mn))$.

Reduction to LearnOrCover!

S_1 ●

S_2 ●

S_3 ●

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S_5 ●

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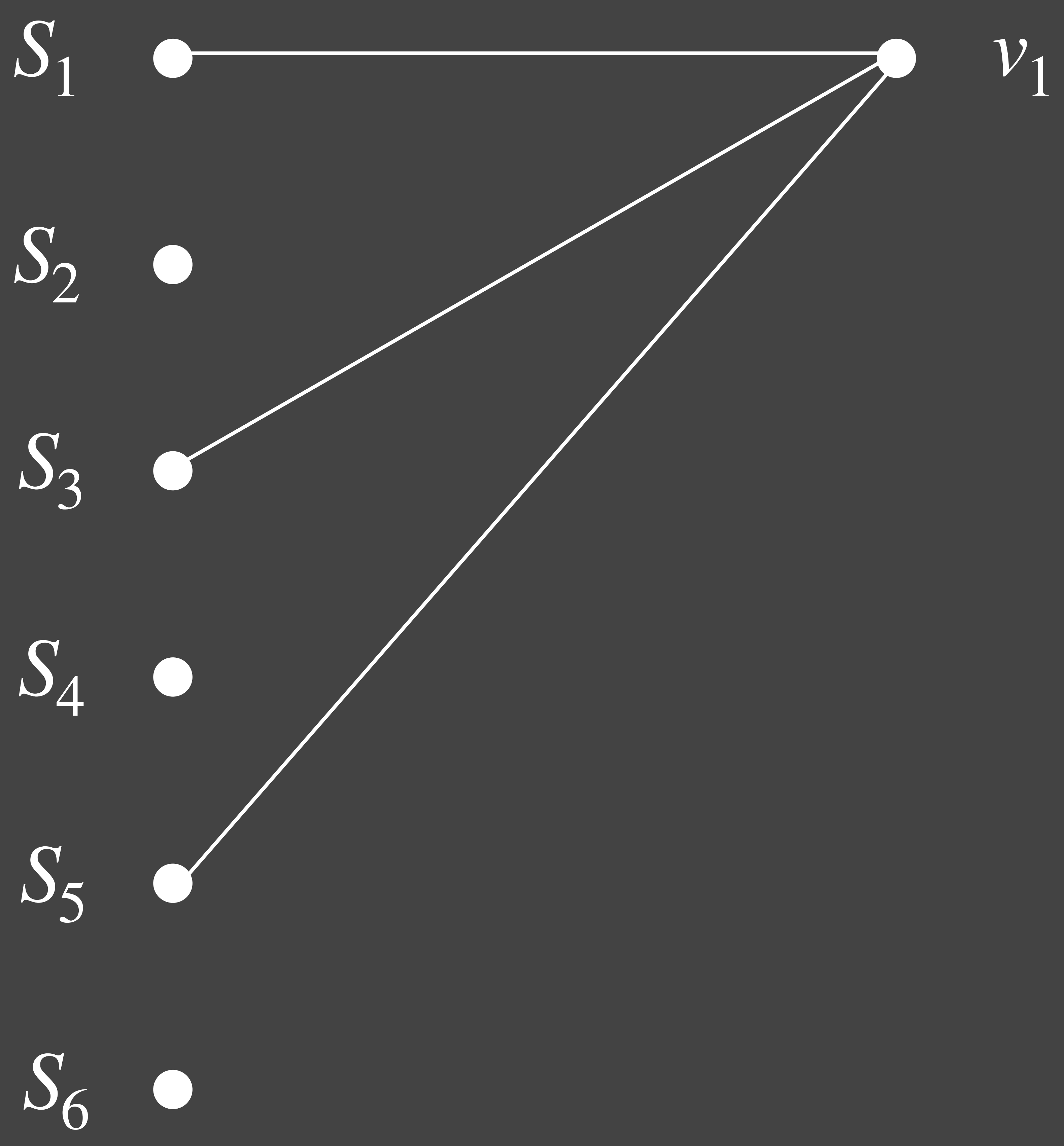

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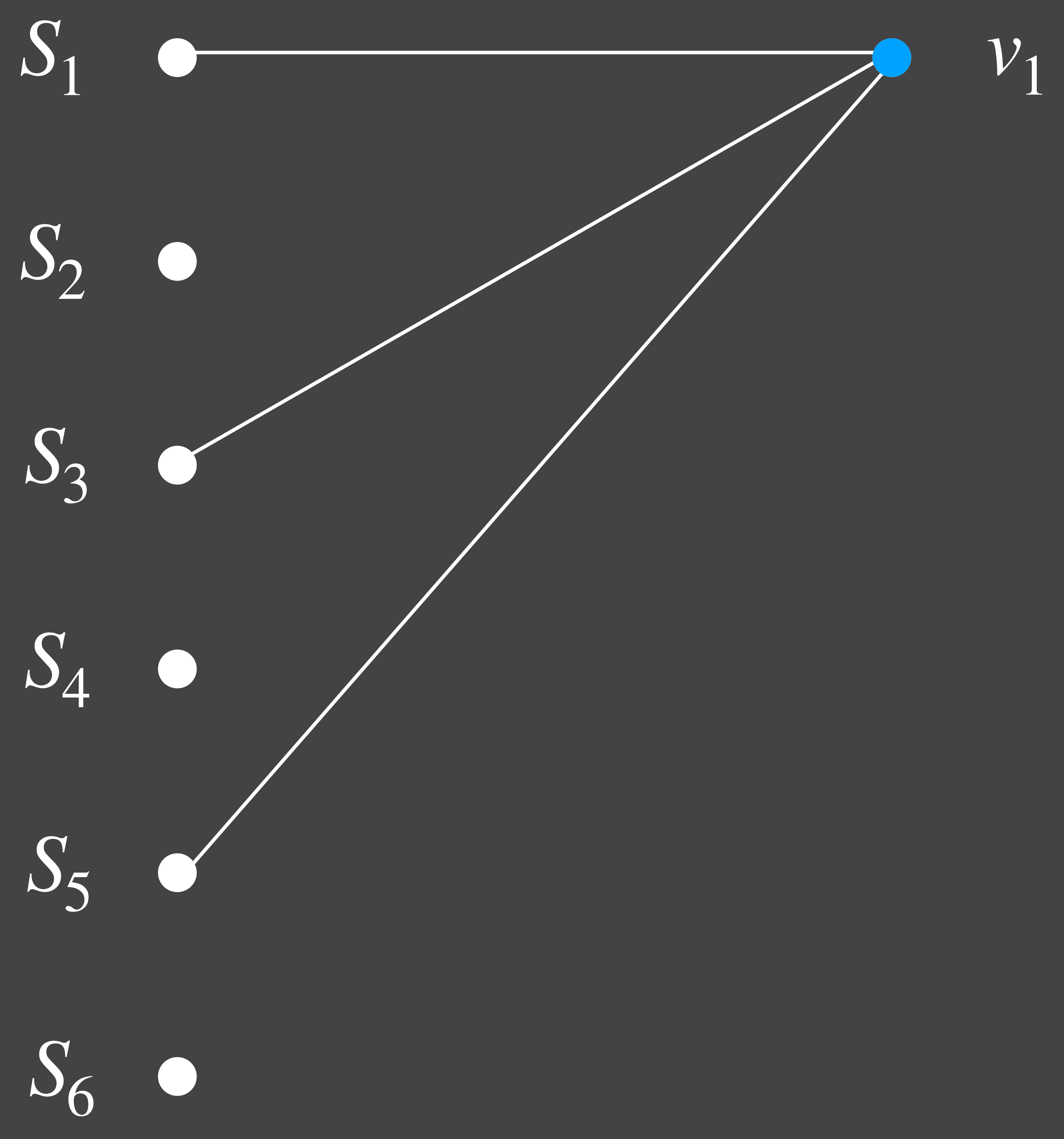

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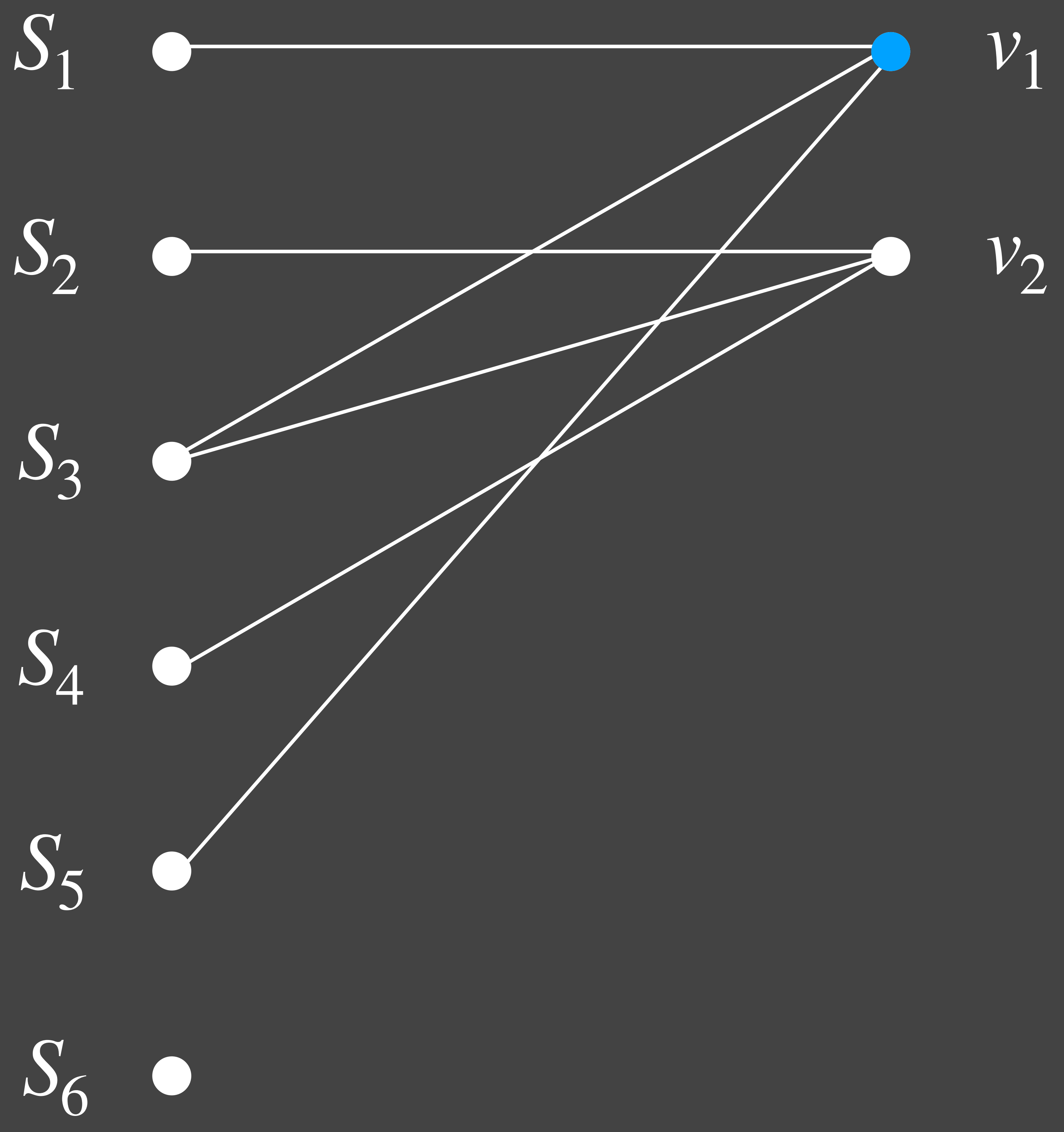

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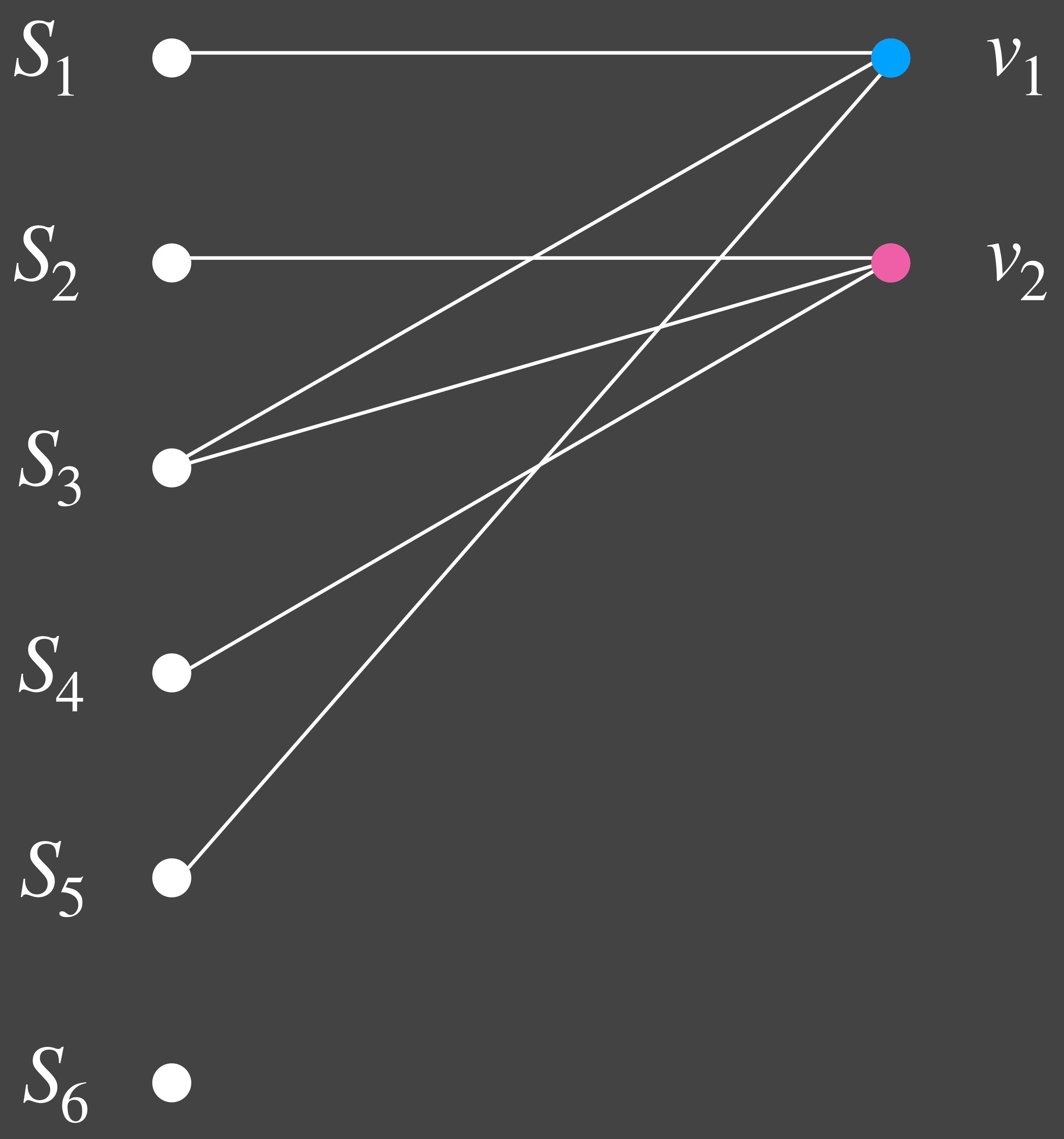

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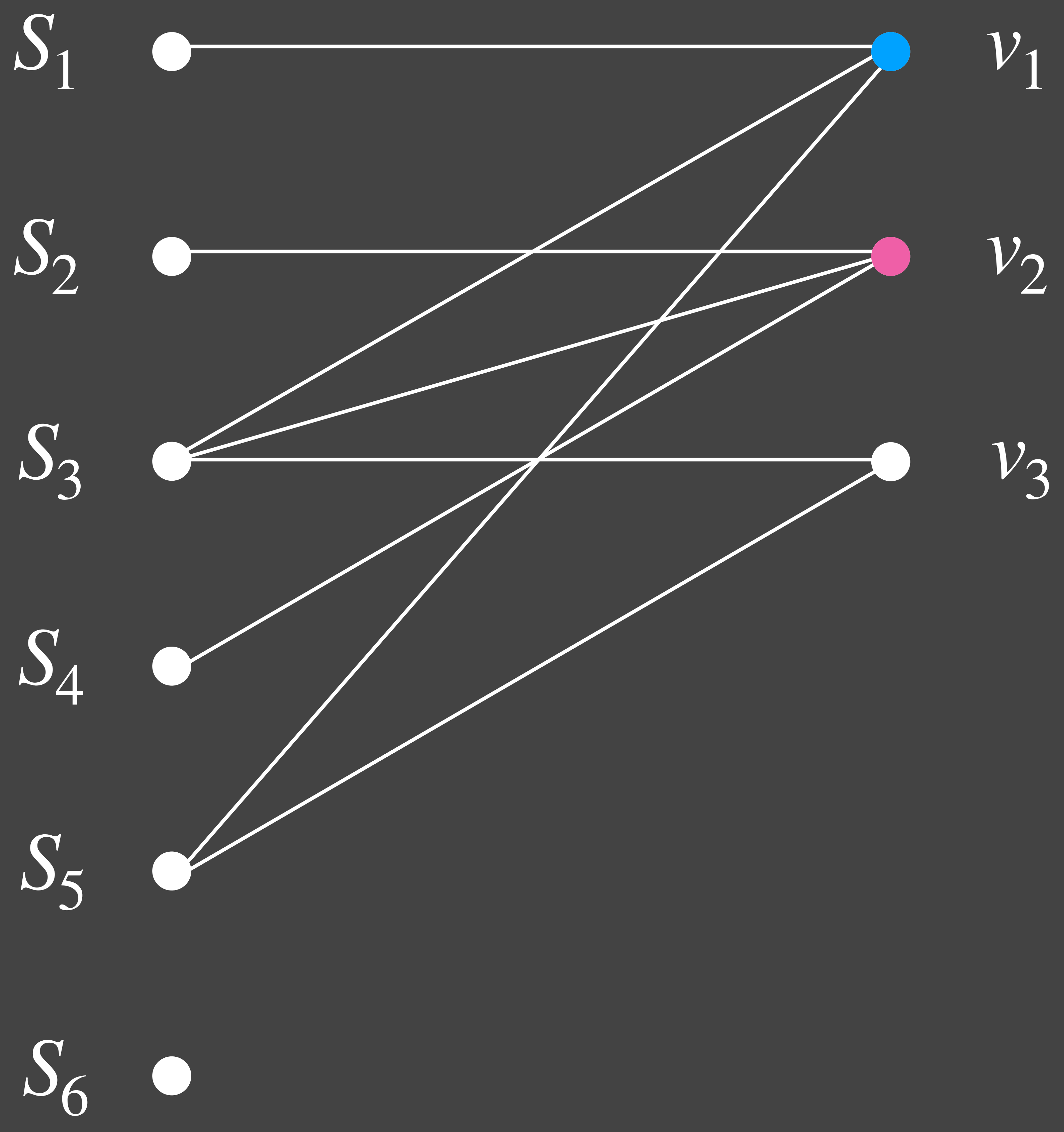

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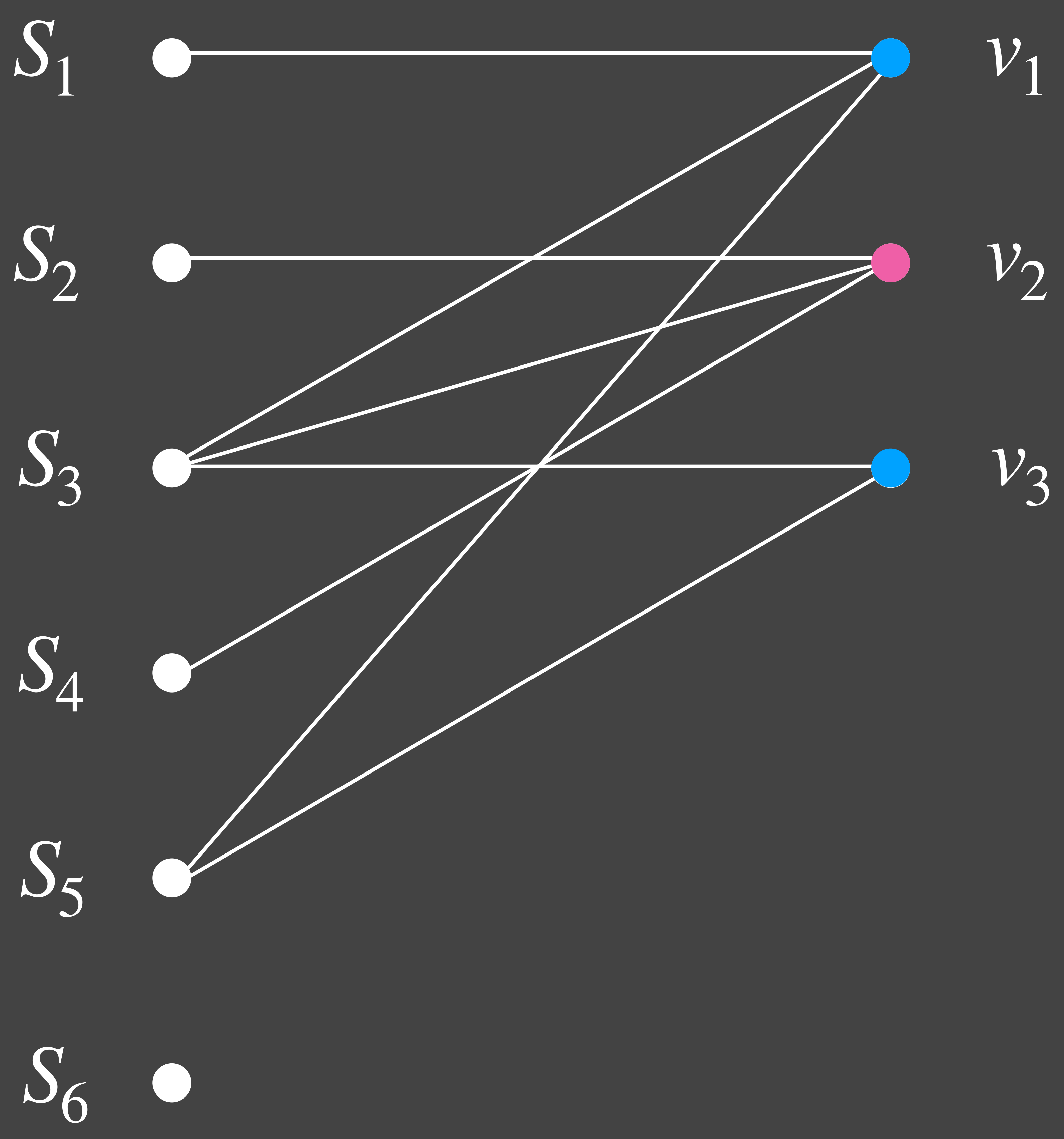

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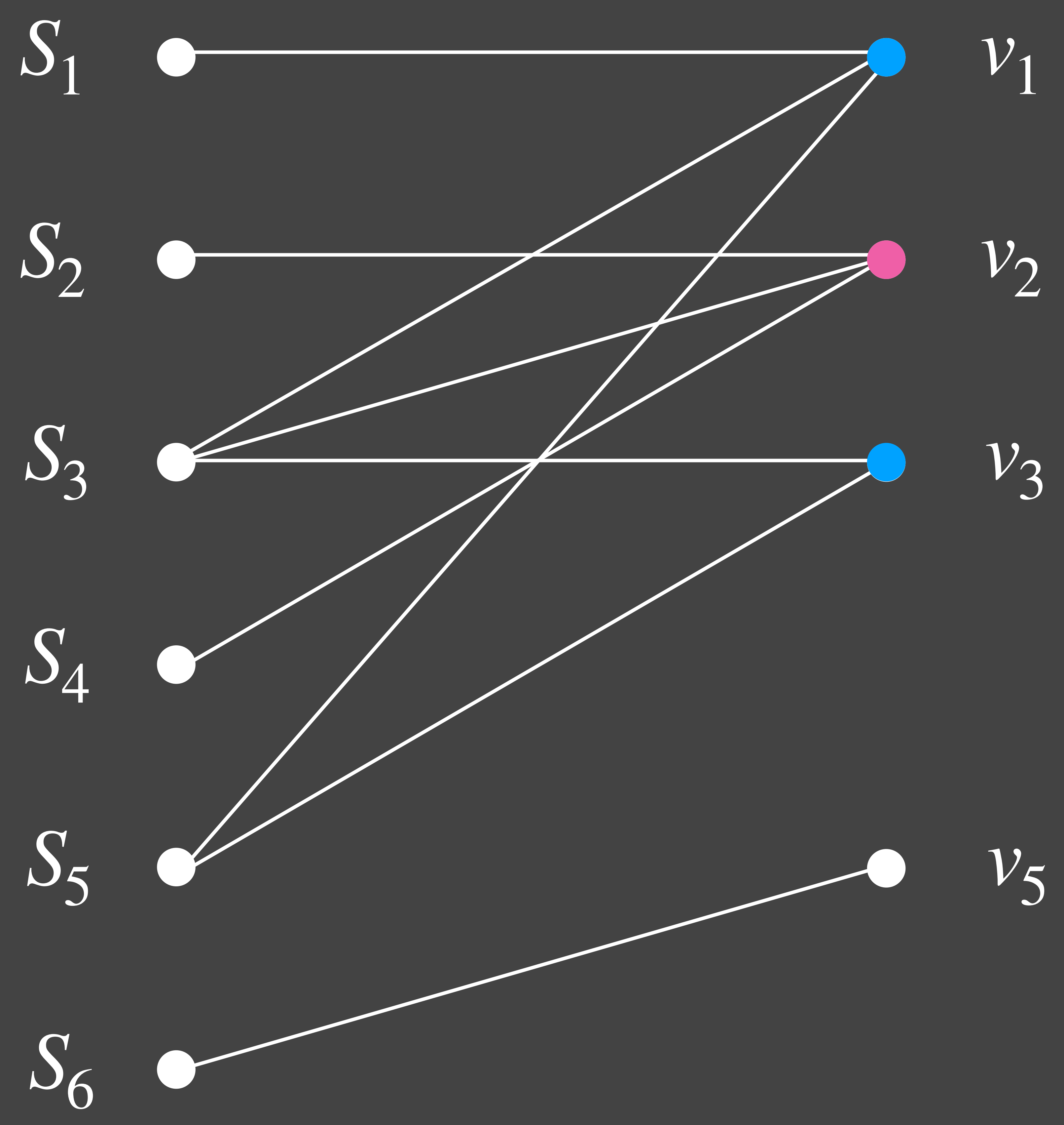

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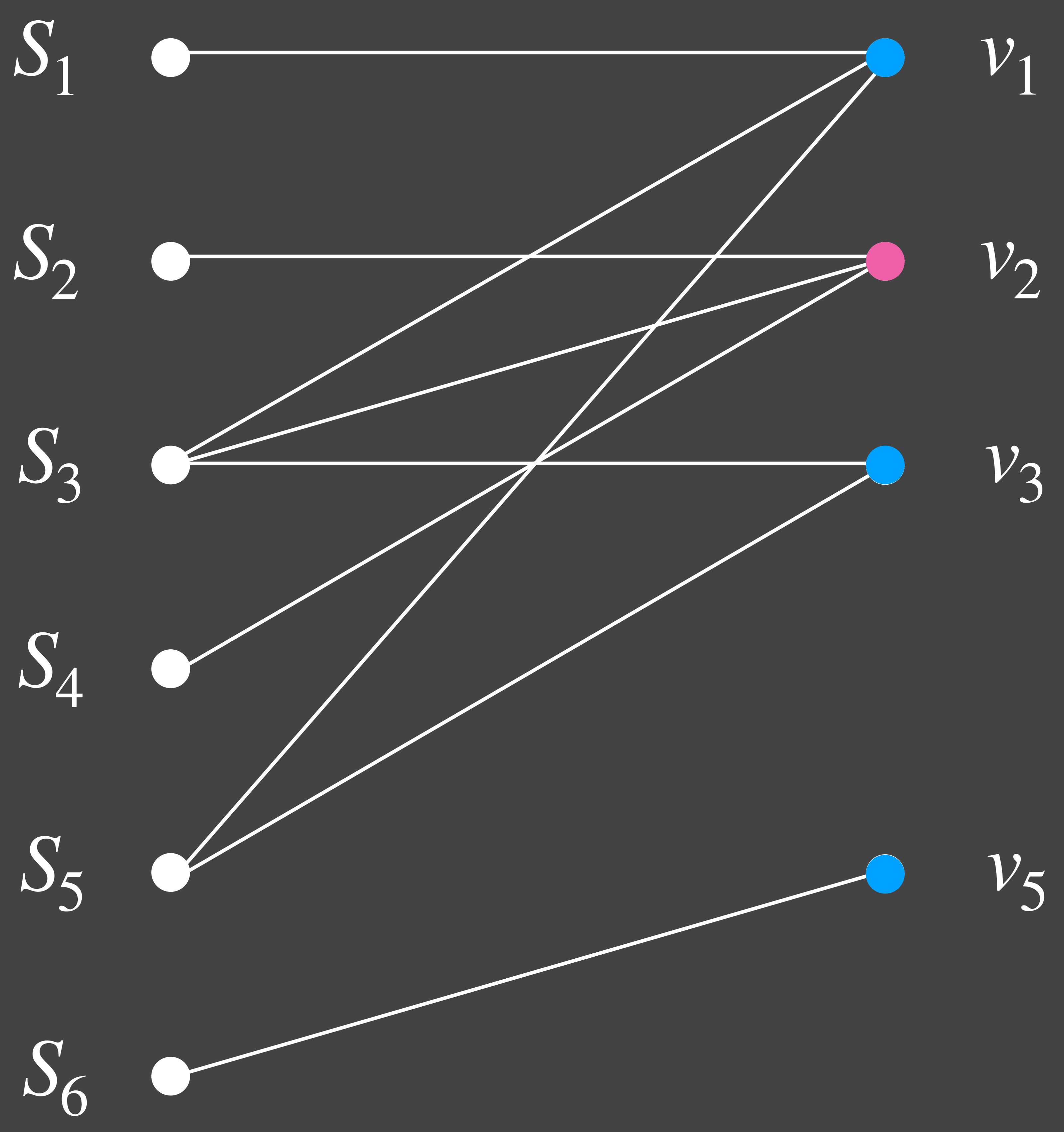

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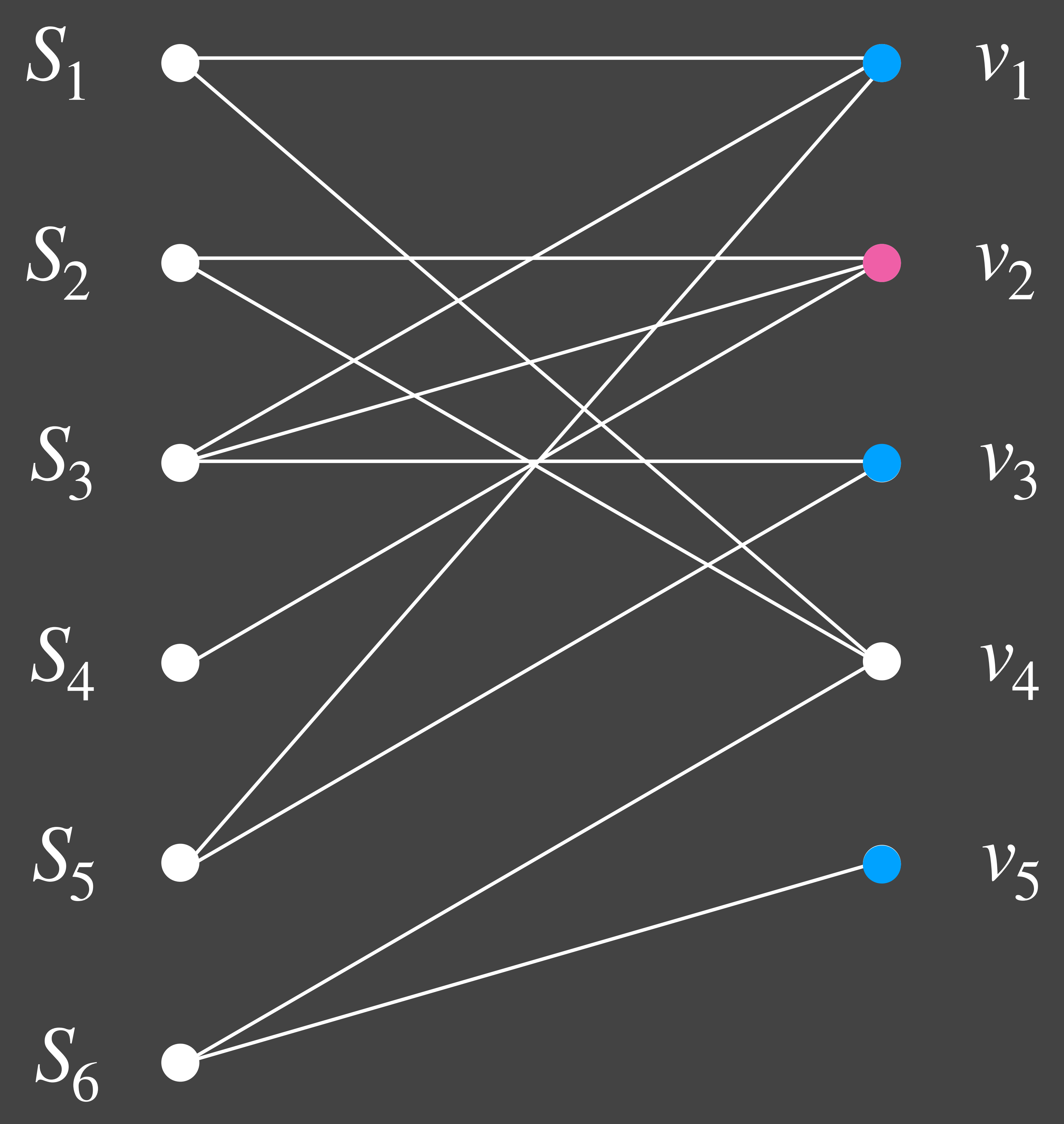

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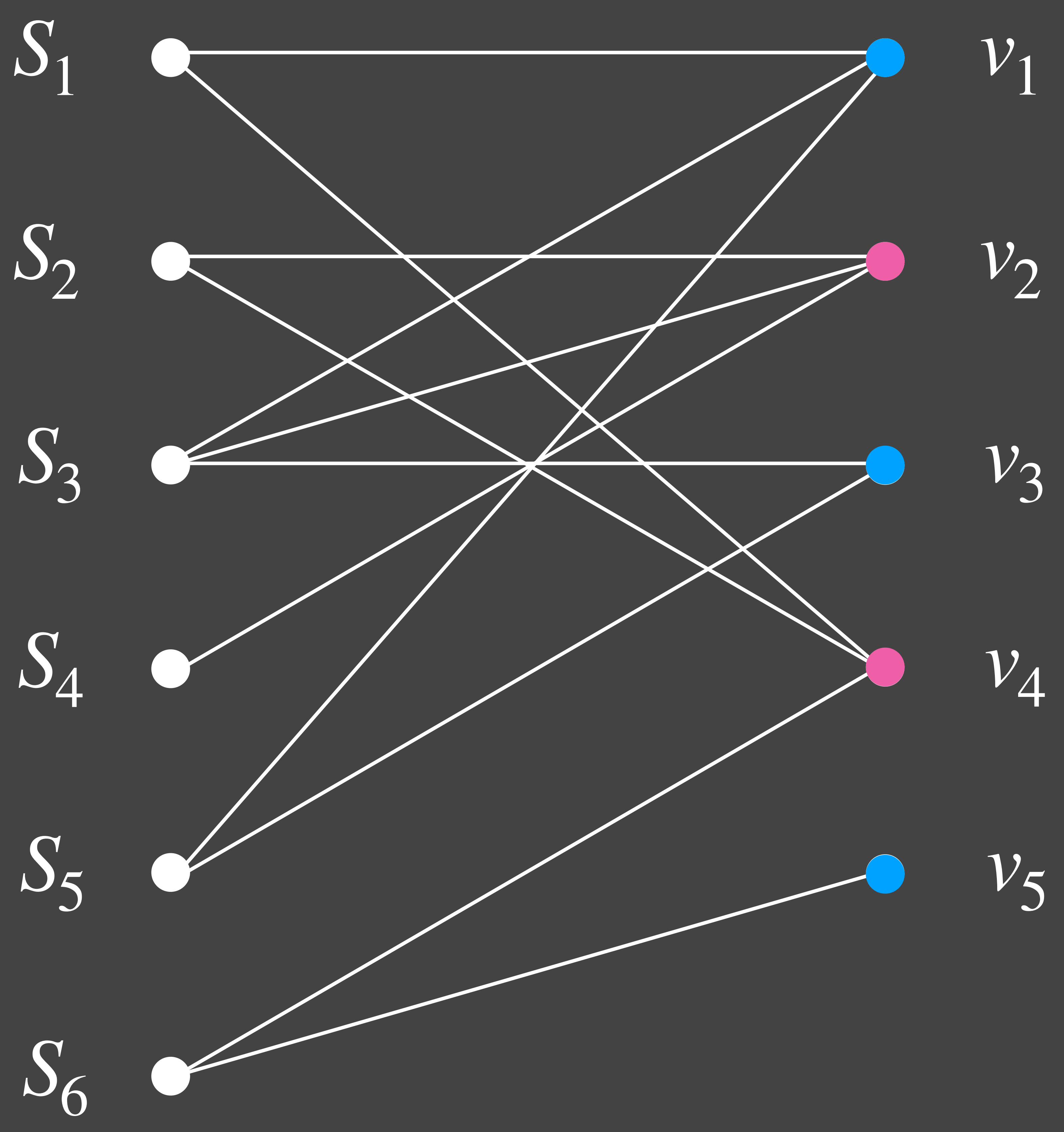

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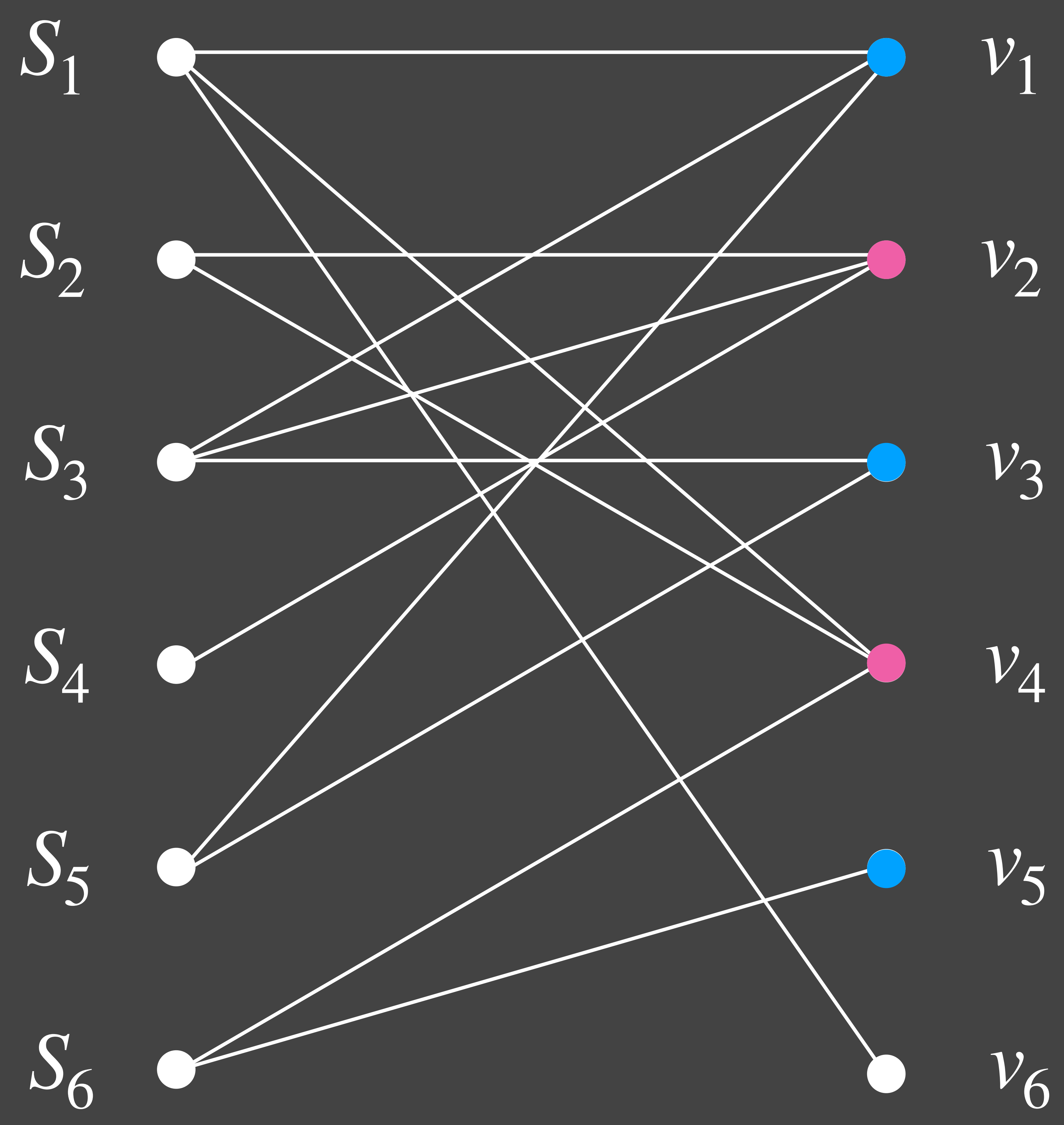

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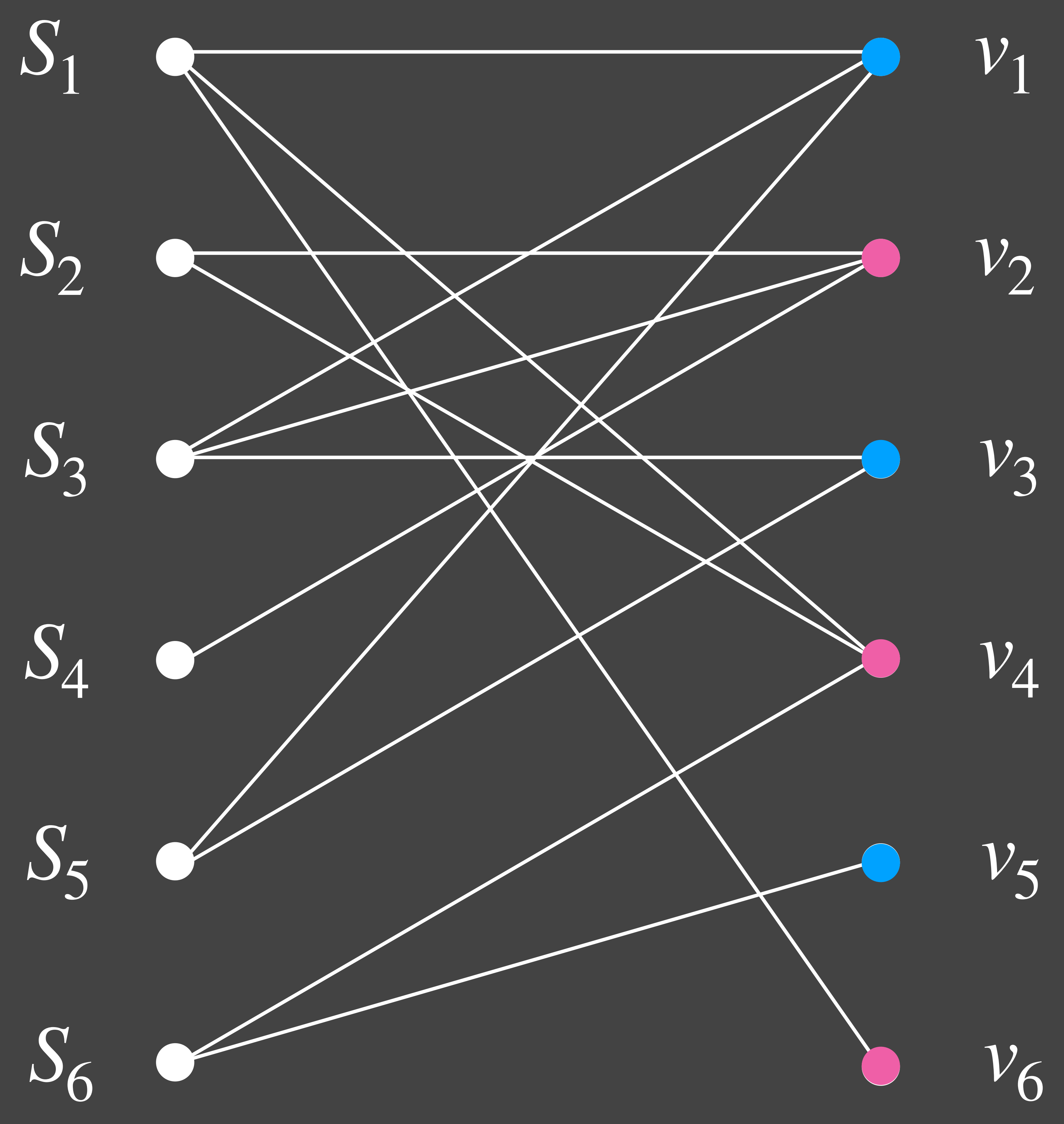

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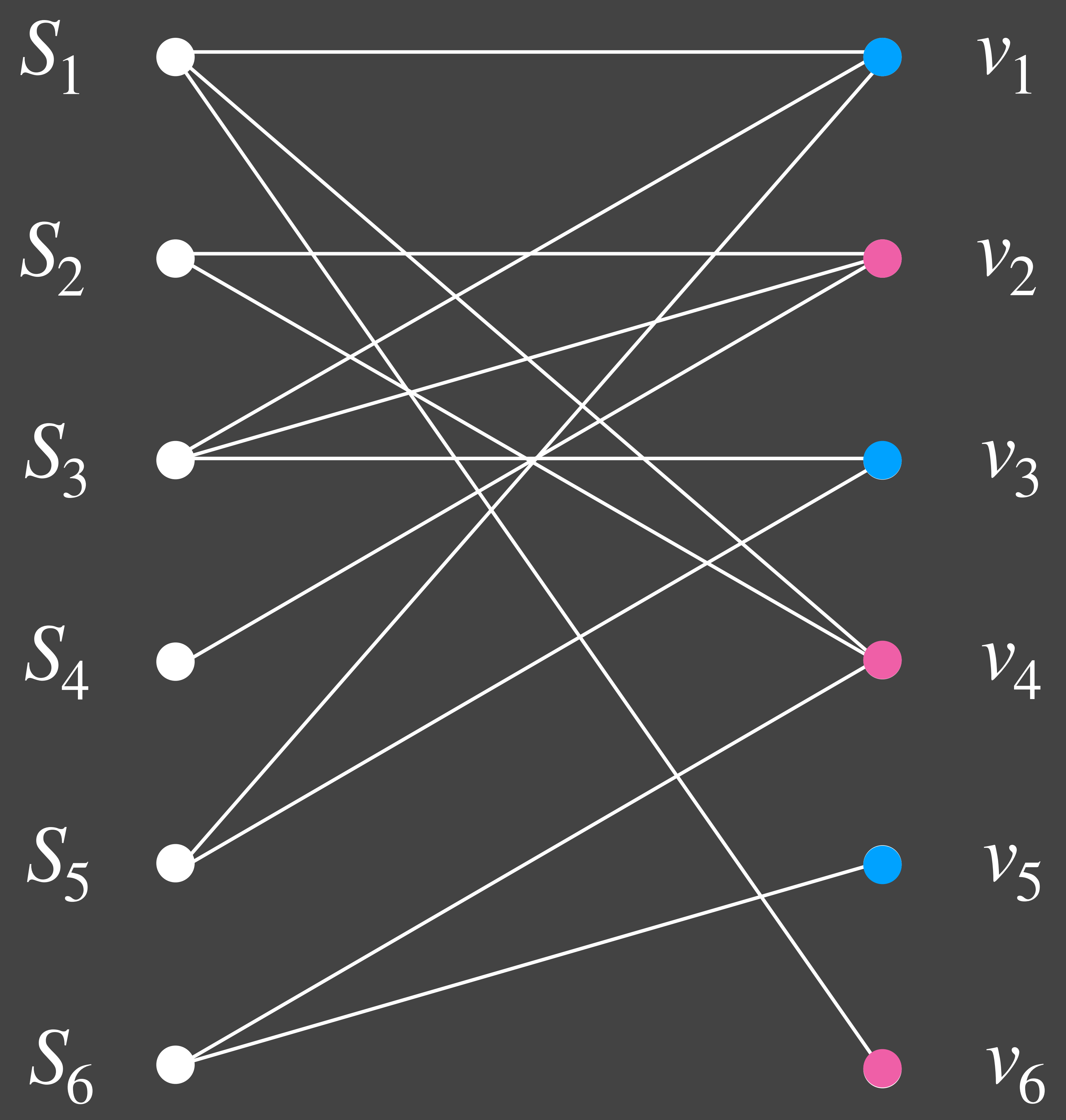

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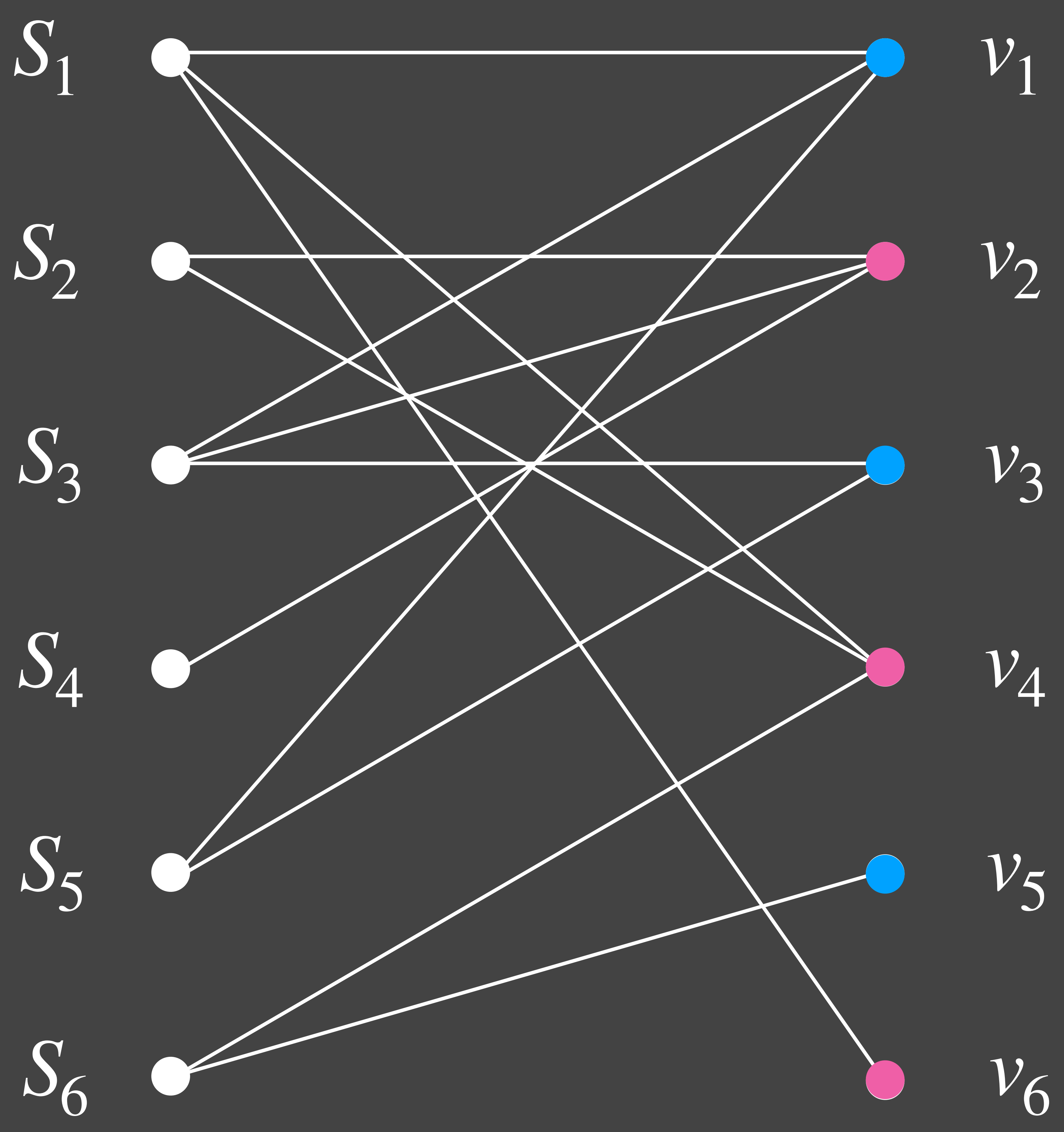

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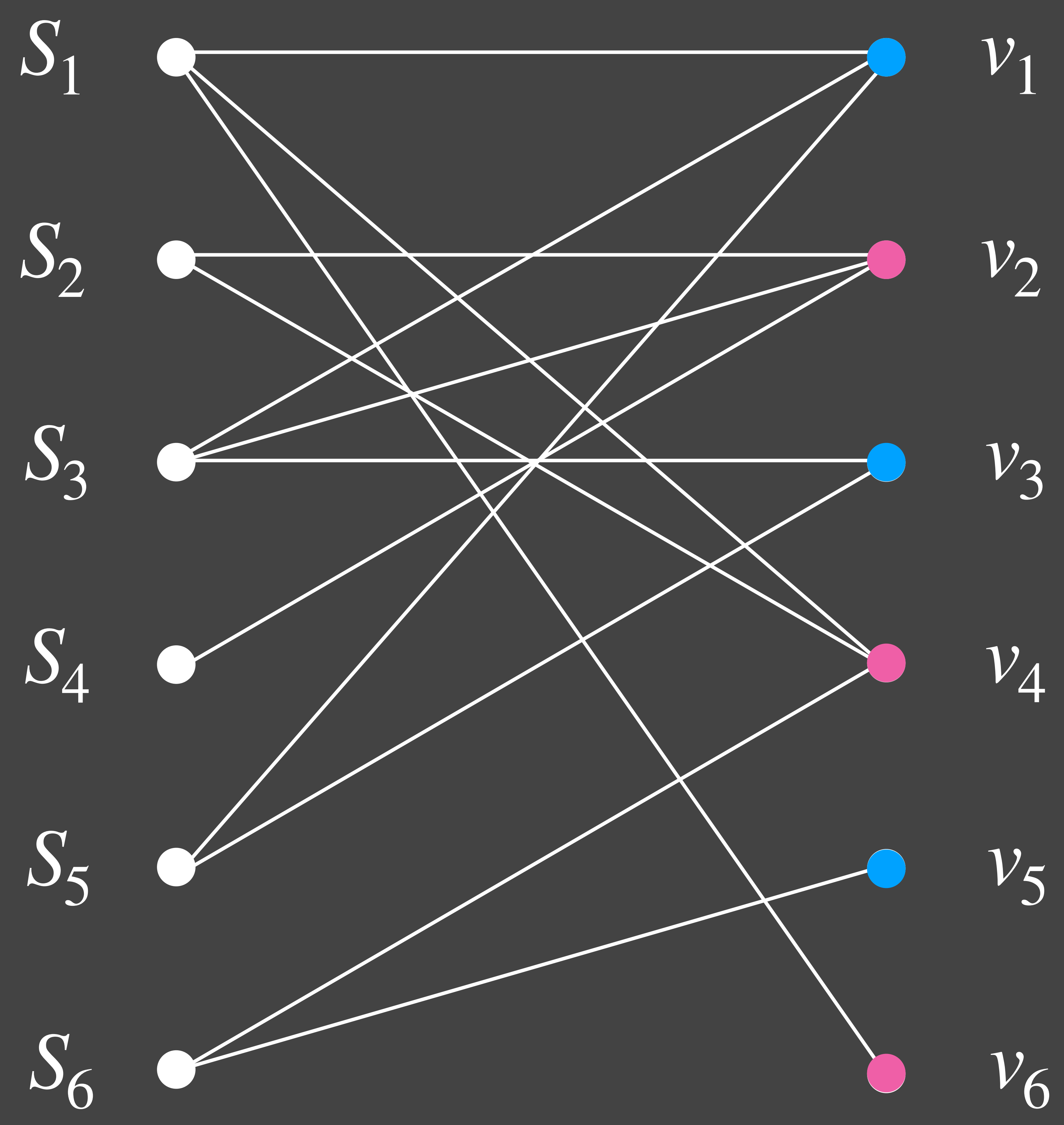

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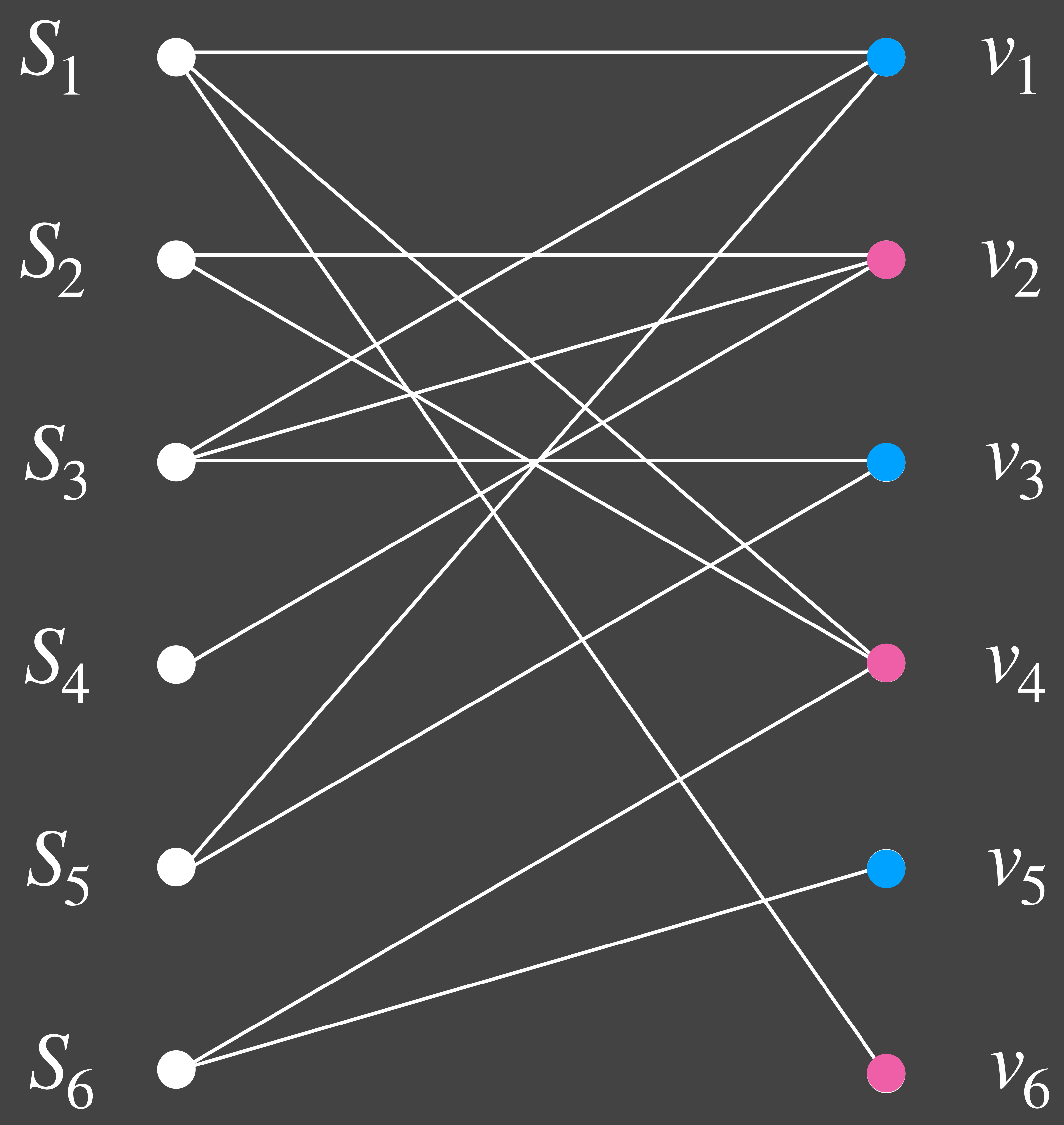

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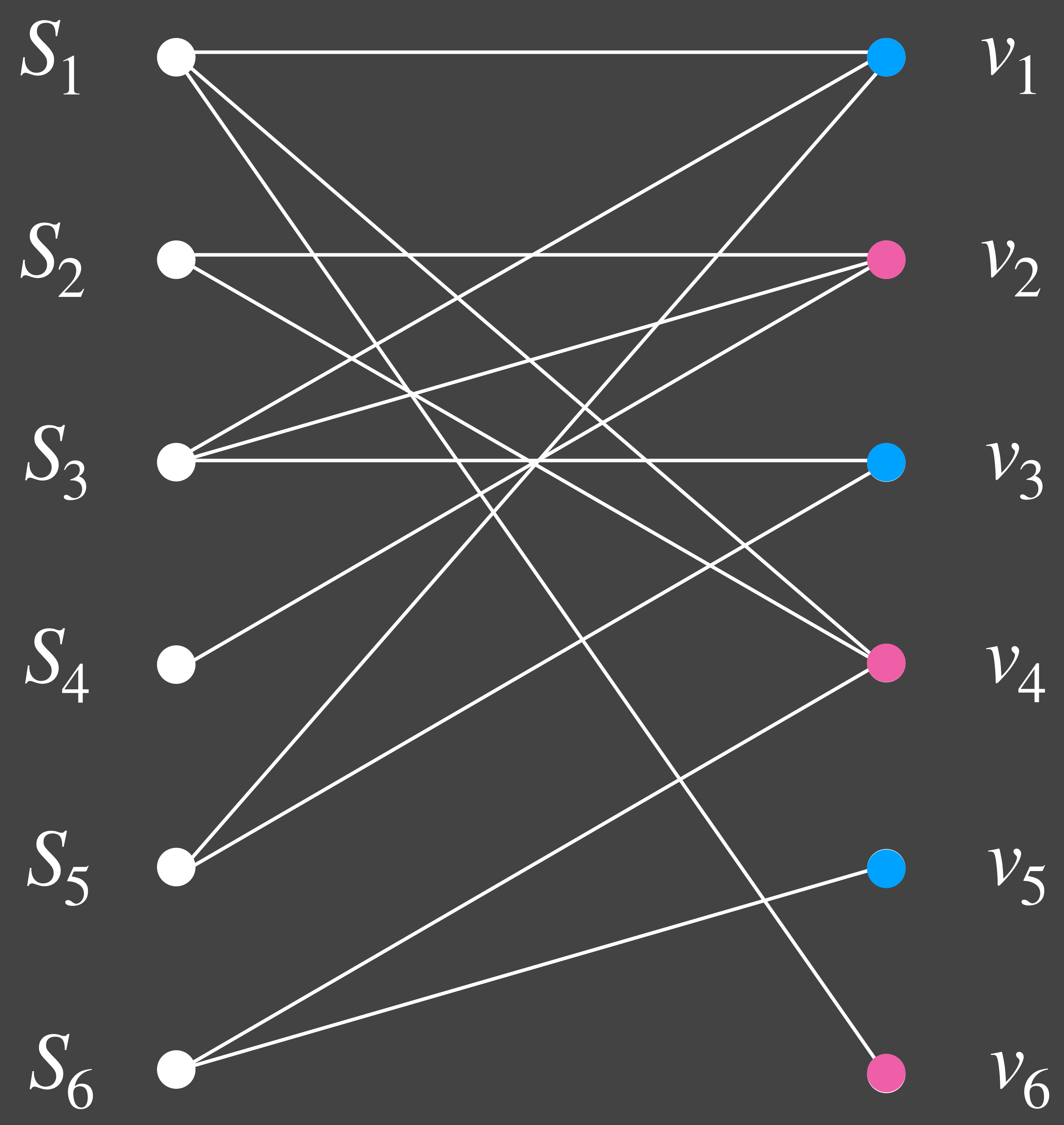

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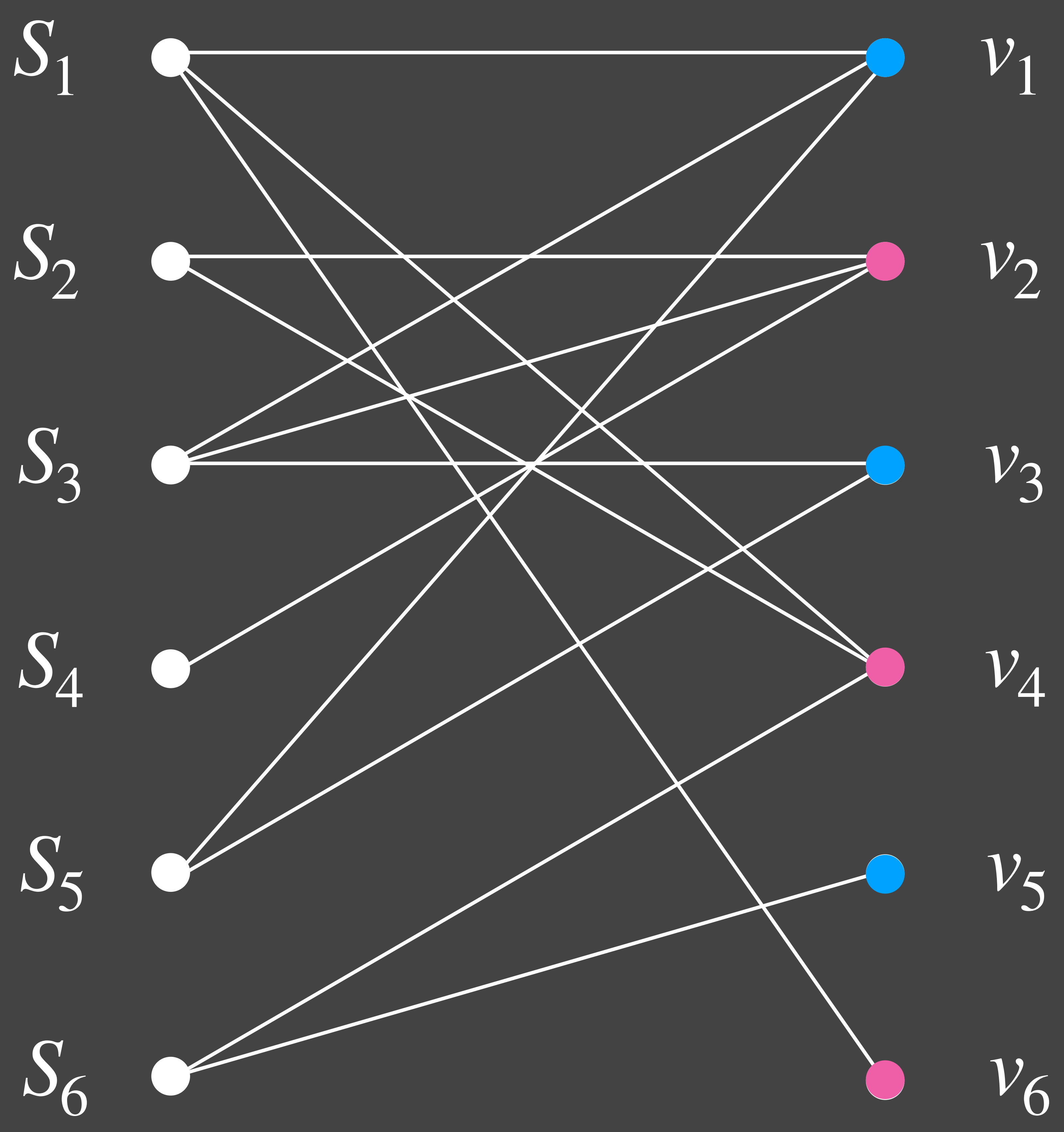

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Claim 1: $\Phi(0) = O(\log mn)$, and $\Phi(t) \geq 0$.

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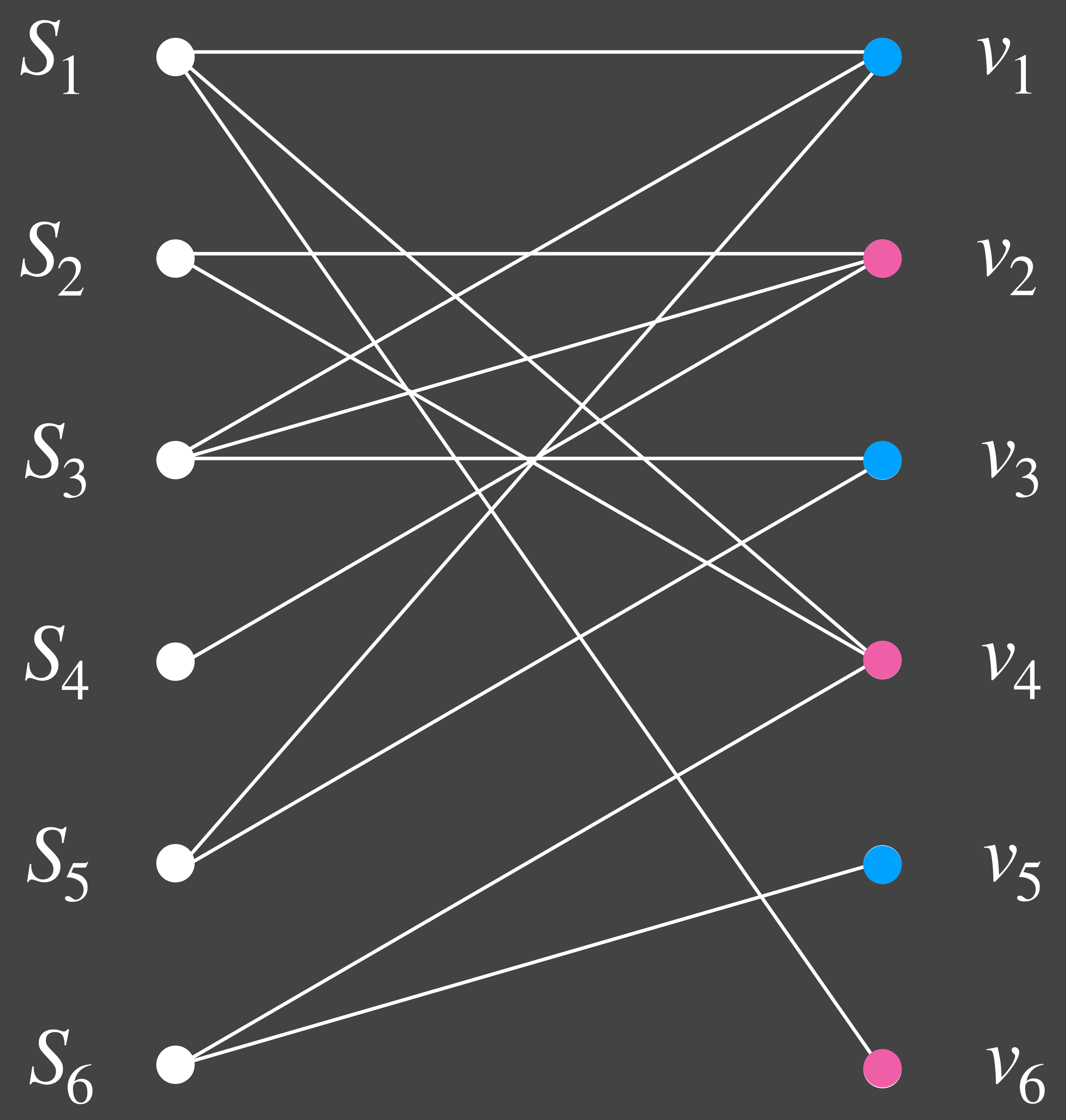

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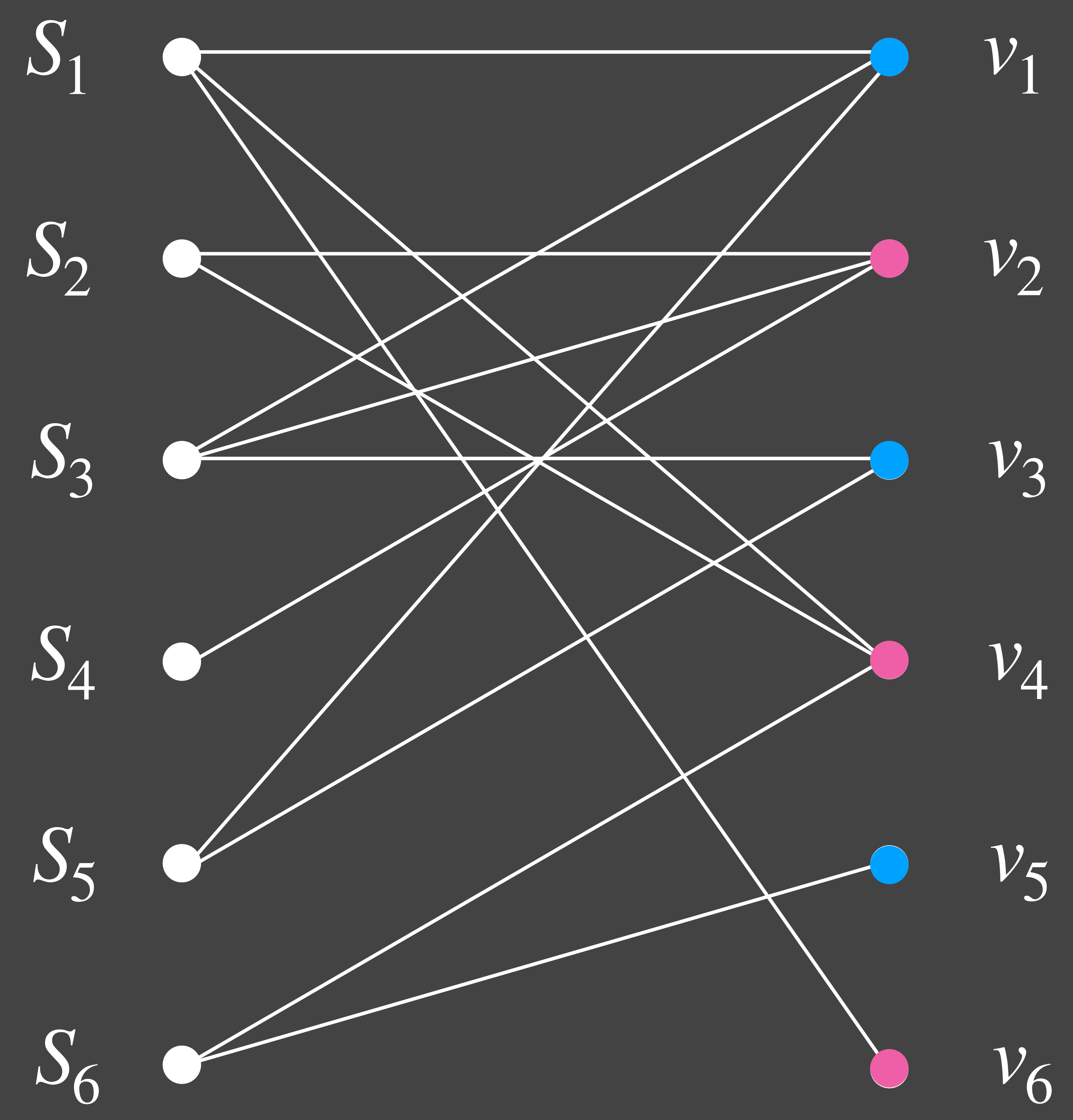

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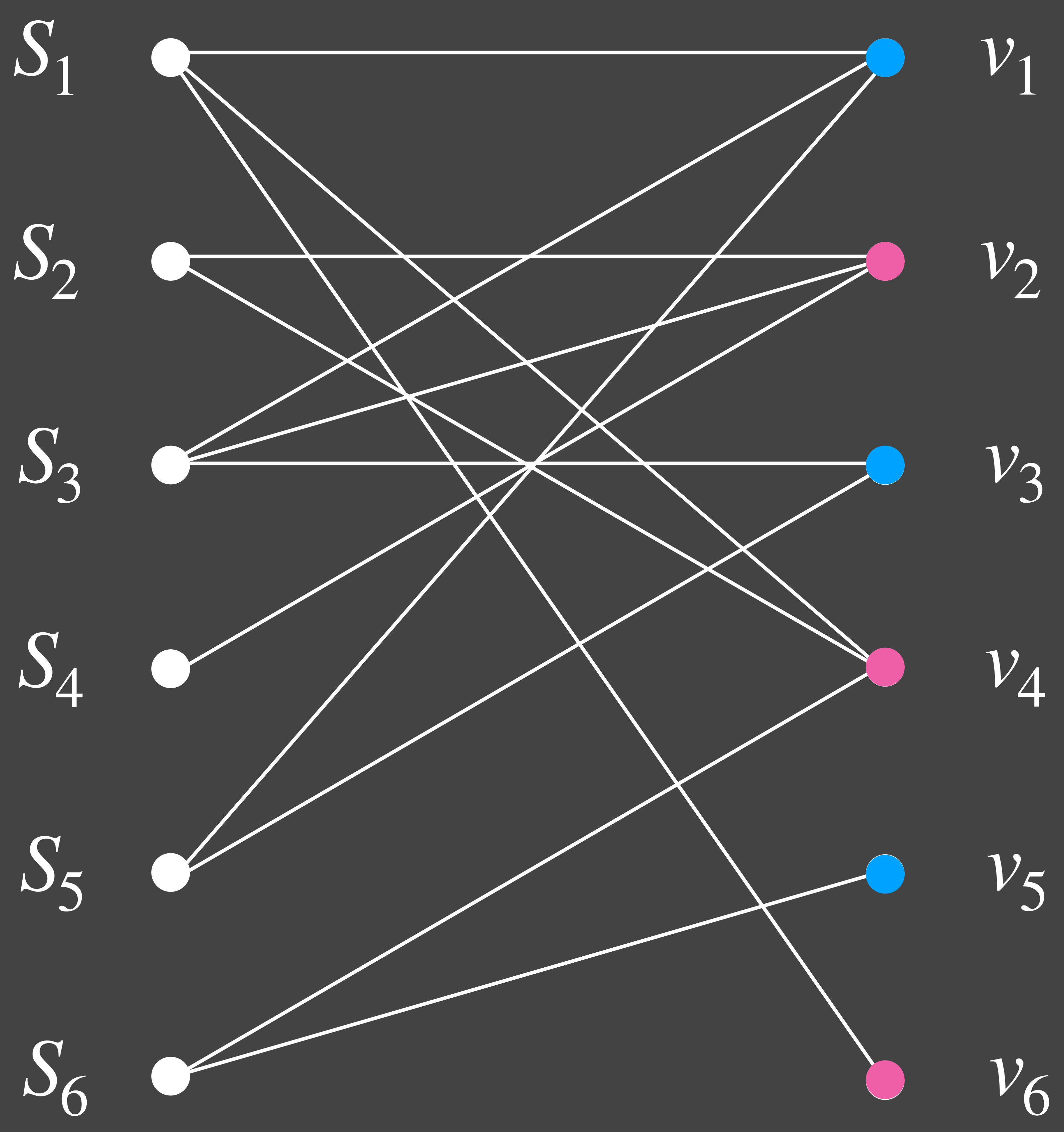

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Talk Outline

Intro

Part I — Online/Dynamic Submodular Cover

Part II — Application: Block-Aware Caching

➡ Part III — Random Order Online Set Cover

Conclusion

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➔ Conclusion

My Amazing Collaborators (so far!)

My Family

Thanks!